

## Phase behaviour of hard board-like particles

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### Supplementary Information

Table 1. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=1$ . Absolute errors are lower than  $5 \cdot 10^{-3}$

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.150	0.202	0.010	0.010	0.010	0.010
I	0.200	0.230	0.020	0.020	0.020	0.011
I	0.250	0.252	0.050	0.030	0.030	0.011
I	0.270	0.262	0.030	0.030	0.030	0.010
I	0.280	0.268	0.020	0.030	0.010	0.010
$N^+$	0.300	0.302	0.830	0.220	0.220	0.001
$N^+$	0.310	0.308	0.850	0.220	0.220	0.010
$N^+$	0.320	0.315	0.880	0.220	0.220	0.010
$N^+$	0.340	0.327	0.901	0.235	0.235	0.020
$N^+$	0.350	0.331	0.905	0.237	0.237	0.010
$N^+$	0.370	0.342	0.920	0.250	0.250	0.010
$N^+$	0.390	0.352	0.935	0.248	0.248	0.010
$N^+$	0.410	0.361	0.945	0.250	0.250	0.010
$N^+$	0.440	0.377	0.960	0.255	0.255	0.020
$Sm^+$	0.480	0.402	0.972	0.257	0.257	0.015
$Sm^+$	0.490	0.408	0.974	0.260	0.255	0.015
$Sm^+$	0.500	0.412	0.975	0.260	0.260	0.015
$Sm^+$	0.530	0.434	0.978	0.260	0.260	0.015
$Sm^+$	0.550	0.443	0.981	0.261	0.261	0.017
$Sm^+$	0.650	0.482	0.987	0.265	0.265	0.015
$Sm^+$	0.750	0.525	0.991	0.260	0.257	0.015
$Sm^+$	0.800	0.551	0.993	0.260	0.260	0.013
$Sm^+$	0.850	0.566	0.993	0.260	0.260	0.013
$Sm^+$	0.900	0.595	0.996	0.270	0.270	0.028
$Sm^+/Col^+$	1.000	0.620	0.997	0.254	0.254	0.004
$Col^+$	1.500	0.711	0.999	0.991	0.991	0.988
$Col^+$	2.000	0.765	1.000	0.995	0.995	0.993
$Col^+$	2.500	0.803	1.000	0.996	0.996	0.995
$Col^+$	3.000	0.830	1.000	0.998	0.998	0.997
$Col^+$	4.000	0.865	1.000	0.998	0.998	0.998
$Col^+$	5.000	0.890	0.999	0.999	0.999	0.999
K	7.500	0.924	1.000	1.000	1.000	0.998
K	10.000	0.941	1.000	1.000	1.000	0.998

**Table 2. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=2$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.070	0.207	0.001	0.001	0.001	0.001
I	0.100	0.238	0.020	0.020	0.020	0.020
I	0.160	0.287	0.040	0.025	0.022	0.010
I	0.175	0.300	0.050	0.020	0.020	0.020
I	0.200	0.319	0.087	0.023	0.040	0.010
I	0.215	0.332	0.084	0.051	0.032	0.005
I	0.220	0.336	0.100	0.050	0.050	0.000
$N^+$	0.221	0.357	0.760	0.215	0.185	0.010
$N^+$	0.225	0.366	0.840	0.235	0.210	0.012
$N^+$	0.230	0.372	0.850	0.240	0.220	0.010
$Sm^+$	0.235	0.388	0.865	0.223	0.223	0.010
$Sm^+$	0.240	0.391	0.530	0.220	0.210	0.001
$Sm^+$	0.300	0.458	0.950	0.250	0.250	0.005
$Sm^+$	0.350	0.492	0.938	0.250	0.240	0.005
$Sm^+$	0.400	0.533	0.988	0.269	0.269	0.016
$Sm^+$	0.450	0.567	0.990	0.270	0.270	0.031
$Sm^+$	0.500	0.599	0.992	0.281	0.281	0.036
$Sm^+$	0.550	0.643	0.995	0.972	0.972	0.965
$Col^-$	0.600	0.662	0.995	0.987	0.985	0.982
$Col^-$	0.700	0.693	0.997	0.991	0.989	0.988
$Col^+$	0.800	0.720	0.998	0.993	0.992	0.990
$Col^+$	0.900	0.747	0.998	0.994	0.993	0.992
$Col^+$	1.300	0.809	0.999	0.998	0.997	0.997
$Col^+$	1.500	0.828	0.999	0.998	0.998	0.998
$Col^+$	2.000	0.865	1.000	0.999	0.999	0.999
$Col^+$	3.000	0.906	1.000	0.999	0.999	0.999
K	5.000	0.942	1.000	1.000	1.000	1.000
K	10.000	0.971	1.000	1.000	1.000	1.000

