# Dynamical modes of two almost identical chemical oscillators connected via both pulsatile and diffusive coupling

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## Electronic Supplementary Information

### Equations

$$dx_i/dt = -k_1x_iy_i + k_2y_i - 2k_3x_i^2 + k_4x_i(c_0 - z_i)/(c_0 - z_i + c_{\min}) - k_0x_i \equiv X(x_i,y_i,z_i)$$

$$dy_i/dt = -3k_1x_iy_i - 2k_2y_i - k_3x_i^2 + k_7u_i + k_9v_iz_i - k_0y_i \equiv Y(x_i,y_i,z_i,u_i,v_i)$$

$$dz_i/dt = 2k_4x_i(c_0 - z_i)/(c_0 - z_i + c_{\min}) - k_9v_iz_i - k_{10}z_i \equiv Z(x_{i,},z_i,v_i)$$

$$du_i/dt = 2k_1x_iy_i + k_2y_i + k_3x_i^2 - k_7u_i - k_0u_i \equiv U(x_i,y_i,u_i)$$

$$dv_i/dt = k_7u_i - k_9v_iz_i - k_{13}v_i - k_0v_i \equiv V(z_{i,},u_i,v_i)$$

(A):

$$dx_1/dt = X(x_1,y_1,z_1) \qquad \text{(A1)}$$

$$dy_1/dt = Y(x_1,y_1,z_1,u_1,v_1) + C_{\text{inh}} \times P(x_2,\Delta t,\tau), \qquad \text{(A2)}$$

$$dz_1/dt = Z(x_{1,},z_1,v_1) \qquad \text{(A3)}$$

$$du_1/dt = U(x_1,y_1,u_1) + k_u(u_2 - u_1), \qquad \text{(A4)}$$

$$dv_1/dt = V(z_{1,},u_1,v_1) \qquad \text{(A5)}$$

$$dx_2/dt = X(x_2,y_2,z_2) \qquad \text{(A6)}$$

$$dy_2/dt = Y(x_2,y_2,z_2,u_2,v_2) + C_{\text{inh}} \times P(x_1,\Delta t,\tau), \qquad \text{(A7)}$$

$$dz_2/dt = Z(x_{2,},z_2,v_2) \qquad \text{(A8)}$$

$$du_2/dt = U(x_2,y_2,u_2) + k_u(u_1 - u_2). \qquad \text{(A9)}$$

$$dv_2/dt = V(z_{2,},u_2,v_2) \qquad \text{(A10)}$$

(B): $\mathrm{d}x_1/\mathrm{d}t = X(x_1,y_1,z_1) + k_\mathrm{x}(x_2 - x_1)$, (B1)

$\mathrm{d}y_1/\mathrm{dt} = Y(x_1,y_1,z_1,u_1,v_1) + C_\mathrm{inh} \times P(x_2,\Delta t,\tau)$, (B2)

$\mathrm{d}z_1/\mathrm{d}t = Z(x_{1,},z_1,v_1)$ (B3)

$\mathrm{d}u_1/\mathrm{d}t = U(x_1,y_1,u_1)$ (B4)

$\mathrm{d}v_1/\mathrm{d}t = V(z_{1,},u_1,v_1)$ (B5)

$\mathrm{d}x_2/\mathrm{d}t = X(x_2,y_2,z_2) + k_\mathrm{x}(x_1 - x_2)$, (B6)

$\mathrm{d}y_2/\mathrm{dt} = Y(x_2,y_2,z_2,u_2,v_2) + C_\mathrm{inh} \times P(x_1,\Delta t,\tau)$, (B7)

$\mathrm{d}z_2/\mathrm{d}t = Z(x_{2,},z_2,v_2)$ (B8)

$\mathrm{d}u_2/\mathrm{d}t = U(x_2,y_2,u_2)$ (B9)

$\mathrm{d}v_2/\mathrm{d}t = V(z_{2,},u_2,v_2)$ (B10)

(C): $\mathrm{d}x_1/\mathrm{d}t = X(x_1,y_1,z_1)$ (C1)

$\mathrm{d}y_1/\mathrm{d}t = Y(x_1,y_1,z_1,u_1,v_1) - k_\mathrm{diff}[\mathrm{Ag}_1]y_1$ (C2)

$\mathrm{d}z_1/\mathrm{d}t = Z(x_{1,},z_1,v_1)$ (C3)

$\mathrm{d}u_1/\mathrm{dt} = U(x_1,y_1,u_1) + k_\mathrm{u}(u_2 - u_1)$, (C4)

$\mathrm{d}v_1/\mathrm{d}t = V(z_{1,},u_1,v_1)$ (C5)

$\mathrm{d}[\mathrm{Ag}_1]/\mathrm{d}t = C_\mathrm{ex} \times P(x_2,\Delta t,\tau) - k_\mathrm{diff}[\mathrm{Ag}_1]y_1$ (C6)

