Electronic Supplementary Material (ESI) for Soft Matter.

Electronic Supplementary Information

Jumping drops on super-hydrophobic surfaces, controlling energy transfer by timed electric actuation

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ESI 1: Movies of jumping drops

Examples of launching drops can be found in the movies:

../air400Vshortpulse.avi
../decane400Vshortpulse.avi
../decane400Vlongpulse.avi

ESI 2: Equations of motion

To analyze the dynamics of the detachment process in a little more detail we need to derive the equation of motion of the center of mass (CM) of the drop and of the liquid motion with respect to the CM. Therefore we need approximating expressions for the kinetic energy and dissipation involved. The stretching of the drop as described in Section 4, corresponds to an axis-symmetric elongation flow. Because the system is clearly underdamped we assume little dissipation and describe the velocity field inside and outside the droplet by the velocity potentials: $\phi_w(r,\theta) = A_n\,(r/R_0)^n\,P_n(\cos\theta)$ and $\phi_o(r,\theta) = B_n\,(r/R_0)^{-n-1}\,P_n(\cos\theta)$ where R_0 is the radius of the drop and $\phi_{w,o}$ the velocity potential inside and outside the drop, respectively. $P_n(x)$ is the Legendre polynomial of order n. Because the inner and outer radial velocities $\partial_r\phi$ at the surface of the drop near $r = R_0$ should match, the coefficients A_n and B_n obey: $nA_n + (n+1)B_n = 0$. We consider the $n = 1$ and $n = 2$ mode; $n = 1$ describes the motion of the CM and $n = 2$ the stretching deformation of the droplet. With these velocity potentials one calculates the kinetic energy of the internal and external flow due to the deformation, $K_w = \frac{4}{5}\pi R_0\rho_w A_2^2$ and $K_o = \frac{8}{15}\pi R_0\rho_o A_2^2$, respectively. Moreover, we calculate the dissipation in the internal and external flow as: $\dot{W}_w = 16\pi\mu_w A_2^2/R$ and $\dot{W}_o = \frac{16\cdot 72}{35}\,\pi\mu_o A_2^2/R$, respectively. Here we neglect the hydrodynamic interactions with the substrate. So $K_o/\rho_o = \frac{2}{3}\,K_w/\rho_w$ and $\dot{W}_o/\mu_o = \frac{72}{35}\,\dot{W}_w/\mu_w \simeq 2\,\dot{W}_w/\mu_w$. Alternatively, calculating K_w and $\dot{W}_w$ for the 'disc'-geometry yields for small deformations: $K_w = \frac{11}{24}\pi R_c^3\rho_w\,(\partial_t b)^2$ and $\dot{W}_w = 6\pi R_c^3\mu_w\,(\partial_t b/b)^2$. Using these relations to express the kinetic energy and dissipation of the outer flow in terms of the inner flow, all energy and dissipation contributions can be expressed in the parameters b and z and their time derivatives:

$$K_{\text{int}} = \tfrac{11}{24}\pi R_c^3(\rho_w + \tfrac{2}{3}\rho_o)\,(\partial_t b)^2 \quad (1)$$

$$K_{\text{cm}} = \pi R_c^3(\rho_w + \tfrac{1}{2}\rho_o)\,(\partial_t z)^2 \quad (2)$$

$$U_{\text{int}} = \pi\gamma\,((2-\eta)a^2 + 4ab) \quad (3)$$

$$U_{\text{cm}} = 2\pi R_c^3(\rho_w - \rho_o)g\,z \quad (4)$$

The contributions to the dissipation are estimated as:

$$\dot{W}_{\rm int} = 6\pi(\mu_w + 2\mu_o)R_c^3(\partial_t b/b)^2 \tag{5}$$
$$\dot{W}_{\rm cl} = 2\pi\mu_c a^3(\partial_t a/a)^2 \tag{6}$$
$$\dot{W}_{\rm cm} = 6\pi\mu_o R_c\,(\partial_t z)^2 \tag{7}$$

