Focused Ultrasound Actuation of Shape Memory Polymers; Acoustic-Thermoelastic Modeling and Testing

Aarushi Bhargava[1], Kaiyuan Peng[2], Jerry Stieg[2], Reza Mirzaeifar[2,*] and Shima Shahab[1]

[1]Department of Biomedical Engineering and Mechanics, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

[2]Department of Mechanical Engineering, Virginia Polytechnic Institute and State University, Blacksburg, VA 24061, USA

Table of contents:

[*] Corresponding author: rmirzaei@vt.edu

Electronic Supplementary Information (ESI) for RSC Advances

Focused Ultrasound (FU) induced pressure field in medium; linear acoustic model

An analytical model is developed to obtain the ultrasound pressure field from a spherical concave transducer in fluid domain. The model is based on O'Neil's method and is used to calculate a linear pressure field at the focal point of the curved transducer.[1] It is assumed that the amplitude of acoustic waves is sufficiently low so that the acoustic field can be considered as linear where the effects of absorption and nonlinear distortion are negligible. Diffraction effects are also neglected as the diameter of the transducer is assumed to be considerably large as compared to the wavelength of the transducer. The analytical solutions describe the pressure field along the axis of symmetry and in the focal plane of the transducer having the geometric properties as shown in Fig. 2 (main article).[1,2] Under the given assumptions, the velocity potential Φ in the region of a curved transducer is given as

$$\Phi = \iint_S \frac{u}{2\pi l} e^{-ikl} dS$$ * MERGEFORMAT (1)

where u is the harmonic normal velocity of the transducer surface given as $u = u_0 e^{i\omega t}$ and u_0 is the velocity amplitude. The surface of the transducer, *S,* is composed of several point source elements, which focus acoustic waves at the focal point. In Eq. (1), l is the distance from a source point lying in surface element dS, to the point of observation *Q*. The wavenumber k is defined as $k = \omega / c = 2\pi / \lambda$ where c is the sound velocity, ω is the angular frequency of acoustic wave in the domain and λ is the wavelength (*i* is unit imaginary number). Using Eq. (1) the acoustic pressure field is defined as $p = \rho \partial \Phi / \partial t = i\,kc\rho\Phi$, where ρ is the acoustic medium density.

Acoustic pressure along axis of symmetry of transducer and in focal plane. For ease in calculations, the velocity potential in Eq. (1) is converted from cylindrical (Z, R, ψ) to spherical (r, γ, ψ) coordinates. From Fig. 2, the spherical geometry of transducer is related to its dimensions as $Z = r\cos\gamma$ and $R = r\sin\gamma$. The velocity potential in spherical coordinates is

$$\Phi(r,\gamma) = \frac{u}{2\pi} \int_0^{2\pi} \int_0^{r''} l^{-1} e^{-ikl} r'\, dr'\, d\beta'$$ * MERGEFORMAT (2)

where r'' is the distance from the center of the transducer to the edge boundary given as $r''^2 = h^2 + a^2$ and $r' = 2D\cos\alpha$ as shown in Figs. 2 and S1. The term $r'\,dr'\,d\beta'$ in Eq. (2) represents the surface element dS, in Eq. (1), in spherical coordinates. The focal depth is represented with *D*, a is the radius of the transducer and h is the depth of the concave surface. In Eq. (2), $\beta' = \psi' - \psi$ (ψ' is the azimuthal angle at the transducer surface), $d\beta'$ represents a small change in angle β' and l in spherical coordinates is given as $l = (r^2 - 2r\,r'\sin\alpha\sin\gamma\cos\beta' + m\,r'^2)^{1/2}$, where $m = 1 - 2h\,z'/r''^2$ (z' is the axial distance from origin O at the transducer surface to point of observation Q). The wavelength λ and depth h are assumed to be comparatively small as compared to radius a to satisfy the assumptions used in Eq. (1).