$\mathrm{d}x_2/\mathrm{d}t = X(x_2,y_2,z_2)$ (C7)

$\mathrm{d}y_2/\mathrm{d}t = Y(x_2,y_2,z_2,u_2,v_2) - k_\mathrm{diff}[\mathrm{Ag}_2]y_2$ (C8)

$\mathrm{d}z_2/\mathrm{d}t = Z(x_{2,},z_2,v_2)$ (C9)

$\mathrm{d}u_2/\mathrm{dt} = U(x_2,y_2,u_2) + k_\mathrm{u}(u_1 - u_2)$. (C10)

$\mathrm{d}v_2/\mathrm{dt} = V(z_{2,},u_2,v_2)$ (C11)

$\mathrm{d}[\mathrm{Ag}_2]/\mathrm{d}t = C_\mathrm{ex} \times P(x_1,\Delta t,\tau) - k_\mathrm{diff}[\mathrm{Ag}_2]y_2$ (C12)

(D):

$$dx_1/dt = X(x_1,y_1,z_1) + k_x(x_2 - x_1), \tag{D1}$$

$$dy_1/dt = Y(x_1,y_1,z_1,u_1,v_1) - k_{diff}[Ag_1]y_1 \tag{D2}$$

$$dz_1/dt = Z(x_{1,},z_1,v_1) \tag{D3}$$

$$du_1/dt = U(x_1,y_1,u_1) \tag{D4}$$

$$dv_1/dt = V(z_{1,},u_1,v_1) \tag{D5}$$

$$d[Ag_1]/dt = C_{ex} \times P(x_2,\Delta t,\tau) - k_{diff}[Ag_1]y_1 \tag{D6}$$

$$dx_2/dt = X(x_2,y_2,z_2) + k_x(x_1 - x_2), \tag{D7}$$

$$dy_2/dt = Y(x_2,y_2,z_2,u_2,v_2) - k_{diff}[Ag_2]y_2 \tag{D8}$$

$$dz_2/dt = Z(x_{2,},z_2,v_2) \tag{D9}$$

$$du_2/dt = U(x_2,y_2,u_2) \tag{D10}$$

$$dv_2/dt = V(z_{2,},u_2,v_2) \tag{D11}$$

$$d[Ag_2]/dt = C_{ex} \times P(x_1,\Delta t,\tau) - k_{diff}[Ag_2]y_2 \tag{D12}$$

Pure diffusive coupling (PDC) (equations used for simulating results shown in Fig. 1)

(PDC):

$$dx_1/dt = X(x_1,y_1,z_1) + k_x(x_2 - x_1) \tag{PDC 1}$$

$$dy_1/dt = Y(x_1,y_1,z_1,u_1,v_1), \tag{PDC 2}$$

$$dz_1/dt = Z(x_{1,},z_1,v_1) \tag{PDC 3}$$

$$du_1/dt = U(x_1,y_1,u_1) + k_u(u_2 - u_1), \tag{PDC 4}$$

$$dv_1/dt = V(z_{1,},u_1,v_1) \tag{PDC 5}$$

$$dx_2/dt = X(x_2,y_2,z_2) + k_x(x_1 - x_2) \tag{PDC 6}$$

$$dy_2/dt = Y(x_2,y_2,z_2,u_2,v_2), \tag{PDC 7}$$

$$dz_2/dt = Z(x_{2,},z_2,v_2) \tag{PDC 8}$$

$$du_2/dt = U(x_2,y_2,u_2) + k_u(u_1 - u_2). \tag{PDC 9}$$

$$dv_2/dt = V(z_{2,},u_{2,}v_2) \tag{PDC 10}$$

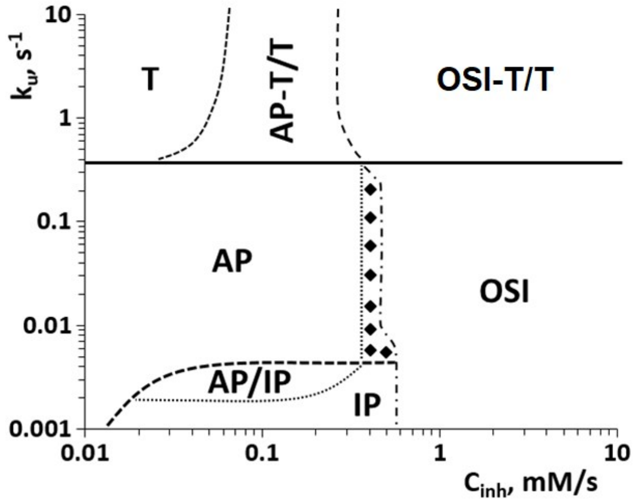


**Figure S1.** The diagram for dynamical modes at mutual inhibitory diffusive and inhibitory pulsatile (IDIP) coupling of two BZ oscillators in the plane $C_{inh}$-$k_u$ at $\tau = 200$ s and $h_2 = 0.051$ M. Black rhombs mark the C mode. Solid horizontal line marks the border of the area of the T and T-induced modes like AP-T and OS-T.