where $\dot{W}_{\rm int}$ represents the dissipation inside the droplet, $\dot{W}_{\rm cl}$ the dissipation due to contact line friction (note: $a = (R_c^3/b)^{1/2}$) and $\dot{W}_{\rm cm}$ is due to the motion of the drop through the surrounding medium. In air density and viscosity are negligible, $\rho_o = \mu_o = 0$. Now we can formulate the following rate equation for the total energy:

$$\partial_t\left(K_{\rm cm} + K_{\rm int} + U_{\rm cm} + U_{\rm int}\right) = -\dot{W}_{\rm int} - \dot{W}_{\rm cl} - \dot{W}_{\rm cm} \tag{8}$$

while the motion of the CM is given by Newton's second law:

$$F_n - 2\pi R_c^3(\rho_w - \rho_o)\,g - 6\pi\mu_o R_c\,\partial_t z = 2\pi R_c^3(\rho_w + \tfrac{1}{2}\rho_o)\partial_t^2 z \tag{9}$$

Because $\partial_t(K_{\rm cm}+U_{\rm cm}) = 2\pi R_c^3(\rho_w+\frac{1}{2}\rho_o)\,\partial_t z\,\partial_t^2 z + 2\pi R_c^3(\rho_w-\rho_o)g\,\partial_t z$ last two equations decouple in an equation for b:

$$\partial_t\left(K_{\rm int} + U_{\rm int}\right) = -\dot{W}_{\rm int} - \dot{W}_{\rm cl} - F_n\,\partial_t b \tag{10}$$

where we used $F_n\,\partial_t z = F_n\,\partial_t b$ (because $F_n = 0$ when $z \neq b$) and one for z:

$$F_n = 2\pi R_c^3(\rho_w + \tfrac{1}{2}\rho_o)\,\partial_t^2 z + 2\pi R_c^3(\rho_w - \rho_o)\,g + 6\pi\mu_o R_c\,\partial_t z \tag{11}$$

To proceed we write Eqs. (1 – 7) in a dimensionless form by scaling the energy on $E_0 = \pi\gamma R_c^2$ and the time on $t_0 = (R_c^3(\rho_w + \frac{1}{2}\rho_o)/\gamma)^{1/2}$. We also define the Bond number, $Bo = g(\rho_w - \rho_o)R_c^2/\gamma$, the Ohnesorge numbers, $Oh_w = \mu_w/(\gamma(\rho_w + \frac{1}{2}\rho_o)R_c)^{1/2}$, $Oh_o = \mu_o/(\gamma(\rho_w + \frac{1}{2}\rho_o)R_c)^{1/2}$, and the Ohnesorge number due to contact line friction, $Oh_c = \mu_c/(\gamma(\rho_w + \frac{1}{2}\rho_o)R_c)^{1/2}$. This results in:

$$K_{\rm int}/E_0 = \nu\beta_\tau^2 \qquad \nu = \frac{11}{36}\,\frac{3\rho_w + 2\rho_o}{2\rho_w + \rho_o} \tag{12}$$
$$K_{\rm cm}/E_0 = \zeta_\tau^2 \tag{13}$$
$$U_{\rm int}/E_0 = u(\beta) \qquad u(\beta) = 4\beta^{1/2} + (2-\eta)\beta^{-1} \tag{14}$$
$$U_{\rm cm}/E_0 = 2Bo\,\zeta \tag{15}$$

for the energy contributions, and

$$t_0\dot{W}_{\rm int}/E_0 = (6\,Oh_w + 12\,Oh_o)\,\beta^{-2}\,\beta_\tau^2 \tag{16}$$
$$t_0\dot{W}_{\rm cl}/E_0 = \tfrac{1}{2}\,Oh_c\,\beta^{-7/2}\,\beta_\tau^2 \tag{17}$$
$$t_0\dot{W}_{\rm cm}/E_0 = 6\,Oh_o\,\zeta_\tau^2 \tag{18}$$

