Supporting Information

Structural Discontinuity induced Surface Second Harmonic Generation in single thin Zinc-blende GaAs Nanowires

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1 Calculation of three Euler angles (α , β , γ) between the lab coordinate system and the crystal axes



Figure S1 Geometries of the lab frame (X,Y,Z) and the crystal frame (CX,CY,CZ). Specifically, β is the angle between cz axis and z axis, α is the angle between x axis and the cz axis's projection on xy plane, and γ is the angle between cx axis and the intersection line of xy and cx-cy planes.

Note that GaAs nanowire of zinc-blende crystal structure is growing along [1¹1], thus $\vec{e}_{z} = (1,-1,1)/\sqrt{3}$. The propagation direction of the excitation light can be set as \vec{e}_{x}

=(a,b,c), where $a^2 + b^2 + c^2 = 1$. Note that a-b+c=0 and $\vec{e_y}$ can be given by the equation $\vec{e_y} = \vec{e_z} \times \vec{e_x} = (-b - c, a - c, a + b)/\sqrt{3}$.

According to Euler's rotation theorem, the laboratory frame (X,Y,Z axes) can be defined by:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = R_z(-\alpha) \cdot R_x(-\beta) \cdot R_z(-\gamma) \begin{pmatrix} CX \\ CY \\ CZ \end{pmatrix}$$
(S1)
where
$$R_z(-\gamma) = \begin{pmatrix} \cos(i)(\gamma) & -\sin(\gamma) & 0 \\ \sin(\gamma) & \cos(\gamma) & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad R_x(-\beta) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos(i)(\beta) & -\sin(i)(\beta) \\ 0 & \sin(\beta) & \cos(i)(\beta) \end{pmatrix}, \quad \text{and} \quad R_z(-\alpha) = \begin{pmatrix} \cos(i)(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{pmatrix}$$
(S2)

So we can get that,

$$\begin{pmatrix} \frac{a}{\sqrt{3}} & \frac{b}{\sqrt{3}} & \frac{c}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{3}} & \frac{-1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

$$= \begin{pmatrix} \cos \alpha \cdot \cos \gamma - \sin \alpha \cdot \cos \beta \cdot \sin \gamma & -\cos \alpha \cdot \sin \gamma - \sin \alpha \cdot \cos \beta \cdot \cos \gamma & -\sin \beta \cdot \sin \gamma \\ \sin \alpha \cdot \cos \gamma + \cos \alpha \cdot \cos \beta \cdot \sin \gamma & -\sin \alpha \cdot \sin \gamma + \cos \alpha \cdot \cos \beta \cdot \cos \gamma & -\cos \beta \cdot \sin \gamma \\ \sin \beta \cdot \sin \gamma & \sin \beta \cdot \cos \gamma \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

that is,

$$\cos\beta = \frac{1}{\sqrt{3}}, \beta = 54.73^{\circ}, \gamma = 135^{\circ},$$

$$\begin{pmatrix}a\\b\\c\end{pmatrix} = \begin{pmatrix}-\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{6}}{5}\sin\alpha\\-\frac{\sqrt{2}}{2}\cos\alpha + \frac{\sqrt{6}}{6}\sin\alpha\\\frac{\sqrt{6}}{3}\sin\alpha\end{pmatrix}, \begin{pmatrix}-c-b\\a-c\\b+a\end{pmatrix} = \begin{pmatrix}\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{6}}{2}\sin\alpha\\-\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{6}}{2}\sin\alpha\\-\frac{\sqrt{2}}{2}\cos\alpha - \frac{\sqrt{6}}{2}\sin\alpha\\-\sqrt{2}\cos\alpha\end{pmatrix}$$
(S3)

2 Theoretical expression of TM-polarized and TE-polarized SHG signals from Zinc-blende GaAs nanowires

The electric field, E_{ω} , is in the y-z plane and the three components on crystal axes (

 $\vec{e_{cx}} \cdot \vec{e_{cy}} \cdot \vec{e_{cz}}$ could be written as,

$$\begin{pmatrix} E_{\omega,cx} \\ E_{\omega,cy} \\ E_{\omega,cz} \end{pmatrix} = E_{\omega} cos\theta_{\omega} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix} + \gamma_{\omega,in} E_{\omega} sin\theta_{\omega} \cdot \frac{1}{\sqrt{3}} \begin{pmatrix} -c - b \\ a - c \\ b + a \end{pmatrix}$$

$$= \begin{pmatrix} \frac{1}{\sqrt{3}} cos\theta_{\omega} + \gamma_{w,in} \frac{-c - b}{\sqrt{3}} sin\theta_{\omega} \\ -\frac{1}{\sqrt{3}} cos\theta_{\omega} + \gamma_{w,in} \frac{a - c}{\sqrt{3}} sin\theta_{\omega} \\ \frac{1}{\sqrt{3}} cos\theta_{\omega} + \gamma_{w,in} \frac{b + a}{\sqrt{3}} sin\theta_{\omega} \end{pmatrix} E_{\omega}$$
(S4)

