Supplementary document

A comprehensive optimization study on Bi_2Te_3-based thermoelectric generator using Taguchi method

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Signal-to-noise ratio

In Taguchi optimization method, the desirable component of the output response is measured in terms of signal; whereas, the undesirable components is measured in terms of noise, which occurs due to variability in the process due to noise factors [1, 2]. Higher the value of signal-to-noise (S/N) ratio, higher is the effect of control factors over the noise factors on the output. Taguchi method tries to maximize the S/N ratio to ensure a certain combination of the control factors that minimizes the effect of the noise factors. This is achieved using a 2-step process. First step aims to reduce variability, whereas the second step targets to bring the mean close to target value. Depending on the goal of the experiments, there are usually three ways to calculate S/N ratio:

Larger is better
$$S/N(dB) = -10\log\left[\frac{1}{r}\sum_{i=1}^{r}\frac{1}{y_i^2}\right] \tag{S1}$$

Smaller is better
$$S/N(dB) = -10\log\left[\frac{1}{r}\sum_{i=1}^{r}y_i^2\right] \tag{S2}$$

Nominal is best
$$S/N(dB) = 10\log\left[\frac{\frac{1}{r}\left(\frac{T^2}{r} - \sigma_{r-1}^2\right)}{\sigma_{r-1}^2}\right] \tag{S3}$$

where r is the number of data points and y_i is the value of i^{th} data point. T denotes sum of all the data points and σ_{r-1}^2 denotes the variance.

$$T = \sum_{i=1}^{r} y_i \tag{S4}$$

$$\sigma_{r-1}^2 = \sum_{i=1}^{r}\frac{(y_i - \bar{y})^2}{r-1} \tag{S5}$$

$$\bar{y} = \frac{\sum_{i=1}^{R} y_i}{r} \tag{S6}$$

In this study, goal is to maximize the output power and the efficiency; therefore, larger is better concept for S/N ratio is used, which is calculated using equation (S1).

Analysis of Variance (ANOVA)

ANOVA stands for Analysis of Variance. It is a statistical technique used to compare the variation in result caused by the control factors and the error term (signifies the collective effect of all external factors) relative to the total variation observed. A typical ANOVA table contains the degrees of freedom (DOF), the sum of squares (SS), variance (V), and percentage contribution (P) by each control factor and by the error term. For a control factor i and the error term e, these parameters are calculated using the equations given below [3, 4].

$$(DOF)_i = k_i - 1 \tag{S7}$$

$$(SS)_i = \sum_{j=1}^{k_i} \frac{S_{ij}^2}{k_i} - S_m \tag{S8}$$

$$V_i = \frac{(SS)_i}{(DOF)_i} \tag{S9}$$

$$P_i = \frac{(SS)_i}{(SS)_T} \tag{S10}$$

$$(DOF)_e = (DOF)_T - \sum_{i=1}^{m} (DOF)_i \tag{S11}$$

$$(SS)_e = (SS)_T - \sum_{i=1}^{m} (SS)_i \tag{S12}$$

$$V_e = \frac{(SS)_e}{(DOF)_e} \tag{S13}$$

$$P_e = \frac{(SS)_e}{(SS)_T} \tag{S14}$$

where k_i is the number of levels for factor i, S_{ij} is the sum of S/N ratios of factor i at level j, and m denotes the total number of control factors. $(DOF)_T$ denotes the total degree of freedom and is equal to $n - 1$. n denotes total number of experiments. $(SS)_T$ is called the total sum of squares and is calculated as

$$(SS)_T = \sum_{i=1}^{n} (S/N)_i^2 - S_m \tag{S15}$$

S_m is a correction factor, which is calculated using

$$S_m = \frac{1}{n}\left(\sum_{i=1}^{n} (S/N)_i\right)^2 \tag{S16}$$

An ANOVA table may also contain F-value (or F-ratio) for every control factor, which is defined as a ratio of the variance for a factor i over the error variance.

$$(F - value)_i = \frac{V_i}{V_e} \tag{S17}$$

Table S1. Taguchi standard L_{25} (5^5) standard orthogonal array.