**Table 3. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=3$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.040	0.193	0.025	0.028	0.020	0.005
I	0.050	0.212	0.030	0.028	0.018	0.010
I	0.100	0.280	0.040	0.040	0.022	0.015
I	0.150	0.340	0.060	0.050	0.040	0.015
I	0.160	0.351	0.070	0.065	0.030	0.015
I	0.162	0.354	0.050	0.050	0.030	0.015
I	0.163	0.355	0.070	0.070	0.070	0.030
Sm <sup>+</sup>	0.164	0.399	0.802	0.220	0.225	0.020
Sm <sup>+</sup>	0.165	0.395	0.785	0.220	0.220	0.020
Sm <sup>+</sup>	0.167	0.410	0.850	0.230	0.250	0.010
Sm <sup>+</sup>	0.170	0.419	0.930	0.260	0.265	0.015
Sm <sup>+</sup>	0.200	0.461	0.965	0.260	0.260	0.015
Sm <sup>+</sup>	0.250	0.518	0.980	0.280	0.280	0.000
Sm <sup>+</sup>	0.300	0.571	0.989	0.295	0.295	0.006
Sm <sup>+</sup>	0.330	0.598	0.990	0.357	0.357	0.137
Col <sup>l</sup>	0.380	0.645	0.992	0.957	0.954	0.927
Col <sup>l</sup>	0.400	0.657	0.993	0.977	0.974	0.979
Col <sup>l</sup>	0.450	0.683	0.994	0.988	0.986	0.954
K	0.500	0.706	0.995	0.990	0.988	0.961
K	0.600	0.723	0.990	0.993	0.985	0.989

**Table 4. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=3.5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.110	0.315	0.000	0.000	0.000	0.000
I	0.130	0.342	0.050	0.090	0.010	0.010
I	0.140	0.356	0.060	0.060	0.030	0.030
I	0.143	0.363	0.050	0.050	0.030	0.030
I	0.145	0.366	0.040	0.050	0.038	0.030
I	0.147	0.370	0.050	0.060	0.030	0.030
$N^-$	0.150	0.390	0.200	0.490	0.100	0.280
$N^-$	0.152	0.395	0.230	0.670	0.150	0.400
$Sm^+$	0.155	0.436	0.900	0.270	0.245	0.020
$Sm^+$	0.160	0.442	0.934	0.280	0.260	0.030
$Sm^+$	0.170	0.462	0.958	0.273	0.273	0.020
$Sm^+$	0.200	0.498	0.975	0.255	0.255	0.010
$Sm^+$	0.240	0.556	0.984	0.754	0.312	0.072
$Sm^+$	0.250	0.566	0.984	0.340	0.340	0.118
$Sm^+$	0.260	0.578	0.987	0.270	0.270	0.016
$Col^-$	0.280	0.612	0.988	0.973	0.969	0.965
$Col^-$	0.300	0.623	0.989	0.982	0.979	0.978
$Col^-$	0.310	0.634	0.990	0.983	0.979	0.977
$Col^-$	0.320	0.645	0.991	0.987	0.983	0.982
$Col^-$	0.330	0.653	0.992	0.989	0.985	0.984
$Col^-$	0.350	0.667	0.992	0.992	0.988	0.989
$Col^-$	0.360	0.672	0.993	0.992	0.988	0.989
$Col^-/K$	0.370	0.680	0.994	0.993	0.989	0.990
K	0.380	0.686	0.994	0.994	0.990	0.991
K	0.390	0.690	0.994	0.996	0.990	0.991
K	0.400	0.695	0.994	0.994	0.990	0.992
K	0.45	0.717	0.995	0.992	0.993	0.993

**Table 5. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=4$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.100	0.322	0.050	0.050	0.050	0.000
$N^-$	0.120	0.372	0.250	0.820	0.210	0.550
$N^-$	0.130	0.394	0.260	0.900	0.250	0.630
$Sm^+$	0.140	0.442	0.840	0.280	0.220	0.040
$Sm^+$	0.160	0.479	0.955	0.254	0.254	0.001
$Sm^+$	0.170	0.495	0.953	0.270	0.260	0.030
$Sm^+$	0.190	0.530	0.963	0.270	0.268	0.030
$Sm^+$	0.200	0.545	0.974	0.367	0.367	0.167
$Sm^+$	0.210	0.558	0.966	0.287	0.261	0.048
$Sm_B$	0.220	0.570	0.978	0.630	0.630	0.490
$Col^-$	0.230	0.593	0.980	0.790	0.780	0.716
$Col^-$	0.250	0.616	0.985	0.984	0.976	0.977
$Col^-$	0.260	0.626	0.988	0.982	0.975	0.974
$Col^-$	0.270	0.635	0.990	0.980	0.975	0.973
$Col^-$	0.275	0.639	0.990	0.979	0.975	0.974
$Col^-$	0.280	0.643	0.988	0.981	0.974	0.974
$Col^-/K$	0.290	0.655	0.992	0.985	0.979	0.978
$Col^-$	0.305	0.665	0.992	0.986	0.981	0.979
K	0.350	0.696	0.993	0.995	0.990	0.992
K	0.400	0.722	0.995	0.996	0.996	0.994
K	0.600	0.793	0.997	0.998	0.996	0.997

