Supplementary Information for

Magnetic skyrmion without skyrmion Hall effect

in magnetic nanotrack with perpendicular anisotropy

S1. The simulation method

Our simulation was performed using micromagnetic simulation software called "Object-Oriented Micromagnetic Framework" (OOMMF), which contains the code for the interfacial Dzyaloshinskii-Moriya interaction (DMI).^{S1} The system was composed of a rectangle-shaped PM Co/Pt bilayer and a boundary layer with in-plane magnetic anisotropy (IMA) with widths (w_b) between 0 and 18 nm. The dimensions of the PM region were 400 nm \times 40 nm \times 0.4 nm, and the cell size was 1 nm \times 0.2 nm \times 0.4 nm. In the PM region, the parameters for Co were as follows. The saturation magnetization $(M_{\rm S})$, the exchange stiffness constant (A), the continuous DMI constant (D), the perpendicular magnetocrystalline anisotropy constant (K_u), the spin Hall angle (P), and the damping coefficient (α) were 5.8×10⁵ A/m, 1.5×10⁻¹¹ J/m, 3 mJ/m², 8×10⁵ J/m³, 0.08, and 0.3, respectively.^{S2, S3} The boundary layer shared the same length, thickness, and other parameters, including $M_{\rm S}$, A, P, and α , with the PM region. We assumed that the value of $K_{\rm u}$ for the boundary layer was two orders of magnitude smaller than that in the PM region. This boundary anisotropy energy was much smaller than the demagnetization energy, ensuring effective in-plane boundary magnetic anisotropy. In experiments, the variation in K_{μ} might have been accompanied by a change in the DMI constant at the boundary (D_b) since both parameters are related to interface atomic coupling. However, it is not easy to estimate it. In our simulation, two marginal values were considered for $D_{\rm b}$, which is zero or same as that in the PM track (3 mJ/m^2).

The micromagnetic simulation by OOMMF was carried out based on numerically solving the Gilbert equation:

$$\frac{\partial \dot{m}}{\partial t} = -\gamma_0 m \times H_{\text{eff}} + \alpha (m \times \frac{\partial \dot{m}}{\partial t}) + \gamma_0 H_{\text{SO}} (m \times (\sigma \times m)), \qquad (S1)$$

where \dot{m} , t, γ_0 , and $\dot{\sigma}$ are the unit vector for the orientation of magnetic moments, time, gyromagnetic ratio, and the unit vector for the direction of spin, respectively. H_{SO} is the effective magnetic field for spin-orbit torque (SOT) and it can be expressed as:

$$H_{\rm SO} = \mu_{\rm B} P J / \gamma_0 e M_{\rm S} L_{\rm z} \,, \tag{S2}$$

where $\mu_{\rm B}$, e, and $L_{\rm z}$ are the Bohr magneton, the charge of an electron, and the thickness of the magnetic film, respectively. $H_{\rm eff}$ is the effective magnetic field derived from the free energy density (*E*):

$$\overset{\mathbf{r}}{H}_{\rm eff} = -\frac{1}{\mu_0 M_{\rm S}} \left(\frac{\delta E}{\delta m}\right),\tag{S3}$$

where μ_0 is the vacuum permeability. \dot{H}_{eff} includes the effective field from exchange coupling,

magnetic anisotropy, demagnetization, and DMI, etc.

In a magnetic system with a fixed volume and magnetic parameters, the magnetic moments have a stable distribution with a minimum free energy. This moment distribution can be determined using the calculus of variations.^{S1} Through the known free energy density and moment distribution, \dot{H}_{eff} can be calculated using Eq. (S3). Finally, the dynamical process of magnetic moments driven by a current with SOT can be revealed through resolving Eq. (S1). Therefore, the variation of w_b and magnetic parameters like D_b and K_u modifies the free energy and moment distribution. As a result, the \dot{H}_{eff} and the dynamical behavior of magnetic moments under SOT

are also influenced. The result of the calculation of $\dot{H}_{\rm eff}$ is shown in S7.