Trial number	Factor (A)	Factor (B)	Factor (C)	Factor (D)	Factor (E)
1	1	1	1	1	1
2	1	2	2	2	2
3	1	3	3	3	3
4	1	4	4	4	4
5	1	5	5	5	5
6	2	1	2	3	4
7	2	2	3	4	5
8	2	3	4	5	1
9	2	4	5	1	2
10	2	5	1	2	3
11	3	1	3	5	2
12	3	2	4	1	3
13	3	3	5	2	4
14	3	4	1	3	5
15	3	5	2	4	1
16	4	1	4	2	5
17	4	2	5	3	1
18	4	3	1	4	2
19	4	4	2	5	3
20	4	5	3	1	4
21	5	1	5	4	3

22	5	2	1	5	4
23	5	3	2	1	5
24	5	4	3	2	1
25	5	5	4	3	2

Table S2. Optimization steps, their goal, varying parameters and their range.

Step	**Goal**	**Parameter**	**Range**
Step 1	Optimize geometry of the TEG	Cross-sectional area of p-n legs (L x W)	1.0 x 1.0 – 2.0 x 2.0 mm^2
		Height of p-n legs (H)	1.0 – 2.0 mm
		Resistive load (R)	2.0 – 10.0 Ω
Step 2	Optimize geometry of heat sink	Fin thickness (a)	0.5 - 1.5 mm
		Fin height (c)	10 – 30 mm
		Resistive load (R)	2.0 – 15.0 Ω
Step 3	Study the effect of environmental condition	Total heat transfer coefficient (h_∞)	0 – 60 W/m^2-K
		Ambient temperature (T_∞)	273 – 313 K
		Resistive load (R)	2.0 – 15.0 Ω

Table S3. Response table for the raw data for output power and efficiency.

		Level 1	Level 2	Level 3	Level 4	Level 5
Means of raw data for power, $\bar{P}$ (W)	A	1.5184	1.323	1.2074	1.1064	1.0291
	B	0.7474	1.0766	1.2979	1.4595	1.6029
	C	0.9117	1.3262	1.3859	1.337	1.2235
Means of raw data for efficiency, $\bar{\eta}$ (%)	A	3.370	3.119	2.961	2.822	2.716
	B	2.298	2.802	3.105	3.306	3.478
	C	2.053	3.142	3.379	3.319	3.096

Table S4. Response table for signal-to-noise ratio for output power and efficiency.

		Level 1	Level 2	Level 3	Level 4	Level 5
Means of S/N data for power, $\left(\bar{S}/_N\right)_P$ (dB)	A	3.152	2.2738	1.3735	0.4136	-0.4221
	B	-2.6098	0.2223	1.9735	3.1546	4.0503
	C	-1.1986	2.2238	2.5444	2.0762	1.145
Means of S/N data for efficiency, $\left(\bar{S}/_N\right)_\eta$ (dB)	A	-29.78	-30.22	-30.69	-31.21	-31.68
	B	-32.85	-31.34	-30.39	-29.76	-29.26
	C	-33.88	-30.13	-29.51	-29.71	-30.36

Table S5. ANOVA table highlighting percentage contribution by various factors on the output power.

Source of variation	Degree of freedom (DOF)	Sum of squares (SS)	Variance (V)	F-value (F)	Percentage contribution
(A) Fin thickness	4	0.73858	0.184644	62.23	19.60%
(B) Fin height	4	2.26273	0.565684	190.65	60.06%
(C) Resistive load	4	0.73073	0.182682	61.57	19.40%
Error	12	0.0356	0.002967		0.94%
Total	24	3.7676			100%

Table S6. ANOVA table highlighting percentage contribution by various factors on the efficiency.

