Supplementary Information

Parametric Amplification in MoS_2 Drum Resonator

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Measurement Technique:

A detailed measurement schematic for parametric amplification is shown in the figure S1. We generate two RF signals using Rohde and Schwarz SMA100 signal generators. The RF signal at the frequency ω is used for actuating and another RF signal at the frequency 2ω is used for pumping the resonator. Both the RF signals are combined using a Minicircuit RF combiner. A DC voltage for tuning the resonant frequency is added to the combined RF signal using a Minicircuit bias tee. This RF+DC is applied at the gate of the resonator. At the drain, the output signal is filtered using another bias tee to extract only RF signal. The filtered RF signal is amplified using a MITEQ AU1694 low noise amplifier. The amplified signal is measured using SR844 lock-in at the actuation frequency (ω).

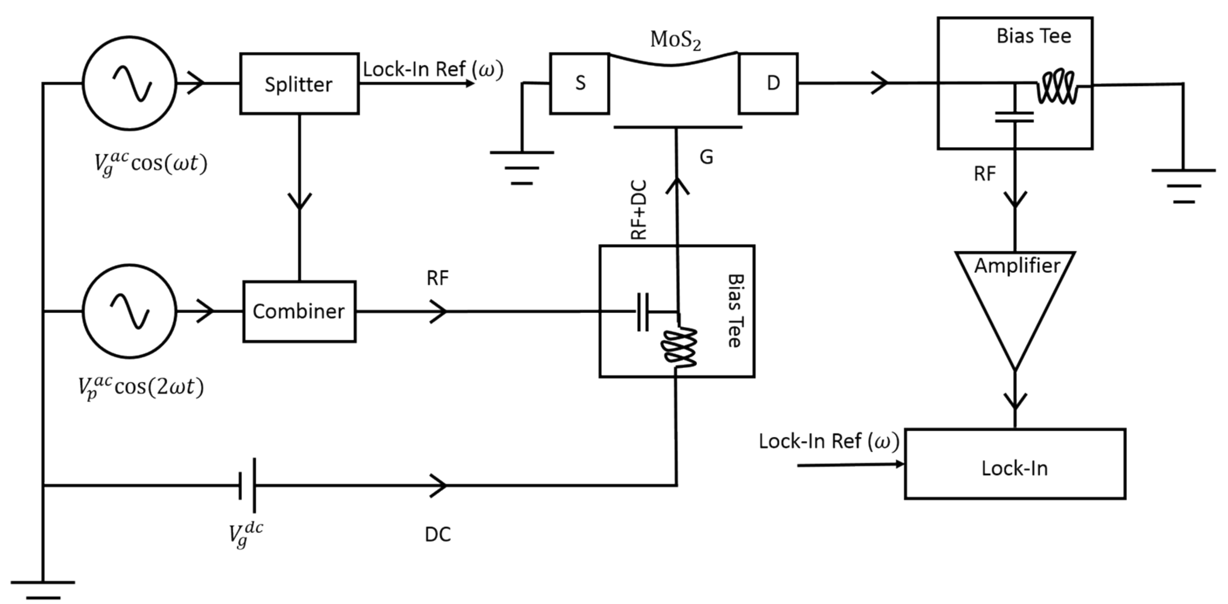


Figure S1 Detailed measurement schematic for performing parametric amplification in the MoS_2 drum resonator. Two signal generators are used, one at frequency ω for actuation and another at frequency 2ω to modulate the spring constant of the resonator. The output RF signal is filtered using a bias tee followed by amplification through a low noise amplifier. The amplified signal is measured using lock-in at the frequency ω.

Estimation of Duffing nonlinearity (α):

Duffing oscillator can be modeled as[1]:

$$\ddot{z} + 2\epsilon\mu\dot{z} + \omega_0^2 z + \epsilon\alpha_m z^3 = f(t) \tag{S1}$$

Where $\mu \geq 0$ is damping coefficient, α_m non-linear Duffing coefficient per unit mass, $f(t) = f_0 cos(\omega t)$ the applied force per unit mass. We define $\omega = \omega_0 + \epsilon\,\sigma$, where ω_0 is the resonance frequency, σ frequency detuning parameter and ϵ small dimensionless parameter.