**Table 6. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=4.5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.030	0.201	0.020	0.020	0.010	0.020
I	0.050	0.251	0.040	0.042	0.033	0.013
I	0.075	0.300	0.030	0.050	0.050	0.050
I	0.090	0.328	0.050	0.050	0.050	0.020
$N^-$	0.100	0.362	0.240	0.850	0.230	0.010
$N^-$	0.110	0.385	0.251	0.910	0.241	0.025
$N^-$	0.120	0.406	0.254	0.941	0.254	0.018
$N^-$	0.125	0.416	0.261	0.948	0.261	0.017
$Sm^+$	0.130	0.455	0.863	0.290	0.217	0.030
$Sm^+$	0.140	0.477	0.872	0.329	0.254	0.076
$Sm^+$	0.150	0.498	0.903	0.290	0.250	0.065
$Sm^+$	0.160	0.518	0.954	0.363	0.350	0.100
$Sm^+$	0.170	0.534	0.894	0.452	0.363	0.125
$Sm_B$	0.180	0.551	0.819	0.727	0.561	0.561
$Sm_B$	0.190	0.573	0.829	0.910	0.753	0.753
$Col^-$	0.200	0.591	0.977	0.984	0.969	0.975
$Col^-$	0.210	0.602	0.985	0.975	0.985	0.977
$Col^-$	0.220	0.615	0.982	0.987	0.975	0.977
$Col^-$	0.230	0.624	0.985	0.987	0.979	0.982
$Col^-$	0.240	0.634	0.988	0.989	0.982	0.984
$Col^-$	0.250	0.642	0.989	0.988	0.983	0.983
$Col^-$	0.280	0.669	0.991	0.991	0.987	0.988
K	0.320	0.700	0.993	0.996	0.991	0.991
K	0.350	0.717	0.994	0.996	0.992	0.994
K	0.400	0.743	0.995	0.994	0.995	0.994

**Table 7. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.060	0.285	0.020	0.020	0.020	0.020
I	0.070	0.307	0.050	0.060	0.030	0.005
I	0.075	0.318	0.050	0.100	0.040	0.010
N-	0.080	0.343	0.220	0.810	0.210	0.002
N-	0.120	0.432	0.267	0.955	0.257	0.023
Sm <sup>-</sup>	0.130	0.456	0.272	0.974	0.270	0.028
Sm <sup>-</sup>	0.140	0.484	0.267	0.977	0.260	0.019
Sm <sup>-</sup>	0.150	0.520	0.329	0.925	0.266	0.094
Sm <sup>-</sup>	0.160	0.543	0.379	0.927	0.302	0.142
Col <sup>-</sup>	0.170	0.570	0.905	0.952	0.867	0.862
Col <sup>-</sup>	0.180	0.590	0.972	0.985	0.966	0.963
Col <sup>-</sup>	0.190	0.602	0.979	0.982	0.966	0.969
Col <sup>-</sup>	0.200	0.618	0.984	0.987	0.978	0.978
Col <sup>-</sup>	0.220	0.639	0.987	0.990	0.983	0.983
Col <sup>-</sup>	0.225	0.643	0.988	0.990	0.984	0.984
Col <sup>-</sup>	0.255	0.671	0.989	0.995	0.987	0.985
K	0.300	0.709	0.993	0.996	0.991	0.994
K	0.400	0.761	0.996	0.998	0.995	0.997

**Table 8. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=5.5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.030	0.217	0.001	0.001	0.001	0.001
I	0.050	0.275	0.050	0.078	0.048	0.030
I	0.060	0.300	0.020	0.020	0.020	0.020
I	0.065	0.313	0.050	0.100	0.040	0.010
$N^-$	0.070	0.336	0.210	0.800	0.210	0.010
$N^-$	0.080	0.365	0.240	0.900	0.240	0.001
$N^-$	0.100	0.414	0.270	0.960	0.270	0.020
$N^-$	0.110	0.435	0.265	0.962	0.260	0.030
$Sm^-$	0.120	0.460	0.265	0.977	0.265	0.021
$Sm^-$	0.130	0.493	0.268	0.990	0.265	0.020
$Sm^-$	0.140	0.524	0.270	0.988	0.262	0.023
$Sm^-$	0.150	0.548	0.267	0.992	0.267	0.020
$Col^-$	0.155	0.573	0.958	0.982	0.949	0.941
$Col^-$	0.160	0.584	0.974	0.983	0.967	0.965
$Col^-$	0.180	0.613	0.981	0.987	0.976	0.976
$Col^-$	0.200	0.640	0.987	0.990	0.984	0.984
$Col^-$	0.230	0.673	0.990	0.994	0.988	0.987
$Col^-$	0.250	0.690	0.992	0.996	0.990	0.989
K	0.280	0.714	0.993	0.997	0.992	0.995
K	0.300	0.725	0.994	0.997	0.993	0.995



**Table 9. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=7$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.020	0.199	0.001	0.000	0.001	0.001
I	0.030	0.242	0.025	0.040	0.025	0.010
I	0.035	0.263	0.040	0.070	0.038	0.010
I	0.038	0.274	0.040	0.060	0.040	0.010
I	0.041	0.285	0.040	0.100	0.040	0.002
$N^-$	0.050	0.330	0.210	0.880	0.210	0.000
$N^-$	0.080	0.425	0.262	0.965	0.257	0.010
$N^-$	0.090	0.452	0.260	0.980	0.270	0.010
$Sm^-$	0.100	0.496	0.269	0.983	0.263	0.027
$Sm^-$	0.110	0.524	0.258	0.988	0.258	0.019
$Sm^-$	0.120	0.551	0.276	0.992	0.276	0.040
Col	0.135	0.598	0.951	0.989	0.949	0.936
Col	0.150	0.632	0.984	0.993	0.982	0.980
Col	0.170	0.657	0.987	0.993	0.985	0.983
Col	0.180	0.670	0.989	0.995	0.989	0.986
K	0.200	0.695	0.991	0.996	0.991	0.989
K	0.210	0.705	0.992	0.997	0.997	0.989