for the dissipation. Here we used $V = 2\pi R_c^3$, $\alpha = a/R_c$, $\beta = b/R_c$ (such that $\alpha^2\beta = 1$), $\zeta = z/R_c$, $\beta_\tau = \partial_\tau\beta$, $\beta_{\tau\tau} = \partial_\tau^2\beta$, and so on. The differential equations now read:

$$\partial_\tau\left(\nu\,\beta_\tau^2 + u(\beta)\right) = -\left((6\,Oh_w + 12\,Oh_o)\,\beta^{-2}\,\beta_\tau^2 + \tfrac{1}{2}\,Oh_c\,\beta^{-7/2}\,\beta_\tau^2\right) - \tilde{F}_n\,\beta_\tau \tag{19}$$
$$\tilde{F}_n = 2\,(Bo + \zeta_{\tau\tau}) + 6\,Oh_o\,\zeta_\tau \tag{20}$$

Performing the differentiation with respect to τ and defining:

$$f(\beta,\beta_\tau) = \partial_\beta u + (6\,Oh_w + 12\,Oh_o)\,\beta^{-2}\beta_\tau + \tfrac{1}{2}\,Oh_c\,\beta^{-7/2}\beta_\tau \tag{21}$$

we finally obtain after division by β_τ :

$$\beta_{\tau\tau} = \frac{2\,Bo + f(\beta,\beta_\tau) + 2\,\zeta_{\tau\tau} + 6\,Oh_o\,\zeta_\tau}{-2\nu} \tag{22}$$

$$\zeta_{\tau\tau} = \begin{cases} \dfrac{2\,Bo + f(\beta,\beta_\tau) + 6\,Oh_o\,\zeta_\tau}{-2(\nu+1)} & \text{if} \quad \zeta_{\tau\tau} + 3\,Oh_o\,\zeta_\tau + Bo > 0 \\ -Bo - 3\,Oh_o\,\zeta_\tau & \text{if} \quad \zeta - \beta > 0 \end{cases} \tag{23}$$

where the condition $\zeta_{\tau\tau} + 3\,Oh_o\,\zeta_\tau + Bo > 0$ is equivalent with: $f(\beta,\beta_\tau) < 2\nu\,Bo$. These ODE's have been solved using an rk4 integration routine with $\Delta\tau = 0.05$. During free-flight, when $\zeta > \beta$ and $F_n = 0$, the motion of the CM is given by:

$$\zeta_{\tau\tau} + \tau_{\rm dsp}^{-1}\zeta_\tau + Bo = 0 \tag{24}$$

where $\tau_{\rm dsp} = (3\,Oh_o)^{-1}$ is the characteristic dissipation time of the CM motion. The solution of this equation is given by:

$$\zeta(\tau) = \zeta_{\max} + \tau_{\rm dsp}^2 Bo\,\left(1 - e^{-(\tau-\tau_{\max})/\tau_{\rm dsp}} - (\tau - \tau_{\max})/\tau_{\rm dsp}\right) \tag{25}$$

Last equation describes the free-flight trajectory of the droplet in oil, *i.e.* the black dotted curve in Fig. 4 of the manuscript.

ESI 3: Additional Figures

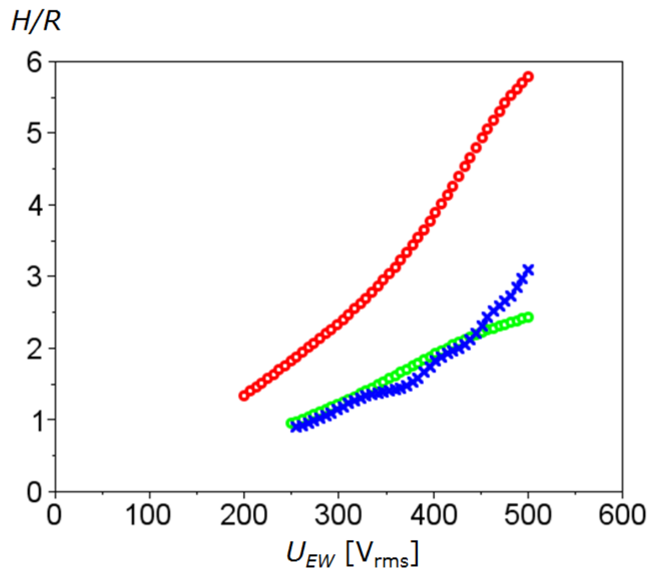

Fig. ESI 1: *Jump height versus applied voltage as calculated with our model. Green curve: water drop launched in air after t_p=8 ms, red curve: water drop launched in n-decane after t_p = 10 ms and blue curve: water drop launched in n-decane after t_p = 200 ms.*