S2. The initial state for the magnetization distribution of moments at the edge layer

As shown in Fig. S1a, there are four types of orientations for magnetic moments at the edge layer with IMA before the formation of a skyrmion by current. The boundary moments spontaneously tilt along the *y*-axis with no dependence on their initial orientations. After injecting a current of $J=8\times10^{14}$ A/m² for 0.3 ns, Néel-typed skyrmions are generated in the track when the w_b is 10 nm, 20 nm, and 40nm, respectively, for both D_b (Fig. S1b-d). In the simulation, when w_b is smaller than about 20 nm for $D_b=0$ mJ/m² and 28 nm for $D_b=3$ mJ/m², a skyrmion and an edge layer with moments tilting along the -y direction can be created synchronously. However, when w_b is large, a skyrmion has to be created by two steps. At first, the moment distribution in the edge-modified nanotrack needs to reach its stable state spontaneously, and a skyrmion can be formed subsequently by injecting a current. Additionally, the edge modification does not change the size of skyrmion. The skyrmions created under all the conditions share the same diameter of about 15 nm.

When the moments in the PM track are along +z-direction, the boundary moments are toward the inner part of track. When the PM moments are along -z-direction, the moments at the edge layer are in the +y-direction, pointing towards the outside of the track. In both cases, the moment orientation of the skyrmion at the perimeter that is closer to the edge is opposite that at the edge.



Fig. S1 Four types of magnetization states (a) before current injection. The stable magnetization state after injecting current for 0.3 ns in the track with (b) 10-nm w_b , (c) 20-nm w_b , and (d) 40-nm w_b .

S3. Calculation of the Skyrmion number Q and the dissipative tensor D_{ij}

The skyrmion number, Q, is an integer that represents the number of times the spin direction wraps the unit sphere and is defined by the integral equation, Eq. (2), in the paper. This integral can be discretized as

$$Q = \frac{1}{4\pi} \sum_{i=1}^{N} \sum_{j=1}^{M} \overset{\mathbf{r}}{\mathbf{m}}_{i,j} \cdot \left(\frac{\overset{\mathbf{h}}{\mathbf{m}}_{i+1,j} - \overset{\mathbf{h}}{\mathbf{m}}_{i,j}}{\Delta x} \times \frac{\overset{\mathbf{h}}{\mathbf{m}}_{i,j+1} - \overset{\mathbf{h}}{\mathbf{m}}_{i,j}}{\Delta y}\right) \Delta x \Delta y$$
(S4)

where $m_{i,j} = (m_x, m_y, m_z)_{i,j}$ is a unit vector representing the direction of the moment of the cell,

which can be obtained from the ODT files generated by OOMMF and extracted by MATLAB. Using MATLAB, we calculated the sum formula (S4) throughout the square-shaped region that contained the entire skyrmion but excluded the edge layer (Fig. S2). $N \times M$ is the number of moments in this region, and the difference between N and M is due to the size difference for the cell along the *x*- and *y*-directions in the simulation.



Fig. S2 The region for the numerical calculation of the skyrmion number and the dissipative tensor.

The dissipative tensor $D_{ij}=D_{xx}=D_{yy}$ was determined using a method similar to the sum formula (S5).

$$D_{yy} = \sum_{i=1}^{N} \sum_{j=1}^{M} \left(\frac{\mathbf{m}_{i,j+1} - \mathbf{m}_{i,j}}{\Delta y} \right) \cdot \left(\frac{\mathbf{m}_{i,j+1} - \mathbf{m}_{i,j}}{\Delta y} \right) \Delta x \Delta y$$
(S5)

As shown in Fig. S3a, D_{yy} increase with w_b when D_b is zero but changes non-monotonically with w_b when D_b is 3 mJ/m². However, the change in D_{yy} is still very small. As shown in Fig. S3b, the Q of the skyrmion in the track without an IM edge layer is between -0.98 and -0.99 which is very close to the standard -1. The addition of an edge layer with w_b that varies from 0 nm to 18 nm has little impact on Q. Thus, the addition of an edge layer with IMA can alter the local magnetization distribution of a skyrmion to a small extent but does not affect the topological properties of the entire skyrmion.



Fig. S3 Boundary width w_b dependence of (a) the dissipative tensor D_{yy} , and (b) the skyrmion number Q for $D_b=0$ and 3 mJ/m².

S4. The evolution of the displacement and velocity of a skyrmion in the PM track with an edge layer with IMA with different boundary widths under a current of 2.5×10^{11} A/m²

As shown in Fig. S4a and c, the evolution of the transverse displacement of a skyrmion becomes slower with time, which shows that the motion of the skyrmion approaches stability. This stable motion can also be observed in the approximately linear Δx -t relationship shown in Fig. S4b and d when the duration is sufficiently long.