**Table 10.** Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=8$ . Absolute errors are lower than  $5 \cdot 10^{-3}$

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.020	0.212	0.050	0.050	0.050	0.000
I	0.025	0.237	0.030	0.020	0.030	0.020
I	0.030	0.262	0.035	0.050	0.030	0.010
N-	0.035	0.295	0.210	0.750	0.210	0.020
N-	0.050	0.358	0.280	0.920	0.280	0.050
N-	0.060	0.392	0.250	0.950	0.250	0.010
N-	0.075	0.442	0.272	0.975	0.272	0.040
Col	0.080	0.486	0.255	0.980	0.255	0.020
Col	0.085	0.507	0.290	0.980	0.290	0.050
Col	0.090	0.522	0.269	0.985	0.269	0.028
Col	0.100	0.553	0.560	0.987	0.560	0.418
Col	0.110	0.574	0.560	0.990	0.560	0.420
Col	0.120	0.597	0.560	0.990	0.560	0.420
Col	0.125	0.607	0.560	0.990	0.560	0.430
Col	0.130	0.616	0.560	0.990	0.560	0.430
Col	0.150	0.662	0.988	0.996	0.987	0.985
K	0.170	0.689	0.991	0.998	0.990	0.987
K	0.190	0.710	0.992	0.998	0.992	0.990
K	0.210	0.731	0.993	0.998	0.993	0.991

**Table 11.** Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=9$  and  $W^*=9$ . Absolute errors are lower than  $5 \cdot 10^{-3}$

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.010	0.170	0.000	0.000	0.000	0.000
I	0.015	0.193	0.025	0.040	0.025	0.020
I	0.020	0.224	0.050	0.040	0.030	0.000
I	0.025	0.254	0.050	0.100	0.050	0.010
$N^-$	0.027	0.272	0.198	0.710	0.190	0.001
$N^-$	0.028	0.280	0.200	0.750	0.200	0.001
$N^-$	0.030	0.292	0.210	0.820	0.210	0.000
$N^-$	0.040	0.340	0.240	0.930	0.240	0.020
$N^-$	0.050	0.382	0.250	0.950	0.250	0.010
$N^-$	0.060	0.420	0.250	0.975	0.250	0.001
$N^-$	0.065	0.445	0.254	0.973	0.250	0.001
$N^-$	0.070	0.466	0.254	0.982	0.254	0.001
$N^-$	0.072	0.473	0.255	0.981	0.255	0.010
$N^-$	0.073	0.478	0.250	0.983	0.248	0.005
$N^-$	0.074	0.481	0.256	0.982	0.256	0.012
Col	0.075	0.515	0.650	0.978	0.650	0.540
Col	0.085	0.550	0.648	0.970	0.648	0.540
Col	0.110	0.600	0.700	0.990	0.700	0.600
Col	0.115	0.611	0.692	0.995	0.692	0.588
Col	0.120	0.633	0.983	0.993	0.984	0.980
Col	0.125	0.645	0.987	0.994	0.986	0.983
Col	0.130	0.653	0.987	0.994	0.987	0.984
K	0.150	0.687	0.990	0.998	0.990	0.987
K	0.170	0.712	0.992	0.998	0.992	0.990

**Table 12. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=1$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.100	0.179	0.036	0.022	0.027	-
I	0.130	0.203	0.048	0.023	0.024	-
$N^+$	0.140	0.235	0.758	0.202	0.202	0.014
$N^+$	0.150	0.246	0.794	0.212	0.212	0.014
$N^+$	0.200	0.290	0.918	0.244	0.244	0.015
$N^+$	0.300	0.366	0.976	0.261	0.261	0.018
$N^+$	0.350	0.402	0.986	0.262	0.262	0.017
$Sm^+$	0.370	0.413	0.988	0.263	0.263	0.018
$Sm^+$	0.400	0.434	0.990	0.265	0.265	0.014
$Sm^+$	0.500	0.489	0.994	0.259	0.259	0.013
$Sm^+$	0.530	0.533	0.997	0.260	0.260	0.016
$Col^+$	0.550	0.555	0.998	0.266	0.266	0.024
$Col^+$	0.600	0.578	0.998	0.275	0.275	0.032
$Col^+$	0.750	0.630	0.999	0.968	0.968	0.957
$Col^+$	1.000	0.692	0.999	0.989	0.898	0.986
$Col^+$	1.400	0.759	0.999	0.995	0.995	0.993
K	1.500	0.772	0.999	0.994	0.994	0.994
K	2.000	0.826	0.999	0.997	0.997	0.996

