Supplementary Material to the manuscript: Fracture Toughness of Wet and Dry Particulate Materials: Role of Plastic Deformation (M. L. Sesso and G. V. Franks)

Below are the complete solutions for the stresses along the principle axes as presented in reference 63, (G. Hondros, Aust. J. Appl. Sci., 1959, 10, 243–268) for a disk in plane stress (or plane strain) of radius R in diametral compression (see Figure S1). The analysis is conducted in polar coordinates (r = radius and θ = angle form the vertical axis). α is the half angle of the flattened area and p is the pressure applied over that area.

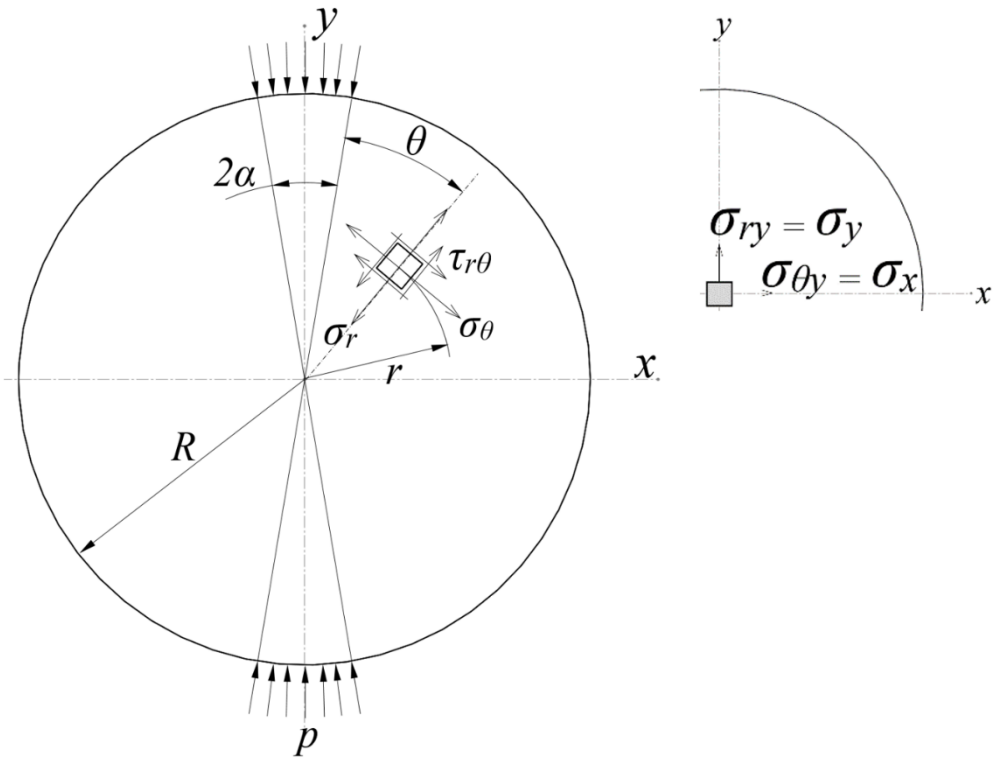

Figure S1. ***Detailed figure of the diametral compression test. The notation is for the polar stress components that occur in an element compressed along the diametral axis. Adapted from Hondros in reference 63.***

Stresses along the **vertical** diameter:

$$\sigma_{\theta y} = +\frac{2p}{\pi}\left[\frac{\left(1-\frac{r^2}{R^2}\right)\sin 2\alpha}{1-\frac{2r^2}{R^2}\cos 2\alpha+\frac{r^4}{R^4}}-\tan^{-1}\frac{\left(1+\frac{r^2}{R^2}\right)}{\left(1-\frac{r^2}{R^2}\right)}\tan\alpha\right] \quad (1)$$

$$\sigma_{ry} = -\frac{2p}{\pi}\left[\frac{\left(1-\frac{r^2}{R^2}\right)\sin 2\alpha}{1-\frac{2r^2}{R^2}\cos 2\alpha+\frac{r^4}{R^4}}+\tan^{-1}\frac{\left(1+\frac{r^2}{R^2}\right)}{\left(1-\frac{r^2}{R^2}\right)}\tan\alpha\right] \quad (2)$$

$$\tau_{r\theta} = 0 \quad (3)$$

Stresses along the **horizontal** diameter:

$$\sigma_{\theta x} = -\frac{2p}{\pi}\left[\frac{\left(1-\frac{r^2}{R^2}\right)\sin 2\alpha}{1+\frac{2r^2}{R^2}\cos 2\alpha+\frac{r^4}{R^4}}+\tan^{-1}\frac{\left(1-\frac{r^2}{R^2}\right)}{\left(1+\frac{r^2}{R^2}\right)}\tan\alpha\right] \quad (4)$$

$$\sigma_{rx} = +\frac{2p}{\pi}\left[\frac{\left(1-\frac{r^2}{R^2}\right)\sin 2\alpha}{1+\frac{2r^2}{R^2}\cos 2\alpha+\frac{r^4}{R^4}}-\tan^{-1}\frac{\left(1-\frac{r^2}{R^2}\right)}{\left(1+\frac{r^2}{R^2}\right)}\tan\alpha\right] \quad (5)$$

$$\tau_{r\theta} = 0 \quad (6)$$

In the case of the vertical diameter, (parallel to the loading direction) as in the work presented in the present paper, $\sigma_{\theta y} = \sigma_x$, $\sigma_{ry} = \sigma_y$ and $\tau_{r\theta} = \tau_{xy} = 0$ resulting in Equations 3, 4 and 5 in the paper.

Figure S2 below shows the normalised stress distributions ($2p/\pi = 1$) along the both principle axes using the above equations.

