

Supplemental Material S1: Elastic models for tip displacement and blocking force, normal and shear stress distribution and shear strength of chucked interfaces

S1.1 Elastic model for tip displacement and blocking force of DEA units

Deformation and blocking force of the VSDEA is determined by the performance of its constituent DEA units. In this section, an elastic model predicting the tip displacement and blocking force of the DEA units is described. An elastic model for the curved unimorph DEA to predict the displacement and generated force was developed based on previous models for piezoelectric benders, linear elasticity, and beam theory by Kadooka et al¹⁴. This model was modified for use with flat unimorph DEA by Imamura et al²¹ and extended to account for viscoelasticity of the DEA materials by Kadooka et al¹². This study considers a multilayer actuator consisting of n lamina in the cantilever configuration shown in Figure S1. The following expression for tip displacement u_p as a function of tip load P and applied voltage V was developed using methodology described by Imamura²¹:

$$u_p = \alpha P + \beta V^2 \quad (\text{S1})$$

$$\alpha = -\frac{\sum_{i=1}^n L^3 Y_i'(z_{i+1} - z_i)}{3bK} \quad (\text{S2})$$

$$\beta = \frac{L^2 \left(\sum_{i=1}^n Y_i'(z_{i+1} - z_i) \sum_{i=1}^n \frac{\nu_i'(z_{i+1} + z_i) \epsilon_i \epsilon_0}{2(z_{i+1} - z_i)} - \sum_{i=1}^n \frac{1}{2} Y_i'(z_{i+1}^2 - z_i^2) \sum_{i=1}^n \frac{\nu_i' \epsilon_i \epsilon_0}{z_{i+1} - z_i} \right)}{2K} \quad (\text{S3})$$

$$K = \left(\sum_{i=1}^n \left(\frac{1}{2} Y_i' z_{i+1}^2 - \frac{1}{2} Y_i' z_i^2 \right) \right)^2 - \left(\sum_{i=1}^n Y_i'(z_{i+1} - z_i) \sum_{i=1}^n \left(\frac{1}{3} Y_i' z_{i+1}^3 - \frac{1}{3} Y_i' z_i^3 \right) \right) \quad (\text{S4})$$

For the bending actuator shown in Figure 4, the preferred axis of bending is about the y -axis, and bending about the x -axis as well as strain along the y -axis may be neglected. Thus, a plane strain condition on the x - z plane may be considered by replacing Young's modulus Y and Poisson's ratio ν by their equivalent plane strain values as follows:

$$Y' = \frac{Y}{1-\nu^2}, \nu' = \frac{\nu}{1-\nu} \quad (\text{S5})$$

The blocking force (reaction force generated when the actuator's tip is held fixed) is determined by setting $u_p = 0$ in equation (S1) and solving for P :

$$P_{Block} = -\frac{\beta}{\alpha} V^2 \quad (\text{S6})$$

S1.2 VSDEA load-displacement behavior and normal and shear stress distribution

In this section, expressions for force-displacement behavior of the VSDEA and its normal and shear stress distribution are derived. Such expressions are useful to predict the bending stiffness of the VSDEA in the fully chucked state, and estimate the shear stress that must be supported by the chucked interfaces to support a given load. For this analysis, the VSDEA is configured as shown in Figure 4, such that the top end is fixed and its length is oriented parallel to the direction of gravity. It is assumed that (i) static equilibrium holds, (ii) Euler-Bernoulli beam theory holds, and (iii) no slip occurs at all bonding surfaces. Considering a VSDEA loaded in the configuration shown in Figure 4c, the stiffness is predicted in the deformed shape as shown in Figure S2a, with radius of curvature R and subtended angle θ . The actuator is fixed at S ($\alpha = \theta$) with a concentrated load due to weight of a supported object acting at the free end ($\alpha = 0$).

The VSDEA consists of n lamina, as shown in Figure S2b. An expression for the radius η of the neutral axis is a function of the equivalent plane strain Young's modulus Y'_i and laminar radii r_i :

$$\eta = \frac{\sum_i^n Y'_i (r_{i+1}^2 - r_i^2)}{2 \sum_i^n Y'_i (r_{i+1} - r_i)} \quad (\text{S7})$$

Bending stiffness of the actuator D is derived as follows, where b denotes the width of the VSDEA and I is the area moment of inertia about the neutral axis:

$$D = \sum_i^n Y'_i I_i = \frac{b}{3} \sum_i^n Y'_i \left\{ (r_{i+1} - \eta)^3 - (r_i - \eta)^3 \right\} \quad (\text{S8})$$