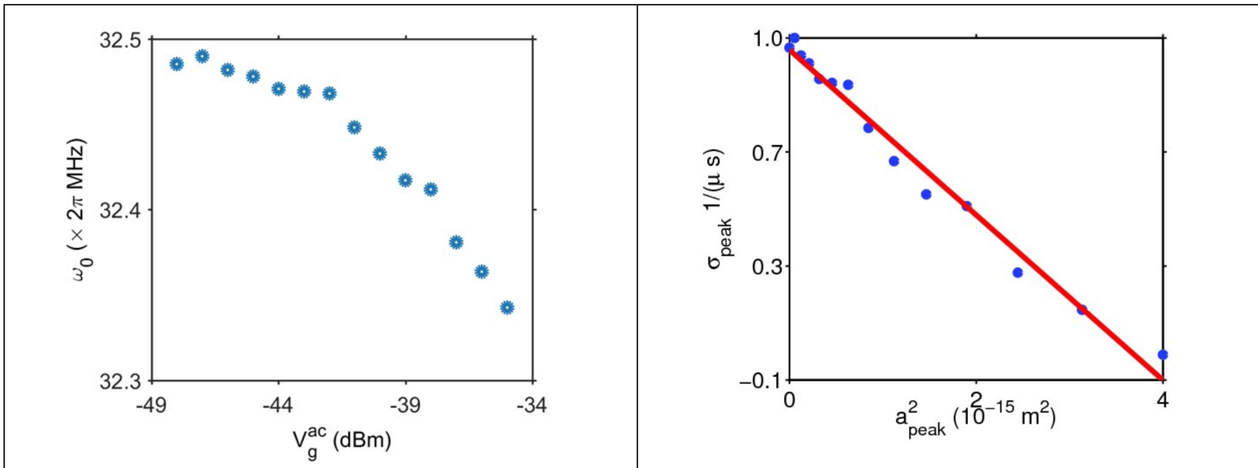
From the backbone curve (the curve that connects the peak (a_{peak}) of the response functions for different drives):

$$\sigma_{peak} = \frac{3\alpha_m}{8\omega_0} a_{peak}^{2} \tag{S2}$$

a_{peak} is estimated as[2]:

$$a_{peak} = - \frac{C_g V_g^{dc} V_g^{ac}}{Q m z_0 \omega_0^2} \tag{S3}$$

σ_{peak} as a function of a_{peak}^{2} is plotted in the figure S2b.



(a) (b)

Figure S2 (a) Variation of frequency corresponding to the peak of the amplitude with V_g^{ac}. Decreasing peak frequency with increasing force indicates negative cubic nonlinearity. (b) Detuning parameter as function of square of the peak amplitude.

The estimated $\alpha = m.\alpha_m$ is $\sim\ -2.3\times\ 10^{12}\ kg.m^{-2}.s^{-2}$.

Simulation Details:

Simulations are performed using MATLAB 2014 by solving the governing differential equation as described by the equation (3) in the main text. Duffing coefficients α and quality factor Q used in the simulations are obtained from the experimental observations.

$$m\ddot{z} + m\frac{\omega_0}{Q}\dot{z} + [k + k_p\,cos(2\omega_0 t)]z + \alpha z^3 = F(t) \tag{S4}$$

where $k_p = (\partial^2 C_g/\partial z^2)V_g^{dc}V_p^{ac}$ is the modulating spring constant and $C_g = A\epsilon_0/(z_0 - z)$ is the capacitance between the gate and the membrane, z_0 is the gap between membrane and the gate, ϵ_0 is the permittivity of free space. The equation above can be re-written as

$$m\ddot{z} + \frac{m\omega_0}{Q}\dot{z} + [m\omega_0^2 + \frac{2A\epsilon_0 V_g^{dc}V_p^{ac}}{(z_0 - z)^3}cos(2\omega_0 t)]z + \alpha z^3 = \frac{A\epsilon_0 V_g^{dc}V_g^{ac}}{(z_0 - z)^2}cos(\omega_d t + \phi) \tag{S5}$$