For obtaining acoustic pressure on the axis of symmetry, β' vanishes and Eq. (2) becomes

$$\Phi = \frac{u}{m}\int_{z'}^{D'} e^{-ikl}\,dl = \frac{1}{ikm}u\,(e^{-ikz'} - e^{-ikD'})$$ * MERGEFORMAT (3)

where D' is the distance of Q (located on symmetrical axis) from the edge boundary at r'' (Fig. S1) and given as $D' = (z'^2 + m\,r''^2)^{1/2} = [(z'-h)^2 + a^2]^{1/2}$. To separate the amplitude and the phase factors in Eq. (3), two parameters δ and C are introduced as $\delta = D' - z'$ and $C = (D' + z')/2$. Equation (3) in terms of δ and C is $\Phi = u(e^{ik\delta/2} - e^{-ik\delta/2})\,e^{-ikC}/ikm = u_0 P\,e^{i(\omega t - kC)}/k$, where $P = 2\sin(k\,\delta/2)/m$. The corresponding axial acoustic pressure is given as $p = i\,k\,\rho\,c\,\Phi = i\rho\,c\,u_0 P e^{i(\omega t - kC)}$. Figure S1b shows the relative pressure along symmetrical axis where p_0 is the characteristic acoustic pressure at the surface of the transducer; the maximum pressure is achieved at focal point $z'/D = 1$. For a transducer with an input power, P', the characteristic source pressure, p_0, is calculated as $p_0 = (2\rho c P'/\pi a^2)^{1/2}$.[3]

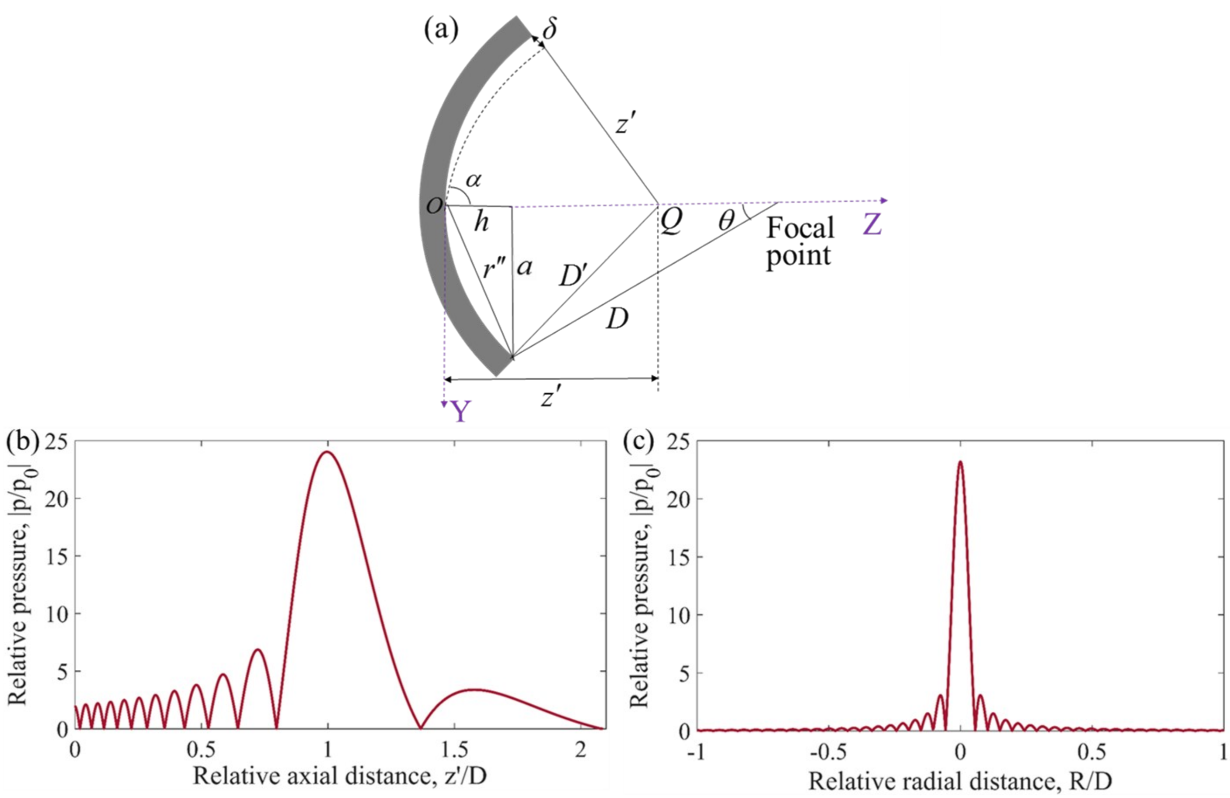

Fig. S1. (a) Geometrical details of the transducer with point of observation on the axis of symmetry, (b) relative acoustic pressure along axis of symmetry and (c) in focal plane

