Electronic Supplementary Material (ESI) for RSC Advances.

Supplementary Information

for Derivation from Equation (new 2) to Equation (new 6)

Equation (10, new 6) is the variant of Equation (6, new 2) and has been derived from H. L. Ong's work [1,2].

In equation (10, new 6),

$$f_\theta = \frac{1}{2}\chi_a H^2 sin^2\theta_m - 2\chi_a H^2 \int_0^d sin^2\theta\, dz + W_2 \sin^2\theta_o \quad (10, new 6)$$

Here we had omitted the complex derivation and caused your confusion. In short, term $\boldsymbol{\theta}_m$ contains the elastic constants and we just give the compact expression.

We have already know Eq. (6, new 2) as total free energy function under external magnetic field per area on substrate[3, 22]:

$$f = \int_0^d [(K_{11} sin^2\theta + K_{33} cos^2\theta)\left(\frac{d\theta}{dz}\right)^2 - \chi_a H^2 sin^2\theta)\, dz + \frac{1}{2}[\delta(z)+\delta(dz)]\sum_{n=1}^{\infty} W_{2n}\, sin^{2n}\theta \quad (6)$$

K_{11} and K_{33} are splay and bend elastic constants, χ_a is the magnetic anisotropy of NLC, we can define $\boldsymbol{\theta} = \boldsymbol{\theta}_m$ when $z=\frac{d}{2}$.

By using variational method, based on $\frac{\partial f}{\partial \theta} - \frac{d}{dz}\left(\frac{\partial f}{\partial(\frac{\partial\theta}{\partial z})}\right) = 0$, we can get the system equilibrium state's Euler equation:

$$(K_{11} sin^2\theta + K_{33} cos^2\theta)\frac{d^2\theta}{dz^2} - (K_{33} - K_{11})\, cos\theta\, sin\theta \left(\frac{d\theta}{dz}\right)^2 - 2\chi_a H^2 sin\theta\, cos\theta = 0 \quad (7)$$

And boundary condition: $(K_{11} sin^2\theta + K_{33} cos^2\theta)\frac{d\theta}{dz}|_{z=0.d} = -2W_2 sin\theta_0\, cos\theta_0$ (8)

By doing further Dimensionality reduction of Eq. (7)

We can have $(K_{11} sin^2\theta + K_{33} cos^2\theta)\left(\frac{d\theta}{dz}\right)^2 = \chi_a H^2 (cos^2\theta_m - cos^2\theta)$ (18)

And $$\left(\frac{d\theta}{dz}\right)^2 = \frac{\chi_a H^2(\cos^2\theta_m - \cos^2\theta)}{(K_{11}\sin^2\theta + K_{33}\cos^2\theta)} \quad (19)$$

Because LC cell is symmetrical around z=d/2, So we only need to consider $0 \le z \le \frac{d}{2}$,

$$\frac{d\theta}{dz} = -\sqrt{\frac{\chi_a}{K_{11}}} H \sqrt{\frac{\cos^2\theta_m - \cos^2\theta}{1 + k\cos^2\theta}} \quad (20)$$

In which $k = \frac{K_{33} - K_{11}}{K_{11}}$

Based on Eq. (18), we have $H\sqrt{\frac{\chi_a}{K_{11}}}\int_0^{d/2} dz = -\int_{\theta_0}^{\theta_m}\sqrt{\frac{1+k\cos^2\theta}{\cos^2\theta_m-\cos^2\theta}}d\theta$ (21)

And we can get $H = -\frac{2}{l}\sqrt{\frac{K_{11}}{\chi_a}}\int_{\theta_0}^{\theta_m}\sqrt{\frac{1+k\cos^2\theta}{\cos^2\theta_m-\cos^2\theta}}d\theta$ (22)

Then, based on Eq. (6) and Eq. (22)

$$f_\theta=\frac{1}{2}\ \chi_a H^2 sin^2\theta_m - 2\chi_a H^2\int_0^d sin^2\theta\, dz + W_2\sin^2\theta_o \quad \text{(10, new 6)}$$

In which $\theta=\theta_m\sin(\pi z/d)+O(\theta_m^3)$, $\theta_m\approx[(H/H_0)^2-1]/2B$, $B=(1-k-9u/4)/4$, $u=1-\chi_\perp/\chi_\parallel$, H_0=(π/d)($K_{33}/\chi_a)^{1/2}$.θ_o is the director orientation of the LC at the substrates deviates from the easy direction which provides resistance to hold back LC's tendency to align at magnetic field's direction.

So equation (10, new 6) is dependent on the elastic splay or bend constants. It is just author's simplification that causes careful reader's confusion. Based on derivation above, we can know the first two part of equation (10, new 6) precisely and it is very useful for our combination of theory and experiment results.

Reference

1. Optically induced Freedericksz transition and bistability in a nematic liquid crystal, Hiap Liew Qng, PHYSICAL REVIEW A, VOLUME 28, NUMBER 4 OCTOBER 1983,
2. "Optical-field-enhanced and static-field-induced first-order Freedericksz transitions in a planar parallel nematic liquid crystal", Phys. Rev. A 33, 3550 (1986)