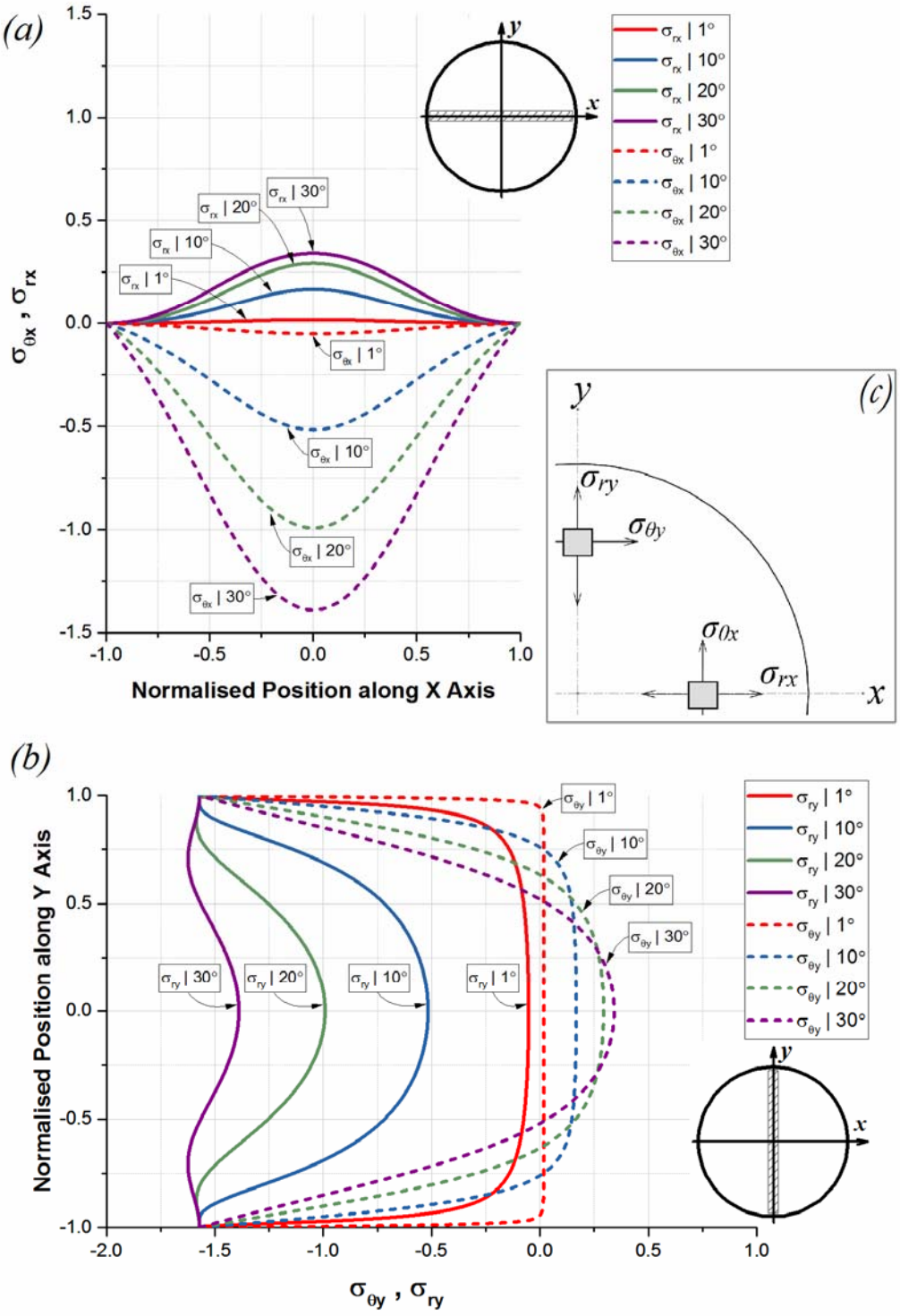

Figure S2. ***Theoretical stress distribution in a diametraclly compressed sample along the horizontal axis (a) and along the verticle axis (b).***

The Table S1 below lists the complete dataset of the fracture toughness measurements, densities and saturation of the wet Al_2O_3 samples. N = number of samples tested.

Table S1

	a/R	N	Mode I Fracture Toughness KIC	Minimum	Median	Maximum	Saturation Values	Mean Saturation ± SD
				(MPa·m½)			%	
35 Wet	0.1	4	0.01462 ± 0.00089	0.01378	0.01446	0.01578	0.934	0.877 ± 0.046
	0.2	7	0.02527 ± 0.00145	0.02362	0.0248	0.02785	0.870	
	0.3	4	0.033 ± 0.00182	0.03037	0.03356	0.03452	0.864	
	0.5	6	0.02884 ± 0.00265	0.02505	0.02876	0.03208	0.860	
							0.875	
							0.785	
							0.918	
							0.910	

	a/R	N	Mode I Fracture Toughness KIC	Minimum	Median	Maximum	Density Values	Mean Density ± SD
				(MPa·m½)			g/cm^3	
35 Dry	0.1	8	0.0038 ± 0.00085	0.0029	0.00351	0.00497	2.719	2.677 ± 0.111
	0.2	6	0.00461 ± 0.0014	0.00202	0.00487	0.00588	2.690	
	0.3	7	0.00697 ± 0.00162	0.00455	0.0072	0.00887	2.653	
	0.5	4	0.00696 ± 0.00138	0.00567	0.00668	0.00883	2.922	
							2.758	
							2.482	
							2.686	
							2.689	
							2.659	
							2.620	
							2.570	

	a/R	N	Mode I Fracture Toughness KIC	Minimum	Median	Maximum	Saturation Values	Mean Sat. ± SD
				(MPa·m½)			%	
50 Wet	0.1	8	0.01795 ± 0.00527	0.01174	0.01772	0.02469	0.817	0.83 ± 0.022
	0.2	8	0.02303 ± 0.00697	0.01443	0.02465	0.03211	0.854	
	0.3	8	0.02059 ± 0.00418	0.0157	0.02048	0.02595	0.825	
	0.5	8	0.03417 ± 0.00735	0.02685	0.03167	0.04313	0.825	
							0.801	
							0.866	
							0.823	

	a/R	N	Mode I Fracture Toughness KIC	Minimum	Median	Maximum	Density Values	Mean Density ± SD
				(MPa·m½)			g/cm^3	
50 Dry	0.1	4	0.00337 ± 0.00076	0.00273	0.00317	0.00444	2.511	2.697 ± 0.087
	0.2	4	0.00275 ± 0.0008	0.00219	0.00275	0.00332	2.717	
	0.3	4	0.00585 ± 0.00161	0.00459	0.0053	0.0082	2.717	
	0.5	4	0.00869 ± 0.00416	0.00575	0.00869	0.01163	2.706	
							2.730	
							2.803	
							2.603	
							2.746	
							2.737	

	a/R	N	Mode I Fracture Toughness KIC	Minimum	Median	Maximum	Saturation Values	Mean Sat. ± SD
				(MPa·m½)			%	
60 Wet	0.1	8	0.01838 ± 0.00137	0.01581	0.01865	0.02017	0.819	0.802 ± 0.025
	0.2	16	0.0239 ± 0.00304	0.01914	0.02301	0.0286	0.814	
	0.3	8	0.02428 ± 0.00269	0.02059	0.02471	0.02707	0.815	
	0.5	16	0.03003 ± 0.00482	0.02064	0.03059	0.03905	0.758	