Fig. S4 Boundary width dependence of the evolution of the transverse displacement for (a) $D_b=0$ mJ/m² and (c) $D_b=3$ mJ/m² and that of the longitude displacement for (b) $D_b=0$ mJ/m² and (d) $D_b=3$ mJ/m².

When the D_b is zero, the addition of an edge layer leads to a monotonic reduction in Δy . However, when D_b is 3 mJ/m², the addition of an edge layer results in a non-monotonic variation in Δy , and the smallest Δy is found when w_b is 10 nm.

The time-dependent velocity was determined by differentiation with respect to the displacement-time curves and is shown in Fig. S5. At all w_b values, when time increases from 0 to 1 ns, v_y continues to decrease and approaches zero, while v_x continues to increase and approaches its stable value. The w_b -dependence of v_x and v_y is similar to that of Δx and Δy . When D_b is 3

mJ/m², the v_x values at 1 ns for different w_b are close. However, when D_b is 0 mJ/m², there is a difference between them because the longitude motion approaches stability soon after the beginning of motion when w_b is greater than 10 nm.



Fig. S5 Boundary width dependence of the evolution of the transverse velocity for (a) $D_b=0$ mJ/m² and (c) $D_b=3$ mJ/m² and that of the longitude velocity for (b) $D_b=0$ mJ/m² and (d) $D_b=3$ mJ/m².

S5. The evolution of the *y*-component of the Magnus force and that of edge force with different boundary widths under a current of 2.5×10^{11} A/m²

The *y*-components of the Magnus force $((F_M)_y)$ and the edge force (F_y) were calculated using Eqs. (3) and (4) in the paper based on the values of Q, D_{yy} , v_x , and v_y . The evolution of both $(F_M)_y$ and F_y are shown in Fig. S6. Both $(F_M)_y$ and F_y increase monotonically with *t* and are on the order of 10^{-13} N. The difference between them is reduced with *t*.



Fig. S6 (a) The time dependence of $(F_M)_y$ and F_y when w_b is 0 nm. (b,c) The time dependence of $(F_M)_y$ and F_y when w_b is 10 nm for $D_b = 0$ and 3 mJ/m², respectively. (d,e) The time dependence

of $(F_{\rm M})_{\rm y}$ and $F_{\rm y}$ when $w_{\rm b}$ is 18 nm for $D_{\rm b} = 0$ and 3 mJ/m², respectively.

S6. The transverse displacement dependence of the *y*-components of the Magnus force and edge force with different boundary widths under a current of 2.5×10^{11} A/m²

It can be seen that the approximately linear relationship between $(F_M)_y$ or F_y and Δy is satisfied, especially when the motion is far from stable. When there is no DMI at the edge layer, the increase in w_b causes a monotonic increase in both forces. However, when D_b is 3 mJ/m², the increase in w_b results in a non-monotonic variation in both $(F_M)_y$ and F_y , and they reach their maximum values when w_b is around 10 nm.



Fig. S7 The transverse displacement dependence of the *y*-component of the Magnus force $((F_M)_y)$ for (a) $D_b=0$ mJ/m² and (b) $D_b=3$ mJ/m². The transverse displacement dependence of the *y*-component of the edge force (F_y) for (c) $D_b=0$ mJ/m² and (d) $D_b=3$ mJ/m².

S7. The calculation of effective field ($\dot{H}_{\rm eff}$)

The H_{eff} contributed from exchange coupling (H_{ex}) , DMI (H_{DMI}) , and magnetic anisotropy (H_{a}) was numerically calculated based on Eq. (S3) and the moment distribution obtained from the

micromagnetic simulation.

