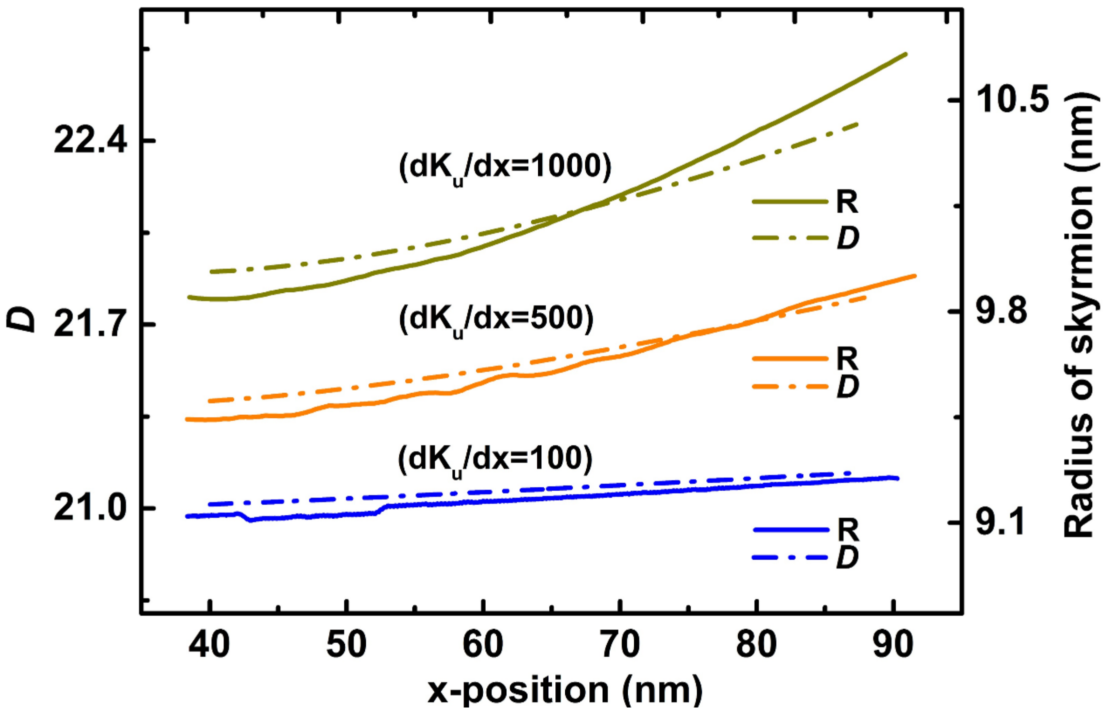
Electronic Supplementary Material (ESI) for Nanoscale.

Supplementary 1: Calculation of dissipative force tensor *D*.

The dissipative tensor *D* in Thiele equation can be written as,[1]

$$D_{ij} = \int_{sk} d^2 r \partial_i m \cdot \partial_j m = \begin{cases} D & (i,j) = (x,x),(y,y), \\ 0 & \text{otherwise.} \end{cases} \tag{1}$$

In the numerical simulations, *D* was observed to change as the skyrmions were driven to the lower K_u regions, as shown in Fig. S1. This was not considered in the original manuscript for simplicity. Therefore, a small difference between the simulated and analytical trajectories was observed in Fig. 4(d).



Supplementary Figure S1. Dissipative force tensor and radius at different *x*-position during skyrmion motion under various K_u gradients.

Supplementary 2: Calculation of the coefficient *C(x)* in the equation of F_y.

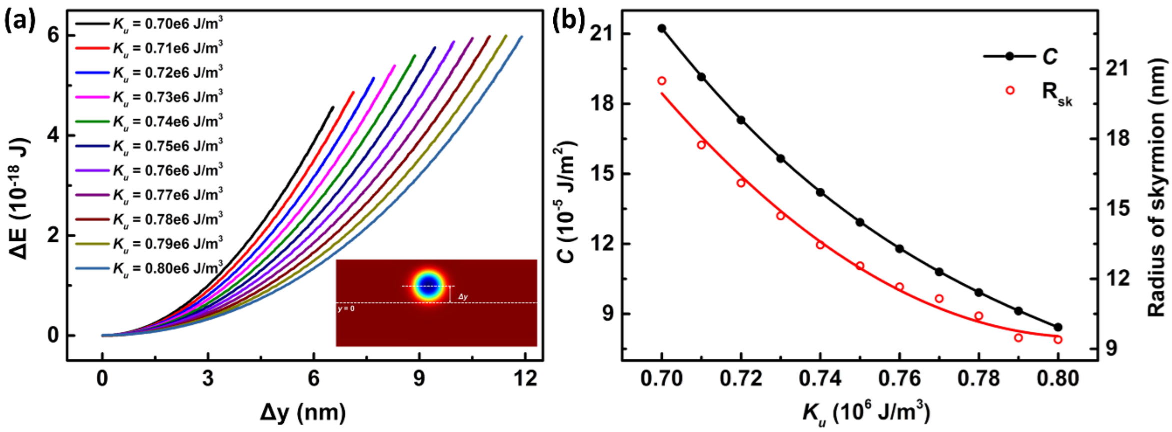
For a finite nanowire, the skyrmion trajectory along the *y*-axis is determined by the repulsion between the edge and the skyrmion. The repulsive potential acts on the skyrmion as a force (F_y). The change in magnetostatic energy along the *y*-axis was found by applying a spin-current (7×10^{11} A/m^2) along the *x*-axis to drive the skyrmion. Under the applied current, the skyrmion experiences a skyrmion Hall effect and drifts in the *y*-axis. The system's total magnetostatic energy was then calculated at every *y*-position as shown in Fig. S2-1(a), which can be written as,