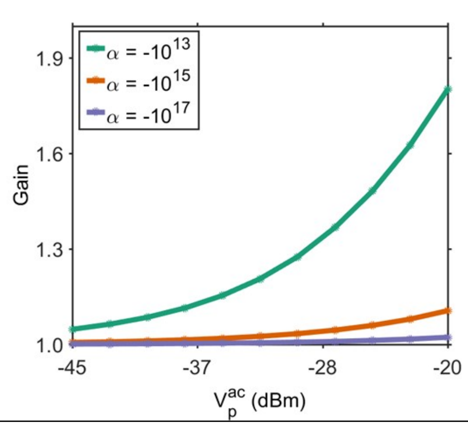
In equation (S5), ϕ is the phase between drive frequency and pump frequency. Since $z \ll z_0$, we assume $\partial C_g/\partial z = A\epsilon_0/z_0^2$ and $\partial^2 C_g/\partial z^2 = 2A\epsilon_0/z_0^3$. Further, we non-dimensionalize the above equation as:

$$\ddot{\tilde{z}} + Q^{-1}\dot{\tilde{z}} + [1 + Hcos(2\tau)]\tilde{z} + \tilde{z}^3 = Gcos(\omega\tau + \phi) \tag{S6}$$

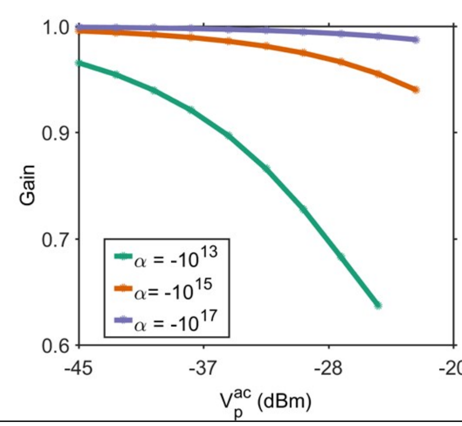
Where $H = \frac{2A\epsilon_0 V_g^{dc}V_p^{ac}}{z_0^3 m\omega_0^2}$, $G = \frac{A\epsilon_0 V_g^{dc}V_g^{ac}}{z_0^2\omega_0^3}\sqrt{\frac{\alpha}{m^3}}$, $\tilde{z} = z\sqrt{\frac{\alpha}{m\omega_0^2}}$ and $\omega = \omega_d/\omega_0$

In the equation, the derivatives are with respect to the new variable τ. We perform the simulation using the following parameters:

Parameter	**Value**
m	$1.6 \times 10^{-17} kg$
z_0	$270 \times 10^{-9} m$
t_h	$6.5 \times 10^{-10} m$
V_g^{dc}	$-23\ V$
V_g^{ac}	$-49\ dBm$
Q	500
ω_0	$2\pi \times 32.5\ MHz$



(a)



(b)

Figure S3 Gain for different Duffing non-linearity, it shows the gain for (a) maxima (b) minima. With increase in the nonlinearity, both the maxima and the minima limit towards unity gain.

The calculated critical voltage V_c using the above parameters is -22 dBm, beyond which self-oscillation is observed. Figures S3a and b show the effect of cubic nonlinearity on the gain with varying V_p^{ac} for different values of α. The figures illustrate, the cubic nonlinearity plays a significant role in limiting the parametric gain of the system[3]. Figure S4 is simulated with experimentally found $\alpha \sim -2 \times 10^{12}\ kg.m^{-2}.s^{-2}$. It shows the maximum gain is nearly 3 (when V_p^{ac} is close to the critical voltage) which matches with the experimentally observed value mentioned in the main text. Figure S5a shows the experimentally observed change in phase difference ($\Delta\phi$) between two successive maxima (or minima) for different value of V_p^{ac}. We observe $\Delta\phi$ changes from 180° to 360° at about -17 dBm V_p^{ac}. This is verified using simulation as shown in the figure S5b.