Concretely, H_a , H_{DMI} , and H_{ex} are expressed as:^{S4}

$$\overset{\mathbf{r}}{H_{a}} = \frac{2(K - \frac{1}{2}\mu_{0}M_{s}^{2})}{\mu_{0}M_{s}}m_{z}\overset{\mathbf{r}}{e_{z}},$$
(S6)

$$\overset{\mathbf{f}}{H}_{\rm DMI} = \frac{2D}{\mu_0 M_{\rm S}} \left[\frac{\partial m_z}{\partial x} \overset{\mathbf{f}}{e}_{\rm x} + \frac{\partial m_z}{\partial y} \overset{\mathbf{f}}{e}_{\rm y} - \left(\frac{\partial m_x}{\partial x} + \frac{\partial m_y}{\partial y} \right) \overset{\mathbf{f}}{e}_{\rm z} \right], \tag{S7}$$

$$\overset{\mathbf{r}}{H}_{\mathrm{ex}} = \frac{2J}{\mu_0 M_{\mathrm{S}}} \nabla^2 \overset{\mathbf{r}}{m}.$$
(S8)

Without the variation of moment distribution in the z direction, $\nabla^2 \hat{m}$ is determined by

$$\nabla^2 \overset{\mathbf{r}}{m} = \left(\frac{\partial^2 m_x}{\partial x^2} + \frac{\partial^2 m_x}{\partial y^2}\right) \overset{\mathbf{r}}{e}_x + \left(\frac{\partial^2 m_y}{\partial x^2} + \frac{\partial^2 m_y}{\partial y^2}\right) \overset{\mathbf{r}}{e}_y + \left(\frac{\partial^2 m_z}{\partial x^2} + \frac{\partial^2 m_z}{\partial y^2}\right) \overset{\mathbf{r}}{e}_z \tag{S9}$$

The total effective field $\dot{H}_{\rm eff}$ is finally determined by

$$\dot{H}_{\rm eff} = \dot{H}_{\rm a} + \dot{H}_{\rm DMI} + \dot{H}_{\rm ex} \tag{S10}$$

The partial derivative in Eqs. (S7)–(S9) was numerically calculated using the distribution of magnetic moments obtained from simulation. At last, the $\overset{\mathbf{I}}{H}_{eff}$ was calculated using Eqs. (S6)–(S10). The $\overset{\mathbf{I}}{H}_{eff}$ is depicted in Fig. S8. One can see that the *x*-component of $\overset{\mathbf{I}}{H}_{eff}$, $(H_{eff})_x$ is close to zero expect near the two tops of the track, and the variation of w_b makes little impact on the $(H_{eff})_x$. The $(H_{eff})_y$ depends on the *y* coordinate but is not relate to *x*. The *y*-dependent $(H_{eff})_y$ is discussed in detail in Fig. 5 in the paper. $(H_{eff})_z$ is also relevant to the *y* coordinate but weakly depends on *x*. The edge modification also changes the $(H_{eff})_z$ near the track edge.



Fig. S8 The *x*-component of the effective field (H_{eff}) for (a) $D_b=0$ mJ/m² and (b) $D_b=3$ mJ/m².

The y-component of $\overset{1}{H}_{eff}$ for (c) $D_b=0$ mJ/m² and (d) $D_b=3$ mJ/m². The z-component of $\overset{1}{H}_{eff}$ for (e) $D_b=0$ mJ/m² and (f) $D_b=3$ mJ/m².

S8. The influence of thickness dependence of exchange coupling strength

The exchange stiffness constant (*A*) of nanoscale materials is generally size dependent due to a surface effect, and the increase in size may strengthen $A^{S5, S6}$. In experiments, to fabricate an IM edge, the thickness of edge layer needs to be a little larger than that in the inner part of track. However, a precise quantitative estimation of *A* as a function of thickness is difficult ^{S6}. We assume the exchange stiffness constant at the boundary layer (A_b) is twice as that in the inner part (A_i). Based on this assumption, we observed the transverse displacement can be further inhibited when compared to the case that A_b equals A_i , especially when $D_b = 3 \text{ mJ/m}^2$ (Fig. S9(a)). This result is attributed to the modification of moment distribution. The stronger exchange coupling at the edge inhibits the moment vortex due to DMI and increases the projection of moments towards -y direction, resulting in the enhanced repulsive force from edge (Fig. S9(b) and (c)).

In a real PM multilayer such as Co/Pt, the magnetic anisotropy constant is sensitively relevant to the thickness of Co layer ^{S7}. A very small variation of thickness (several Å) of Co may induce the transition between PMA and IMA. Therefore, the change of exchange stiffness constant with the thickness of Co can be also in a small range, which may not influence the trajectory of a skyrmion greatly.



Fig. S9 (a) The trajectory of a skyrmion, (b) the magnetization distribution of nanotracks, and (c) the projection of moments in the *y*-axis direction for different $A_{\rm b}$ and $D_{\rm b}$.

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