$$E = A + By^2 \tag{2}$$

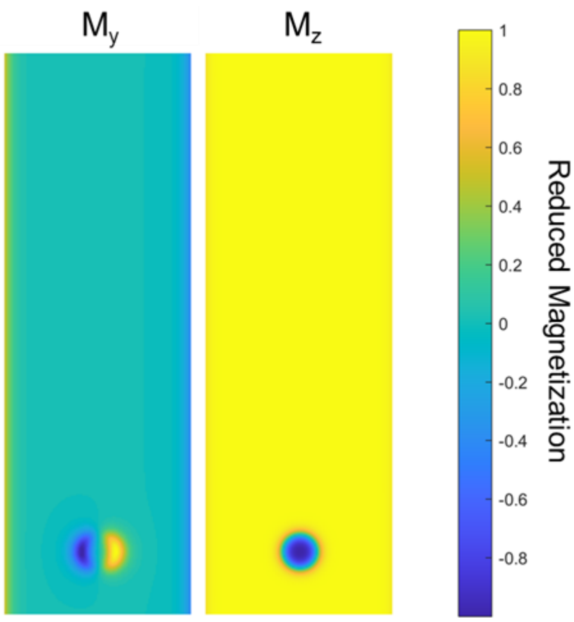
Hence, the energy gradient along the y-axis as well as F_y can be expressed by,

$$F_y = \frac{dE}{dy} = 2By = Cy \tag{3}$$

where C is the coefficient when the skyrmion is driven by spin-current along a nanowire where the K_u remains constant. When a skyrmion is driven by a K_u gradient, the coefficient is not constant due to the change in the size of the skyrmion, which is in turn determined by K_u. Hence, in a nanowire with a K_u gradient along the x-axis, the coefficient C is a function of x, this relation is shown in Fig. S2-1(b). The origins of the inhomogeneous magnetostatic energy landscape can be attributed to the magnetization tilting at the edges. The in-plane magnetization at the edges is opposite of the skyrmion edges. The overlap of the dissimilar magnetization results in a repulsive potential. As shown in Fig. S2-2, an in-plane magnetization exists at the edges which acts to push the skyrmion back to the center of the track.



Supplementary Figure S2-1. (a) Micromagnetic energy as a function of skyrmion displacement along the y-axis from the center of the nanowire with various anisotropy constants. Inset is the schematic diagram of the skyrmion displacement along the y-axis. (b) C and radius of skyrmion depend on the anisotropy constant. This shows that a low/high anisotropy constant, will result in a larger/smaller skyrmion, resulting in a larger/smaller C. This trend indicates that C increases with the increase of x-position in the system applying a negative anisotropy gradient.



Supplementary Figure S2-2. The in-plane (M_y) and out-plane (M_z) components of skyrmion magnetization

Supplementary 3. Energy consumption of a VCMA device

The VCMA effect does not require a current in the magnetic layer and is therefore inherently dissipationless. However, a microscopic amount of electrical energy is needed to charge the capacitive dielectric layer. The VCMA efficiency can be expressed as a ratio between induced PMA change and applied electric field $\frac{\Delta K_u}{\Delta E}$. As reported by Y. Suzuki et al., a small electric field of 100 mV/nm can cause a 40% change in K_u with $\frac{\Delta K_u}{\Delta E} = 210\ \mathrm{fJ/Vm}$.[2] Using a more conservative estimate of $\frac{\Delta K_u}{\Delta E} = 100\ \mathrm{fJ/Vm}$, we find that for the $2\times10^4\ \mathrm{J/m^3}$ change in the K_u required by stepped anisotropy gradient, an electric field of 0.1 V/nm needs to be applied across a gate oxide of 1 nm. Using MgO as a gate oxide with a relative permittivity of 7 and an area of 60 nm$\times$20 nm, the energy required to create the electric field is 0.18 aJ. To achieve a speed of 70 m/s, the gate must be charged at a rate of 3.5 GHz. The resulting power consumption of shifting a single skyrmion at 70 m/s is then 0.65 nW.

The energy dissipation in current-driven skyrmion devices can be calculated from the resistive energy losses through Joule heating. Experimental data from S.Rohart et al. shows that a current density of 5×10^{11} A/m^2 is required to drive a skyrmion at 70 m/s.[3] For comparison purposes, we assume that skyrmions on both systems have a same total magnetic volume. However, for current-driven devices, an additional 4 nm of heavy metal, usually Ta or Pt is added for a strong spin Hall effect required for high speed manipulation. Taking a bulk resistance value of $1\times10^{-7}\ \Omega\,\mathrm{m}$, the power consumed in current-driven skyrmion devices for shifting a single skyrmion at 70 m/s is $0.5\ \mu\mathrm{W}$.

Supplementary 4. Skyrmion motion without annihilation. ($\frac{dK_u}{dx} = 600$ GJ/m^4)

Supplementary 5. Annihilation of skyrmion when the skyrmion center is less than 20 nm from the edge. ($\frac{dK_u}{dx} = 1200$ GJ/m^4)

Supplementary 6. Skyrmion motion induced by stepped anisotropy in a racetrack. The white overlays indicate the position of the potential wells created by the stepped anisotropy.

Reference

1 Seki, S. & Mochizuki, M. *Skyrmions in Magnetic Materials*. (Springer, 2016).

2 Maruyama, T. *et al.* 2009,Large voltage-induced magnetic anisotropy change in a few atomic layers of iron. *Nat. Nanotechnol.* **4**, 158-161.

3 Hrabec, A. *et al.* 2017,Current-induced skyrmion generation and dynamics in symmetric bilayers. *Nature communications* **8**, ncomms15765.