Therefore, the parallel and perpendicular components of bulk SHG are,

$$\begin{split} E_{2w,p}^{\vec{b}} &= \left(E_{2w}^{\vec{b}} \cdot \vec{e_{z}}\right) \vec{e_{z}} = \frac{1}{\sqrt{3}} \left(E_{2\omega,cx}^{\vec{b}} - E_{2\omega,cy}^{\vec{b}} + E_{2\omega,cz}^{\vec{b}}\right) \vec{e_{z}} = \frac{2d_{14}E_{\omega}^{2}}{\sqrt{3}} \left(\frac{1}{2}\gamma_{\omega,in}^{2}sin^{2}\theta_{\omega} - cos^{2}\theta_{\omega}\right) \vec{e_{z}}_{z}, \\ E_{2w,s}^{\vec{b}} &= \gamma_{2\omega,out} \left(E_{2w}^{\vec{b}} \cdot \vec{e_{y}}\right) \vec{e_{y}} = \frac{2d_{14}E_{\omega}^{2}}{\sqrt{3}} \gamma_{\omega,in}\gamma_{2\omega,out} (sin\theta_{\omega}cos\theta_{\omega} - t \cdot \gamma_{2\omega,in}sin^{2}\theta_{\omega}) \vec{e_{y}}_{y}, \\ t &= (a-c)(b+a)(c+b) = \frac{\sqrt{2}(1-2cos2a) \cdot cosa}{2} \end{split}$$

The TM- and TE-polarized SHG intensity becomes, $I_{2\omega,p} = A * |\vec{E_{2\omega,p}}|^2$ and

 $I_{2\omega,s} = B * |\vec{E}_{2\omega,s}|^2$. Thus they can be written as a function of θ_{ω} :

$$\begin{cases} I_{2\omega,p} = A * E_{\omega}^{4} \left(\frac{1}{\sqrt{3}} d_{14} \gamma_{\omega,in}^{2} \sin^{2} \theta_{\omega} - \frac{2}{\sqrt{3}} d_{14} \cos^{2} \theta_{\omega} + d_{zzy}^{s} \gamma_{\omega,in} \sin \theta_{\omega} \cos \theta_{\omega}\right)^{2} \\ \propto \left(A_{1} \sin^{2} \theta_{\omega} - A_{2} \cos^{2} \theta_{\omega} + A_{3} \sin \theta_{\omega} \cos \theta_{\omega}\right)^{2} \\ I_{2\omega,s} = B * \gamma_{\omega,out}^{2} E_{\omega}^{4} \left(\frac{2d_{14}}{\sqrt{3}} \gamma_{\omega,in} \sin \theta_{\omega} \cos \theta_{\omega} - \left(\frac{2}{\sqrt{3}} t \cdot d_{14} - d_{yyy}^{s}\right) \gamma_{\omega,in}^{2} \sin^{2} \theta_{\omega} + d_{yzz}^{s} \cos^{2} \theta_{\omega}\right)^{2} \\ \propto \left(-B_{1} \sin^{2} \theta_{\omega} + B_{2} \cos^{2} \theta_{\omega} + B_{3} \sin \theta_{\omega} \cos \theta_{\omega}\right)^{2} \end{cases}$$
(S5)

2 Detail discussion on the relationship between the polarization and the value of $\gamma_{2w,in}$ and $t(\alpha)$

TM-polarized and TE-polarized SHG is determined by field reduction factors ($\gamma_{2w,in}$, $\gamma_{2w,out}$) and the propagation direction of the incident laser beam (angle α). TM-polarized (black, $I(0^o, \theta_\omega)$) and TE-polarized (red, $I(90^o, \theta_\omega)$) as a function of excitation polarization angle (θ_ω) at different propagation directions of the incident laser beam (angle $\alpha=0^o,15^o,30^o,45^o,60^o,75^o,90^o,105^o,120^o$) are shown in Figure S2 ($\gamma_{2w,in} = \gamma_{2w,out} = 1$) and S3 ($\gamma_{2w,in} = \gamma_{2w,out} = 0.5$).



Figure S2. When $\gamma_{2w,in} = \gamma_{2w,out} = 1$, TM-polarized (blue line) and TE-polarized (red line) SHG as a function of excitation polarization angle (θ_{ω}) at different propagation directions of the incident laser beam (angle $\alpha = 0^{\circ}$, 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°).



Figure S3. When $\gamma_{2w,in} = \gamma_{2w,out} = 0.5$, TM-polarized (blue line) and TE-polarized (red line) SHG as a function of excitation polarization angle (θ_{ω}) at different propagation directions of the incident laser beam (angle $\alpha = 0^{\circ}$, 15°, 30°, 45°, 60°, 75°, 90°, 105°, 120°).