Source of variation	Degree of freedom (DOF)	Sum of squares (SS)	Variance (V)	F-value (F)	Percentage contribution
(A) Fin thickness	4	0.000132	0.000033	57.79	11.40%
(B) Fin height	4	0.000433	0.000108	189.11	37.39%
(C) Resistive load	4	0.000585	0.000146	255.67	50.52%
Error	12	0.000007	0.000001		0.60%
Total	24	0.001158			100%

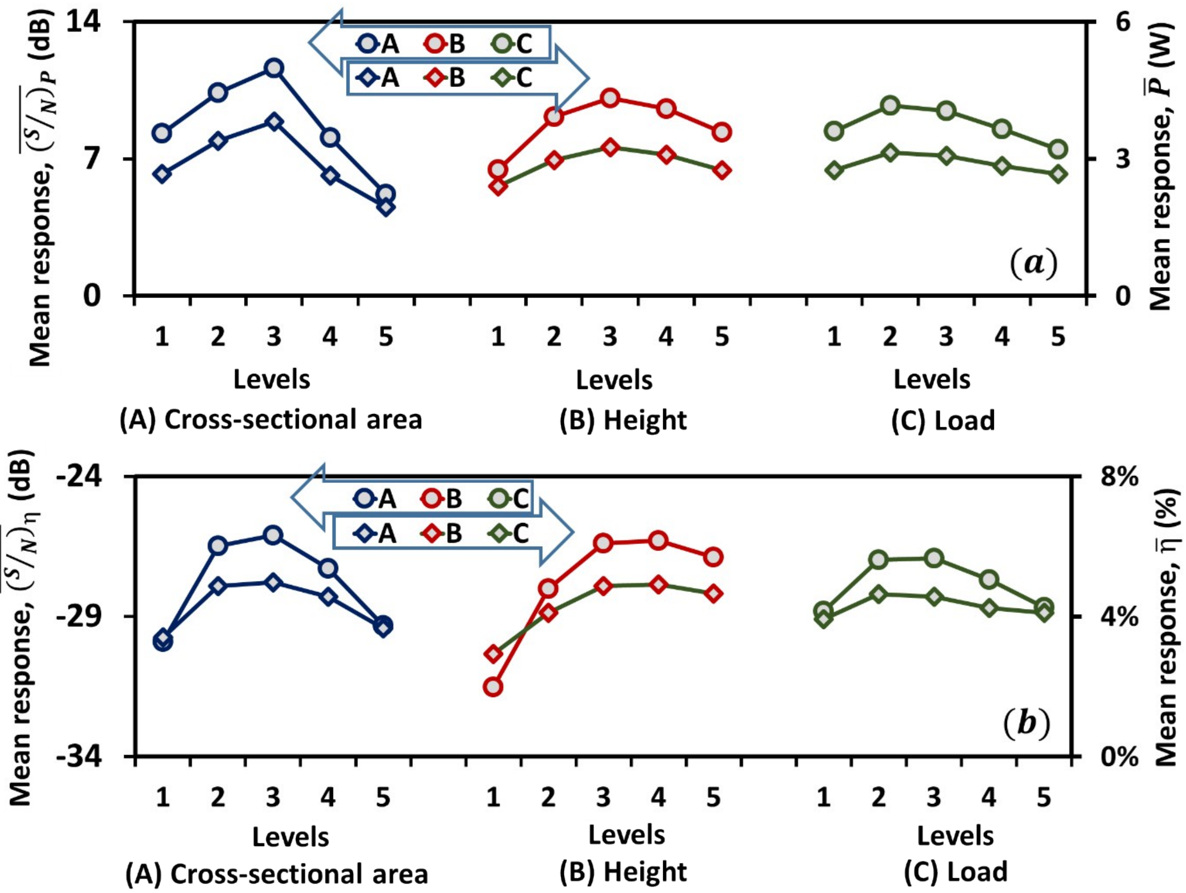

Figure S1. (a) Mean response for raw data and S/N ratio for output power. (b) Mean response for raw data and S/N ratio for efficiency. The level having the highest mean response for the S/N ratio is the optimal level, and the combination of all optimal levels constitutes the optimal setting. Therefore, combination A3B_3C_2 (cross-section area: 1.5 x 1.5 mm^2, height: 1.5 mm, and resistance: 4.0 Ω) is the optimal control factor setting for the power output. Likewise, combination $A_3B_4C_3$ (cross-section area: 1.75 x 1.75 mm^2, height: 1.75 mm, and resistance: 6.0 Ω) is the optimal setting for the highest efficiency.