**Table 13.** Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=2.5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.070	0.243	0.022	0.022	0.014	0.001
I	0.080	0.260	0.085	0.019	0.007	0.002
I	0.090	0.277	0.154	0.079	0.039	0.011
$N^+$	0.095	0.294	0.586	0.139	0.135	0.001
$N^+$	0.100	0.308	0.755	0.231	0.183	0.001
$N^+$	0.120	0.349	0.907	0.251	0.235	0.010
$N^+$	0.130	0.370	0.932	0.250	0.241	0.009
$Sm^+$	0.140	0.400	0.966	0.256	0.251	0.011
$Sm^+$	0.150	0.416	0.971	0.256	0.255	0.012
$Sm^+$	0.200	0.486	0.987	0.267	0.264	0.010
$Sm^+$	0.230	0.523	0.990	0.257	0.256	0.005
$Sm^+$	0.250	0.548	0.991	0.257	0.257	0.019
$Col^+$	0.255	0.588	0.995	0.974	0.972	0.978
$Col^+$	0.270	0.605	0.996	0.985	0.983	0.980
$Col^+$	0.300	0.629	0.997	0.989	0.988	0.985
$Col^+$	0.400	0.693	0.998	0.994	0.991	0.988
$Col^+$	0.500	0.735	0.999	0.996	0.995	0.991
K	0.600	0.768	0.999	0.997	0.996	0.993

**Table 14.** Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=3$ . Absolute errors are lower than  $5 \cdot 10^{-3}$

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.070	0.266	0.0409	0.037	0.0187	0.001
I	0.080	0.285	0.098	0.044	0.0091	0.020
I	0.082	0.289	0.022	0.012	0.034	0.010
$N^+$	0.085	0.308	0.680	0.237	0.161	0.004
$N^+$	0.090	0.314	0.699	0.247	0.172	0.003
$N^+$	0.095	0.327	0.782	0.240	0.195	0.007
$N^+$	0.100	0.348	0.861	0.248	0.218	0.009
$N^+$	0.105	0.360	0.911	0.250	0.231	0.010
$Sm^+$	0.110	0.389	0.954	0.259	0.250	0.012
$Sm^+$	0.150	0.466	0.980	0.264	0.261	0.015
$Sm^+$	0.200	0.537	0.989	0.276	0.274	0.019
$Sm^+$	0.220	0.557	0.975	0.283	0.274	0.045
$Col^+$	0.230	0.607	0.995	0.989	0.984	0.985
$Col^+$	0.250	0.629	0.996	0.991	0.986	0.988
$Col^+$	0.300	0.666	0.997	0.994	0.992	0.991
$Col^+$	0.400	0.725	0.998	0.997	0.995	0.997
$Col^+$	0.430	0.741	0.998	0.997	0.996	0.991
$Col^+$	0.450	0.746	0.998	0.997	0.996	0.992
K	0.500	0.775	0.999	0.998	0.997	0.993

**Table 15. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=3.46$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.070	0.285	0.021	0.017	0.03	0.001
I	0.080	0.306	0.084	0.102	0.021	0.006
I	0.082	0.313	0.214	0.276	0.041	0.010
$N^+$	0.083	0.319	0.508	0.283	0.123	0.068
$N^+$	0.084	0.327	0.669	0.298	0.182	0.048
$N^+$	0.085	0.330	0.677	0.268	0.169	0.060
$N^+$	0.090	0.354	0.864	0.284	0.242	0.051
$Sm^+$	0.095	0.381	0.922	0.263	0.247	0.027
$Sm^+$	0.100	0.394	0.951	0.264	0.255	0.011
$Sm^+$	0.120	0.442	0.969	0.269	0.263	0.016
$Sm^+$	0.150	0.499	0.983	0.266	0.263	0.011
$Sm^+$	0.170	0.534	0.987	0.270	0.268	0.023
$Col^+$	0.190	0.594	0.993	0.984	0.979	0.977
$Col^+$	0.200	0.609	0.994	0.991	0.987	0.987
$Col^+$	0.300	0.697	0.997	0.996	0.994	0.994
$Col^+$	0.400	0.752	0.998	0.998	0.997	0.997
$Col^+$	0.450	0.772	0.999	0.998	0.997	0.996
K	0.500	0.788	0.999	0.998	0.997	0.996

**Table 16. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=3.6$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.060	0.270	0.020	0.01	0.018	0.007
I	0.070	0.291	0.083	0.104	0.028	0.012
I	0.075	0.304	0.20	0.131	0.025	0.013
$N^+$	0.080	0.321	0.212	0.571	0.113	0.01
$N^+$	0.085	0.336	0.244	0.801	0.187	0.013
$N^+$	0.088	0.344	0.266	0.774	0.196	0.012
$Sm^+$	0.089	0.363	0.850	0.257	0.224	0.015
$Sm^+$	0.090	0.374	0.894	0.253	0.237	0.018
$Sm^+$	0.100	0.408	0.960	0.261	0.254	0.011
$Sm^+$	0.150	0.508	0.985	0.276	0.273	0.017
$Sm^+$	0.160	0.533	0.987	0.281	0.278	0.033
$Col^-$	0.170	0.560	0.988	0.911	0.909	0.878
$Col^-$	0.200	0.603	0.991	0.980	0.978	0.973
$Col^+$	0.210	0.627	0.995	0.992	0.989	0.989
$Col^+$	0.250	0.670	0.997	0.995	0.993	0.993
$Col^+$	0.300	0.703	0.997	0.997	0.995	0.995
$Col^+$	0.400	0.761	0.998	0.998	0.997	0.997
$Col^+$	0.450	0.777	0.999	0.997	0.997	0.997
K	0.500	0.795	0.999	0.998	0.998	0.997