To calculate acoustic pressure in the focal plane $z' = D$, Eq. (2) is modified as

$$\Phi(r,\gamma) = \frac{p}{ik\rho c} = u\,S\left(\frac{e^{-ikr}}{2\pi r}\right)F(g)$$ * MERGEFORMAT (4)

where $F(g) = (2/g)\sum_{j=0}^{\infty}(-1)^j (h/a)^{2j} J_{2j+1}(g)$ and the variable g is expressed as $g = (1 - i/kr)\,k\,a\sin\gamma = k\,a\sin\gamma$. Here $J_{2j+1}(g)$ represents the Bessel function. Figure S1c shows relative acoustic pressure at the focal plane where peak pressure occurs at focal point ($R = 0$). The

closed-form solutions are possible only for linear regime. To account for nonlinearity, absorption and diffraction effects in both fluid and polymer domains, numerical approach is adopted to obtain the acoustic pressure field by focused transducers, which is discussed in detail in the following section.

Electronic Supplementary Information (ESI) for RSC Advances

DMA reported loss modulus and stiffness for various TBA-DEGMA compositions

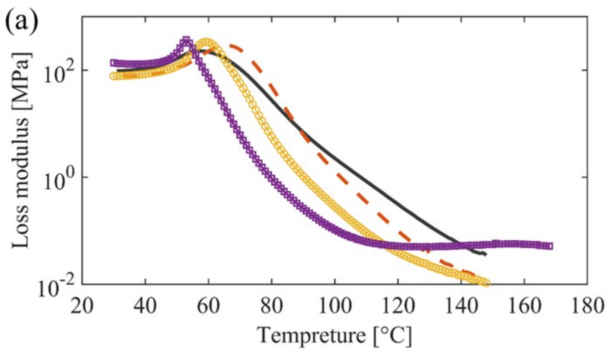

Fig. S2. (a) Loss modulus and (b) stiffness curves obtained from DMA tests for different compositions; the legend used in these two plots is consistent with Fig. 4.

Fast Fourier Transform of pressure waveforms at focal point for various power and frequency; corresponding to Fig. 13

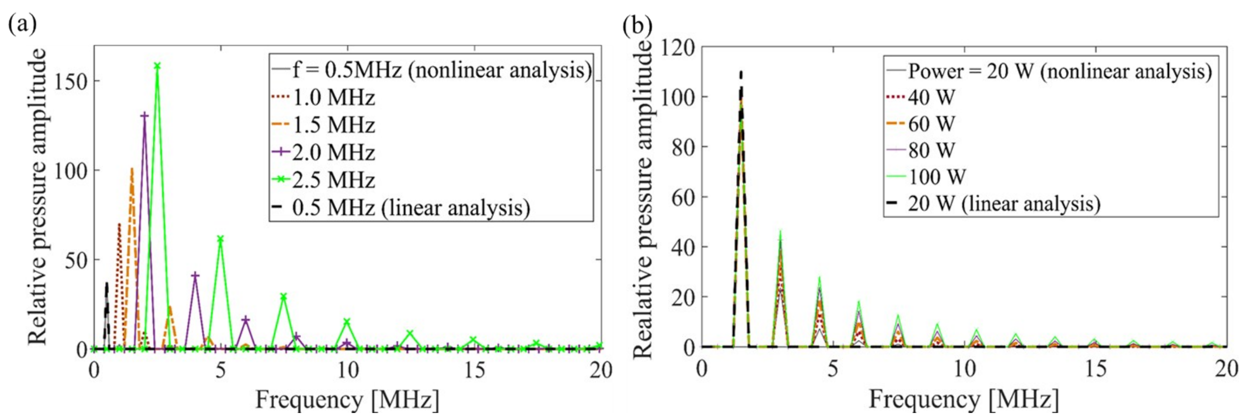

Fig. S3. Relative pressure (with reference as source pressure, p_0) at focal point in frequency domain (a) for various source frequency and 20 W input power (b) for various input power at 1.5 MHz

REFERENCES

1. H. O'Neil, *The Journal of the Acoustical Society of America*, 1949, **21**, 516-526.
2. A. Penttinen and M. Luukkala, *J. Phys. D: Appl. Phys.*, 1976, **9**, 1547.
3. L. E. Kinsler, A. R. Frey, A. B. Coppens and J. V. Sanders, *Fundamentals of Acoustics, 4th Edition, by Lawrence E. Kinsler, Austin R. Frey, Alan B. Coppens, James V. Sanders, pp. 560. ISBN 0-471-84789-5. Wiley-VCH, December 1999.*, 1999, 560.