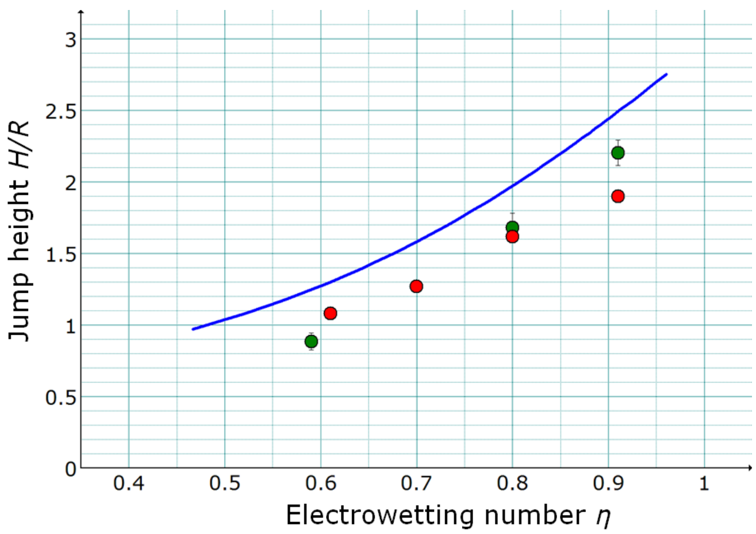

Fig. ESI 2: *Jump height versus applied EW number of a water drop in air. Red symbols: data from Lee et al. [11], green symbols: data from Fig. 5, blue curve: our model calculation.*

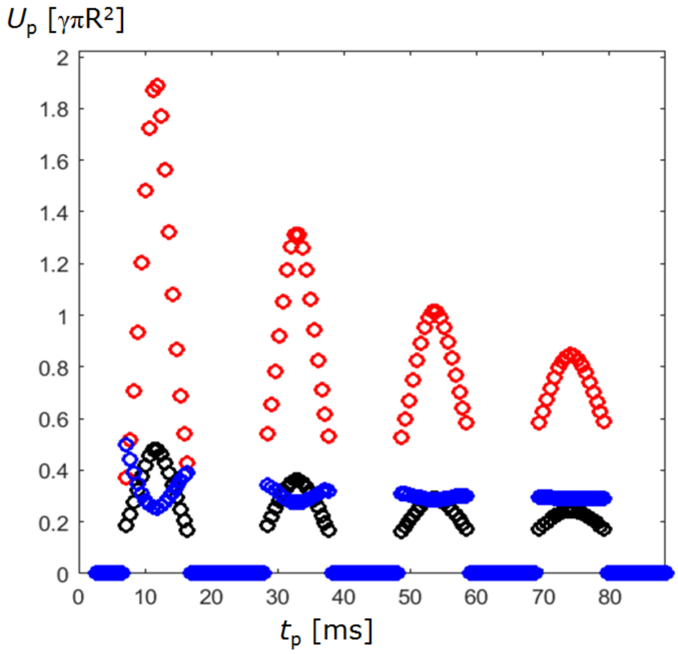

Fig. ESI 3: *Jump height versus EW pulse time as calculated with our model. The black curve represents the gravitational energy U_{gr} in the highest point of the trajectory, while the red curve represents the excess surface energy ΔU_{surf} at detachment. The blue curve represents the ratio $U_{\mathrm{gr}}/\Delta U_{\mathrm{surf}}$, which is approximately 0.3 in stead of 0.25 as observed in the experiments.*