**Table 17. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=3.8$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.040	0.227	0.030	0.033	0.017	0.010
I	0.060	0.279	0.093	0.092	0.0335	0.011
I	0.066	0.293	0.129	0.027	0.32	0.011
$N^-$	0.067	0.297	0.200	0.499	0.103	0.0165
$N^-$	0.068	0.304	0.214	0.583	0.137	0.019
$N^-$	0.070	0.310	0.249	0.692	0.174	0.037
$N^-$	0.080	0.337	0.272	0.790	0.205	0.050
$N^-$	0.085	0.355	0.264	0.861	0.234	0.013
$Sm^+$	0.086	0.372	0.883	0.256	0.240	0.014
$Sm^+$	0.087	0.375	0.881	0.263	0.234	0.033
$Sm^+$	0.088	0.379	0.902	0.266	0.246	0.028
$Sm^+$	0.090	0.396	0.949	0.268	0.257	0.019
$Sm^+$	0.100	0.423	0.964	0.273	0.266	0.023
$Sm^+$	0.150	0.531	0.986	0.264	0.261	0.012
$Col^-$	0.160	0.561	0.950	0.939	0.973	0.934
$Col^-$	0.180	0.598	0.991	0.978	0.974	0.971
$Col^-$	0.190	0.606	0.990	0.979	0.977	0.973
$Col^-$	0.200	0.625	0.993	0.988	0.984	0.982
$Col^+$	0.230	0.662	0.996	0.995	0.992	0.994
$Col^+$	0.300	0.717	0.997	0.997	0.995	0.996
$Col^+$	0.400	0.767	0.998	0.998	0.997	0.997
$Col^+$	0.450	0.789	0.999	0.999	0.999	0.998
K	0.470	0.793	0.999	0.999	0.998	0.998
K	0.500	0.803	0.999	0.999	0.999	0.998

**Table 18. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=4$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.050	0.261	0.028	0.037	0.020	0.001
I	0.060	0.287	0.092	0.123	0.030	0.002
$N^-$	0.061	0.293	0.204	0.564	0.125	0.021
$N^-$	0.065	0.309	0.174	0.506	0.031	0.031
$N^-$	0.070	0.319	0.245	0.745	0.181	0.027
$N^-$	0.080	0.348	0.254	0.853	0.220	0.022
$N^-$	0.084	0.363	0.263	0.898	0.239	0.019
$Sm^+$	0.085	0.391	0.927	0.268	0.251	0.025
$Sm^+$	0.090	0.412	0.959	0.276	0.266	0.041
$Sm^+$	0.100	0.436	0.9655	0.268	0.018	0.020
$Sm^+$	0.120	0.481	0.979	0.271	0.017	0.016
$Sm^+$	0.130	0.503	0.981	0.282	0.278	0.025
$Sm^+$	0.140	0.527	0.984	0.283	0.280	0.034
$Col^-$	0.150	0.560	0.987	0.949	0.940	0.936
$Col^-$	0.175	0.600	0.991	0.982	0.977	0.976
K	0.185	0.617	0.992	0.990	0.985	0.986
K	0.200	0.641	0.993	0.993	0.988	0.990

**Table 19. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=4.5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.020	0.179	0.025	0.028	0.020	0.001
I	0.030	0.217	0.032	0.047	0.022	0.010
I	0.045	0.266	0.044	0.057	0.030	0.005
$N^-$	0.050	0.287	0.204	0.656	0.160	0.012
$N^-$	0.055	0.309	0.243	0.81	0.212	0.015
$N^-$	0.060	0.321	0.234	0.823	0.208	0.019
$N^-$	0.070	0.351	0.255	0.900	0.237	0.018
$N^-$	0.074	0.365	0.256	0.935	0.244	0.012
$Sm^+$	0.075	0.388	0.917	0.285	0.260	0.037
$Sm^+$	0.080	0.408	0.938	0.289	0.268	0.041
$Sm^+$	0.100	0.462	0.967	0.275	0.266	0.012
$Sm^+$	0.110	0.493	0.976	0.277	0.272	0.021
$Col^-$	0.120	0.530	0.981	0.964	0.952	0.934
$Col^-$	0.140	0.572	0.986	0.973	0.965	0.977
$Col^-$	0.150	0.59	0.988	0.974	0.973	0.973
$Col^-$	0.160	0.609	0.991	0.986	0.981	0.986
K	0.165	0.617	0.992	0.990	0.984	0.985
K	0.200	0.665	0.994	0.996	0.991	0.993