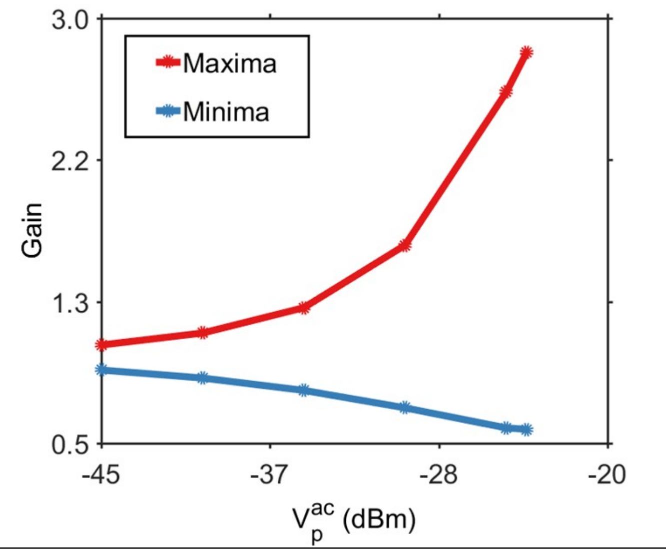
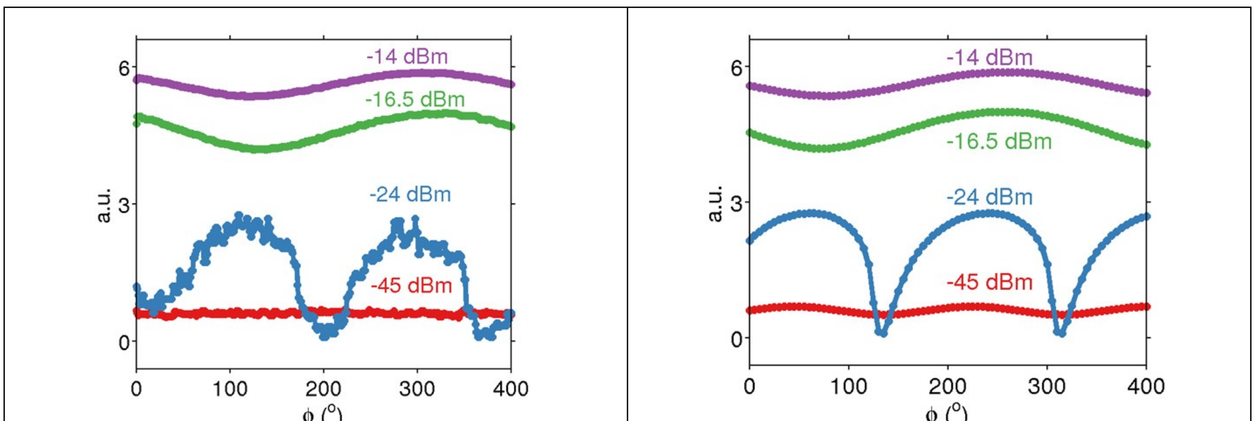


Figure S4 Maximum and Minimum Gain for $\alpha = -2 \times 10^{12}\ kg.m^{-2}.s^{-2}$. It shows that the gain is limited due to the effect of the Duffing nonlineairity.



(a). (b)

Figure S5 (a) Experimental (b) Simulated results, Phase difference $\Delta\phi$ distorts and changes from 180^o to 360^o. The shift occurs at around -17 dBm V_p^{ac}. Y axis has been linearly scaled for better comparison between the experimental and simulated results.

References:

[1] I. Kozinsky, Nonlinear Nanoelectromechanical Systems, California Institute of Technology Pasadena, California, 2007.

[2] Y. Xu, C. Chen, V. V. Deshpande, F.A. Direnno, A. Gondarenko, D.B. Heinz, S. Liu, P. Kim, and J. Hone, Appl. Phys. Lett. **97**, 243111 (2010).

[3] J.F. Rhoads and S.W. Shaw, Appl. Phys. Lett. **96**, 234101 (2010).