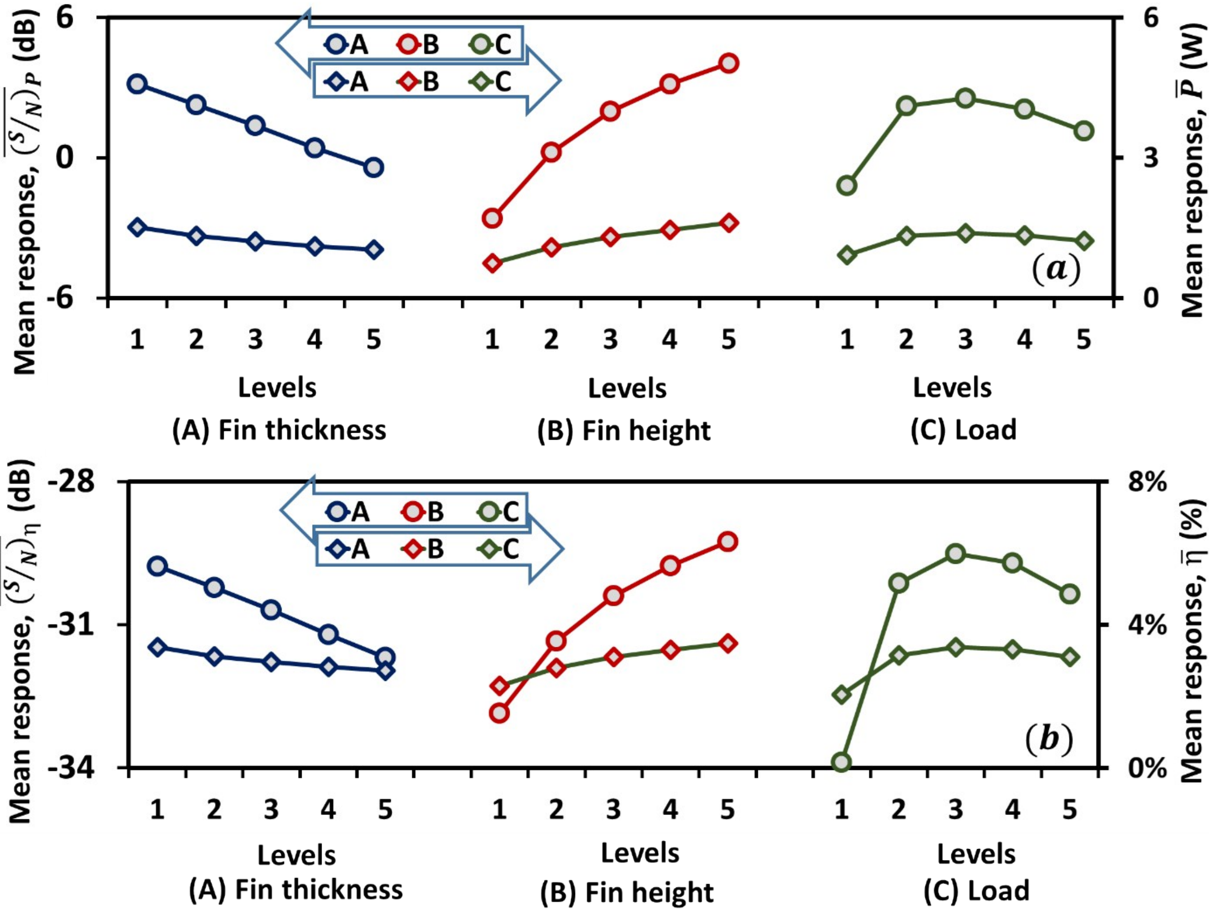

Figure S2. (a) Mean response of raw data and S/N ratio for the output power. (b) Mean response of raw data and S/N ratio for efficiency. Combination $A_1B_5C_3$ (fin thickness: 0.5 mm, fin height: 30 mm, and resistive load: 8.0 Ω) is the optimal control factor setting for output power and efficiency.

References

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[4] Zhang M, Yang B, Wang S, Chen Y. Application of Taguchi method in optimizing the microstructure of 316LN austenitic stainless steel. Materialwissenschaft und Werkstofftechnik. 2016;47:831-8.