**Table 20. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.030	0.229	0.055	0.064	0.024	0.001
I	0.035	0.248	0.043	0.037	0.045	0.010
$N^-$	0.040	0.269	0.185	0.559	0.140	0.020
$N^-$	0.050	0.314	0.242	0.860	0.226	0.019
$N^-$	0.060	0.347	0.257	0.921	0.249	0.024
$N^-$	0.070	0.376	0.260	0.936	0.248	0.018
$N^-$	0.080	0.405	0.265	0.929	0.239	0.021
$Sm^+$	0.085	0.440	0.767	0.591	0.430	0.027
$Sm^+$	0.090	0.464	0.891	0.567	0.487	0.034
$Sm^+$	0.100	0.496	0.905	0.585	0.511	0.061
$Col^-$	0.110	0.532	0.956	0.944	0.924	0.936
$Col^-$	0.120	0.553	0.962	0.963	0.950	0.953
$Col^-$	0.130	0.582	0.986	0.984	0.975	0.979
$Col^-$	0.140	0.602	0.989	0.985	0.979	0.983
K	0.150	0.626	0.992	0.994	0.988	0.988
K	0.200	0.693	0.995	0.997	0.993	0.993

**Table 21. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=5.5$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.030	0.242	0.048	0.098	0.034	0.001
I	0.035	0.271	0.203	0.682	0.175	0.002
$N^-$	0.040	0.297	0.235	0.832	0.220	0.019
$N^-$	0.050	0.333	0.251	0.910	0.243	0.018
$N^-$	0.070	0.397	0.262	0.953	0.256	0.014
$N^-$	0.075	0.412	0.260	0.965	0.256	0.019
$N^-$	0.080	0.434	0.261	0.969	0.254	0.015
$Sm^+$	0.083	0.466	0.543	0.466	0.149	0.04
$Sm^+$	0.085	0.473	0.520	0.480	0.147	0.033
$Sm^+$	0.088	0.474	0.507	0.473	0.139	0.067
$Col^-$	0.090	0.500	0.873	0.57	0.845	0.825
$Col^-$	0.100	0.538	0.969	0.980	0.956	0.956
$Col^-$	0.130	0.606	0.987	0.989	0.980	0.981
$Col^-$	0.14	0.624	0.990	0.990	0.984	0.986
K	0.150	0.642	0.991	0.995	0.988	0.987

**Table 22. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=6$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.020	0.204	0.036	0.047	0.024	0.001
I	0.025	0.230	0.039	0.066	0.032	0.002
I	0.028	0.244	0.09	0.215	0.055	0.011
$N^-$	0.029	0.253	0.155	0.462	0.123	0.017
$N^-$	0.030	0.264	0.220	0.774	0.20	0.013
$N^-$	0.040	0.313	0.238	0.872	0.231	0.018
$N^-$	0.060	0.387	0.262	0.971	0.260	0.022
$N^-$	0.070	0.421	0.268	0.971	0.259	0.013
$N^-$	0.075	0.439	0.263	0.981	0.261	0.025
$N^-$	0.080	0.458	0.286	0.984	0.283	0.047
$N^-$	0.083	0.472	0.242	0.854	0.249	0.038
$Col^-$	0.085	0.509	0.927	0.975	0.949	0.902
$Col^-$	0.090	0.532	0.962	0.981	0.949	0.947
$Col^-$	0.100	0.561	0.979	0.986	0.970	0.968
$Col^-$	0.110	0.589	0.984	0.988	0.977	0.978
K	0.120	0.609	0.988	0.992	0.983	0.983
K	0.150	0.667	0.991	0.997	0.989	0.989

**Table 23.** Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=7$ . Absolute errors are lower than  $5 \cdot 10^{-3}$

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.022	0.235	0.082	0.144	0.063	0.003
$N^-$	0.023	0.245	0.192	0.642	0.172	0.017
$N^-$	0.025	0.266	0.219	0.750	0.204	0.011
$N^-$	0.030	0.295	0.237	0.847	0.228	0.022
$N^-$	0.040	0.343	0.256	0.940	0.254	0.020
$N^-$	0.060	0.421	0.266	0.981	0.265	0.024
$N^-$	0.065	0.437	0.271	0.986	0.270	0.021
$N^-$	0.068	0.450	0.273	0.987	0.273	0.030
$Sm^-$	0.070	0.465	0.264	0.991	0.263	0.016
$Sm^-$	0.075	0.485	0.262	0.993	0.262	0.011
$Sm^-$	0.078	0.502	0.277	0.995	0.276	0.035
$Sm^-$	0.079	0.508	0.281	0.996	0.281	0.034
$Col^-$	0.080	0.533	0.891	0.988	0.886	0.855
$Col^-$	0.090	0.572	0.969	0.988	0.965	0.960
$Col^-$	0.095	0.589	0.978	0.989	0.974	0.971
K	0.100	0.608	0.985	0.995	0.982	0.979
K	0.200	0.753	0.996	0.999	0.995	0.994