						0.786		
						0.823		
	a/R	**N**	**Mode I Fracture Toughness KIC**	**Minimum**	**Median**	**Maximum**	**Density Values**	**Mean Density ± SD**
				(MPa·m½)			g/cm³	
	0.1	8	0.00362 ± 0.00077	0.00219	0.00393	0.00438	2.807	2.751 ± 0.174
	0.2	11	0.00448 ± 0.00152	0.00188	0.00446	0.00712	2.980	
	0.3	8	0.00652 ± 0.00152	0.00438	0.00632	0.00958	2.932	
	0.5	8	0.00607 ± 0.00179	0.00302	0.00634	0.00865	2.418	
60							2.812	
Dry							2.794	
							2.757	
							2.846	
							2.823	
							2.541	
							2.551	

The following Table S2 provides the individual measurements of elastic modulus, the average and standard deviation.

Table S2

	Solids	**Sample**			**Mean Modulus ± SD**
	(Vol %)	1	2	3	(GPa)
	35	6.63	7.53	7.11	7.09 ± 0.45
Wet	50	6.55	5.99	6.07	6.2 ± 0.31
	60	6.24	5.15	5.68	5.69 ± 0.55
	35	2.82	2.96	2.91	2.9 ± 0.07
Dry	50	2.96	2.97	2.97	2.96 ± 0.01
	60	4.44	5.50	4.97	4.97 ± 0.53

The Table S3 below lists the complete dataset of the fracture toughness measurements and saturation of the wet and dry $CaCO_3$ (calcite) samples produced using Omyacarb 2 (2 micron average size) produced from 20 and 30 vol% solids suspensions at pH 10. N = number of samples tested.

Table S3

	a/R	**N**	**Mode I Fracture Toughness KIC**	**Minimum**	**Median**	**Maximum**	**Saturation**
				(MPa·m½)			%
20 Wet	0.1	4	0.00209 ± 0.00013	0.00192	0.00211	0.00223	0.842
	0.2	4	0.00338 ± 0.00018	0.00321	0.00336	0.0036	
	0.3	4	0.00307 ± 0.00040	0.00256	0.00309	0.00353	
	0.5	4	0.00463 ± 0.00041	0.00404	0.00476	0.00495	
	a/R	**N**	**Mode I Fracture Toughness KIC**	**Minimum**	**Median**	**Maximum**	
				(MPa·m½)			
20 Dry	0.1	1	0.00140	0.0014	0.0014	0.0014	
	0.2	3	0.00179 ± 0.00017	0.0016	0.00183	0.00193	
	0.3	3	0.00165 ± 0.00024	0.00146	0.00157	0.00191	
	0.5	3	0.00260 ± 0.00014	0.00251	0.00253	0.00277	
	a/R	**N**	**Mode I Fracture Toughness KIC**	**Minimum**	**Median**	**Maximum**	**Saturation**
				(MPa·m½)			%
30 Wet	0.1	4	0.00203 ± 0.00014	0.00182	0.00207	0.00214	0.844
	0.2						
	0.3	4	0.00277 ± 0.00043	0.00231	0.00276	0.00327	
	0.5	4	0.00363 ± 0.00037	0.0031	0.00374	0.00394	
	a/R	**N**	**Mode I Fracture Toughness KIC**	**Minimum**	**Median**	**Maximum**	
				(MPa·m½)			
30 Dry	0.1	2	0.00156 ± 0.00099	8.62332E-4	0.00156	0.00226	
	0.2	4	0.00206 ± 0.00036	0.00177	0.00195	0.00258	
	0.3	-					
	0.5	1	0.00262				

Figure S3 below plots the mean fracture toughness values of the Omyacarb 2 $CaCO_3$ as a function of dimensionless crack length (a/R).