Comments to Fig. S1.

(*i*) There is a region of birhythmicity between the AP and IP modes.

(*ii*) Complex (C) mode emerges in a narrow area (marked by black rhombs) at middle values of $k_u$. No C mode is found at small (or zero) $k_u$.

(*iii*) New AP-T and OSI-T modes emerge at $k_u > 0.4$ $s^{-1}$, when Turing (T) mode becomes stable at small $C_{inh}$.

(*iv*) New AP-T and OSI-T modes coexist with the T mode.

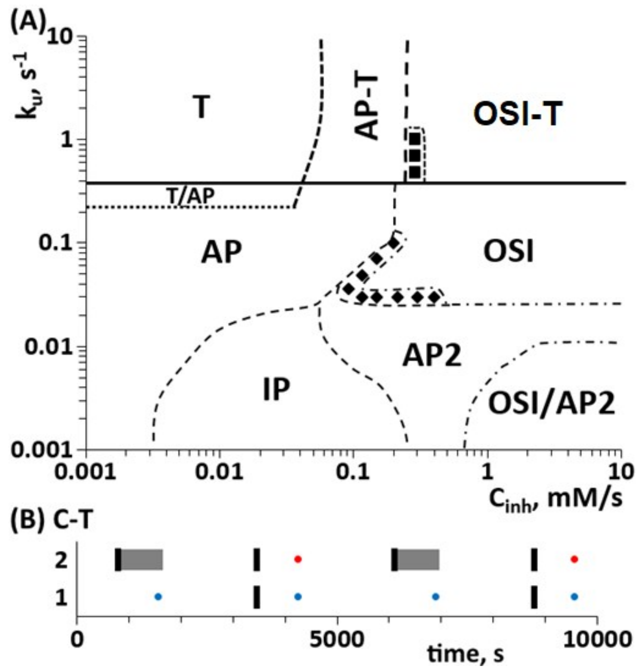


**Figure S2.** (A)The diagram for dynamical modes at mutual inhibitory diffusive and inhibitory pulsatile (IDIP) coupling of two BZ oscillators in the plane $C_{inh}$-$k_u$ at $\tau$ = 800 s and $h_2$ = 0.051 M. Symbols: black rhombs, the C mode; black squares, the C-T mode. Solid horizontal line marks the bottom border of the area of the T and T-induced modes like AP-T and OSI-T. (B) Example of the C-T mode at $C_{inh}$ = 0.3 mM/s and $k_u$ = 1 $s^{-1}$; $T$ = 5331 s.

Comments to Fig. S2.

(*i*) Areas of new AP2, C-T, AP-T, OSI-T, and C modes are seen in Fig. S2(A).

(*ii*) The C-T mode shown in Fig. S2(B) has the characteristic features of both the C and AP-T (or OSI-T) modes. The area of stabile C-T mode is quite narrow. Most likely this area can be made broader by increasing the frequency dispersion.

(*iii*) The OSI aria (which is a continuous area in Fig. S1) is split by the insert aria of the AP2 mode.

(*iv*) New AP-T and OSI-T modes do not coexist with the T mode (like in Fig. S1), but there are two areas of birhythmicity: T/AP and OSI/AP2.

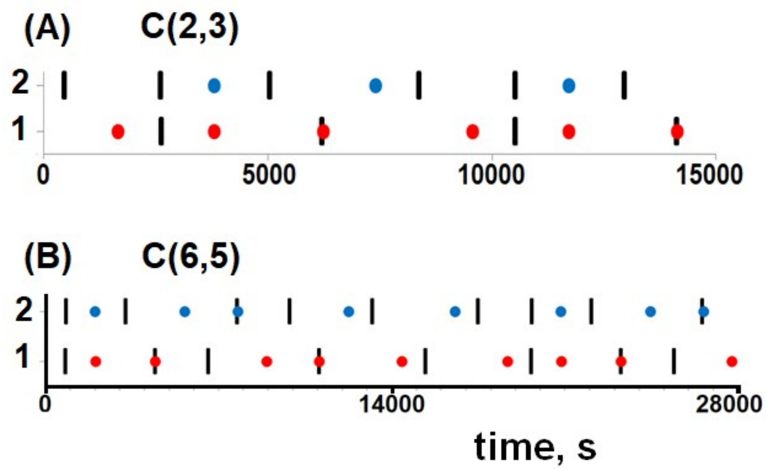


**Figure S3.** Complex modes for mutual inhibitory diffuse and inhibitory pulsatile (IDIP) coupling in the IP region at $\tau = 1200$ s, $C_{inh} = 0.02$ mM/s, and $h_2 = 0.051$ M. (A) The C(2,3) mode at $k_u = 0.08$ s$^{-1}$; $T = 7910$ s. (B) The C(6,5) mode at $k_u = 0.09$ s$^{-1}$; $T = 18825$ s.