**Table 24. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=8$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.010	0.168	0.033	0.045	0.029	0.004
I	0.015	0.205	0.0434	0.072	0.039	0.041
$N^-$	0.017	0.222	0.068	0.120	0.058	0.007
$N^-$	0.018	0.236	0.178	0.612	0.168	0.016
$N^-$	0.02	0.257	0.222	0.795	0.211	0.02
$N^-$	0.04	0.367	0.256	0.964	0.255	0.012
$N^-$	0.050	0.412	0.259	0.977	0.258	0.013
$N^-$	0.055	0.432	0.267	0.984	0.266	0.012
$Sm^-$	0.060	0.455	0.273	0.988	0.272	0.017
$Sm^-$	0.070	0.515	0.281	0.997	0.282	0.020
$Sm^-$	0.075	0.543	0.273	0.998	0.273	0.020
$Sm^-$	0.078	0.551	0.268	0.981	0.268	0.013
$Col^-$	0.079	0.569	0.947	0.989	0.945	0.931
$Col^-$	0.080	0.577	0.973	0.989	0.971	0.967
$Col^-$	0.085	0.586	0.969	0.985	0.975	0.968
$Col^-$	0.088	0.599	0.980	0.986	0.977	0.977
K	0.090	0.614	0.985	0.995	0.983	0.980
K	0.100	0.637	0.987	0.997	0.986	0.983



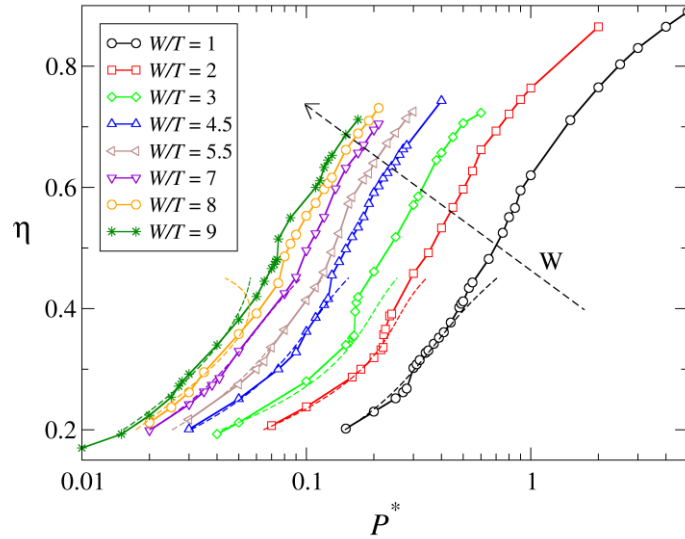
**Table 25. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=10$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.011	0.198	0.0772	0.202	0.067	0.001
$N^-$	0.012	0.215	0.147	0.515	0.146	0.01
$N^-$	0.020	0.392	0.245	0.908	0.244	0.017
$N^-$	0.030	0.361	0.257	0.959	0.257	0.015
$N^-$	0.040	0.416	0.262	0.979	0.261	0.018
$Col^-$	0.043	0.439	0.265	0.985	0.265	0.021
$Col^-$	0.045	0.458	0.268	0.986	0.268	0.026
$Col^-$	0.050	0.496	0.273	0.991	0.273	0.032
$Col^-$	0.080	0.631	0.984	0.995	0.983	0.979
$Col^-$	0.090	0.656	0.989	0.996	0.988	0.986
K	0.095	0.672	0.989	0.998	0.989	0.987
K	0.100	0.685	0.991	0.998	0.990	0.987

**Table 26. Reduced pressure, packing fraction, uniaxial and biaxial order parameters of HBPs with  $L^*=12$  and  $W^*=12$ . Absolute errors are lower than  $5 \cdot 10^{-3}$**

Phase	$P^*$	$\eta$	$S_{2,L}$	$S_{2,T}$	$S_{2,W}$	$B_2$
I	0.005	0.137	0.031	0.04	0.030	0.001
I	0.007	0.168	0.044	0.09	0.046	0.010
I	0.008	0.183	0.075	0.225	0.084	0.015
$N^-$	0.009	0.206	0.193	0.703	0.190	0.011
$N^-$	0.010	0.221	0.246	0.797	0.216	0.016
$N^-$	0.015	0.280	0.245	0.915	0.245	0.015
$N^-$	0.020	0.324	0.253	0.947	0.253	0.015
$N^-$	0.030	0.399	0.265	0.981	0.265	0.020
$CoI^-$	0.033	0.426	0.262	0.984	0.262	0.018
$CoI^-$	0.040	0.500	0.258	0.989	0.258	0.013
$CoI^-$	0.060	0.604	0.977	0.993	0.979	0.988
$CoI^-$	0.070	0.642	0.986	0.995	0.987	0.992
K	0.075	0.661	0.989	0.997	0.989	0.994
K	0.080	0.675	0.990	0.998	0.989	0.995
K	0.090	0.701	0.992	0.999	0.991	0.996
K	0.100	0.721	0.993	0.999	0.997	0.997

**Figure 1.** Equation of state  $\eta$  vs  $P^*$  of HBPs of length  $L^*=9$  at different values of the reduced width. Symbols indicate the simulation results, while dashed lines are theoretical predictions assuming biaxial particles. Solid lines are guides for the eye.



**Figure 2.** Equation of state  $\eta$  vs  $P^*$  of HBPs of length  $L^*=12$  at different values of the reduced width. Symbols indicate the simulation results, while dashed lines are theoretical predictions assuming biaxial particles. Solid lines are guides for the eye.