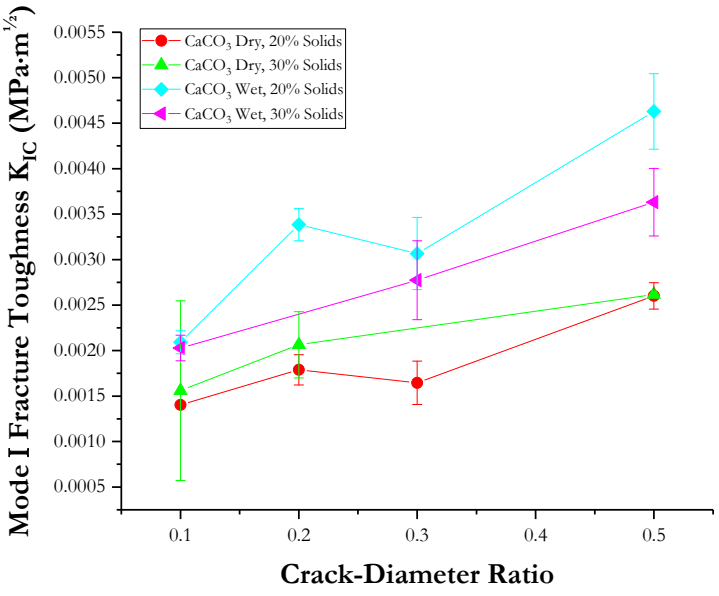

Figure S3. Mode I fracture toughness as a function of the dimensionless crack length for wet and dry $CaCO_3$ samples cast at 20 and 30% solids volume fraction.

This section outlines the procedure used to determine the shape coefficient as per the work of Atkinson *et al.* (C. Atkinson, R.E. Smelser, and J. Sanchez, Int. J. Fract., 1982, 18, 279–291) – reference 33 in the manuscript.

$$F_I = \sum_{i=1}^{n} T_i \left(\frac{a}{R}\right)^{2i-2} A_i(\psi)$$

Where $F_I = K_I/\sigma_f\sqrt{\pi a}$ and the five term truncated solution of T_i is listed in Table S4. The five angular constants $A_i(\psi)$ are determined from Table S5 and notation as per Figure S4.

Table S4. First five coefficients used to determine the shape coefficient.

a/R	T_1	T_2	T_3	T_4	T_5
0.1	1.014998	0.503597	0.376991	0.376991	0.314159
0.2	1.060049	0.514907	0.382430	0.383392	0.318086
0.3	1.135551	0.533477	0.391640	0.393835	0.325033
0.4	1.243134	0.559734	0.404603	0.408597	0.334831
0.5	1.387239	0.594892	0.421949	0.428353	0.347941
0.6	1.578258	0.642124	0.445387	0.454861	0.365559

Table S5. Solutions for the first five angular constants for determine the shape coefficient

A_1	$1 - 4s^2$
A_2	$8s^2(1 - 4c^2)$
A_3	$-4s^2(3 - 36c^2 + 48c^4)$
A_4	$-16s^2(-1 + 24c^2 - 80c^4 + 64c^6)$
A_5	$-20s^2(1 - 40c^2 + 240c^4 - 448c^6 + 256c^8)$

Note: $s = \sin\psi\,;\, c = \cos\psi$

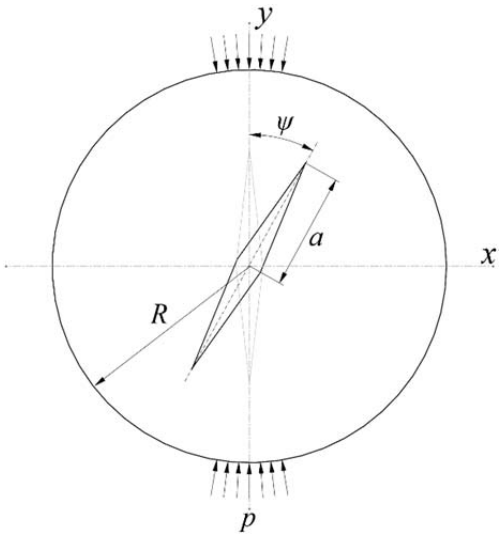

Figure S4

This results in F_I values equal to 1.015 for a/R = 0.1, 1.060 for a/R= 0.2, 1.356 for a/R = 0.3, 1.387 for a/R = 0.5 for a crack aligned parallel to the loading axis, i.e., $\psi = 0$.