The strain energy method is used to predict displacement of curved VSDEA due to the weight of an object. A bending moment M occurs due to radial force P and tangential force T at angular position α as defined in Figure S2a:

$$M = TR(1 - \cos \alpha) - PR \sin \alpha \quad (\text{S9})$$

Strain energy U due to bending moment M is then expressed as:

$$U = \int_0^\theta \frac{M^2}{2D} R d\alpha \quad (\text{S10})$$

Displacements at the tip in the radial and tangential direction caused by T and P are derived as follows:

$$u_v = \frac{\partial U}{\partial T} = \frac{R^3}{2D} \left\{ T \left(3\theta - 4 \sin \theta + \frac{1}{2} \sin 2\theta \right) + P \left(2 \cos \theta - \frac{1}{2} \cos 2\theta - \frac{3}{2} \right) \right\} \quad (\text{S11})$$

$$u_p = \frac{\partial U}{\partial P} = \frac{R^3}{2D} \left\{ T \left(2 \cos \theta - \frac{1}{2} \cos 2\theta - \frac{3}{2} \right) + P \left(\theta - \frac{1}{2} \sin 2\theta \right) \right\} \quad (\text{S12})$$

Since VSDEA is composed of a number of DEA units bonded by electrostatic chucking at various interfaces, knowing the shear stress distribution along the thickness and angular position of the actuator is important to predict the maximum weight that can be supported without exceeding the shear strength of the interfaces. Consider the balance of forces on an arbitrary differential element of the VSDEA with length dx , as shown in Figure S3. Since static equilibrium is assumed, the sum of the force F_1 due to shear stress τ at an arbitrary surface $r = r_c$ and force F_2 due to incremental change in the normal stress σ must vanish:

$$F_1 + F_2 = 0 \quad (\text{S13})$$

$$F_1 = \tau b dx \quad (\text{S14})$$

$$F_2 = \int_{r_1}^{r_c} d\sigma dA \quad (\text{S15})$$

By rearranging equations (S7) - (S9) an expression for the shear stress is given by:

$$\tau = \frac{1}{b} \int_{r_1}^{r_c} \frac{d\sigma}{dx} dA \quad (\text{S16})$$

Axial stress distribution in the radial direction is then expressed as:

$$\begin{aligned} \sigma &= \frac{Y_i M}{D} (r - \eta) \\ &= \frac{Y_i \{ TR(1 - \cos \alpha) - PR \sin \alpha \}}{D} (r - \eta) \end{aligned} \quad (\text{S17})$$

The differential length dx is:

$$dx = R d\alpha \quad (\text{S18})$$

The x differential of stress σ is derived as:

$$\frac{d\sigma}{dx} = \frac{1}{R} \frac{d\sigma}{d\alpha} = \frac{T \sin \alpha - P \cos \alpha}{D} Y_i (r - \eta) \quad (\text{S19})$$

Substituting equation (S19) into (S16) yields:

$$\begin{aligned} \tau(r, \alpha) &= \frac{T \sin \alpha - P \cos \alpha}{D} \int_{r_1}^{r_c} Y_i (r - \eta) dr \\ &= \frac{T \sin \alpha - P \cos \alpha}{2D} \left\{ \sum_{i=1}^{k-1} Y_i (r_{i+1}'^2 - r_i'^2) + Y_k (r_c'^2 - r_k'^2) \right\} \end{aligned} \quad (\text{S20})$$

Where r' denotes the distance from the neutral axis:

$$r' = r - \eta \quad (\text{S21})$$

S1.3 Shear strength of chucked interfaces

Equations (S7)-(S21) assumes that no slip occurs between each DEA unit, while in reality the shear strength of the interface is finite and depends on the degree of electrostatic chucking. To realize the bending stiffness predicted in the former section, it is necessary for the chucking surfaces of the DEA units to exhibit shear strength greater than the shear stress produced by a given load (as predicted in Equation (S20)). The shear strength can be described by the following expression utilizing Coulomb force. In this case, the Johnsen-Rahbek effect is ignored for the sake of simplicity[ref. 1, 2].

$$\tau_r = \frac{\mu \epsilon_0 V^2}{2} \left(\frac{\epsilon_r}{t_D + \epsilon_r (\delta + t_{CL})} \right)^2 \quad (\text{S22})$$

Here, μ is the coefficient of static friction, t_D is the dielectric layer thickness, t_{CL} is the thickness of the contact region, and δ is the physical gap between the interfacial surfaces.

Supplemental References

1. C. J. Fitch, IBM Journal of Research and Development, 1957, 1, 1.
2. M. R. Sogard, A. R. Mikkelsen, M. Nataraju, K. T. Turner, R. L. Engelstad, Journal of Vacuum Science and Technology B, 2007, 25, 2155.

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