Supplementary Information
Tunable hydrodynamics: A field-frequency phase diagram of a non-equilibrium order-to-disorder transition

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I. INTENSITY VARIANCE ANALYSIS

Following the work of Pastore et al. [1], we examine the time dependence of the variance of intensity fluctuations, $V = I(t + \Delta t) - I(t)$, plotted for times from the minimum time of 0.25 s (due to the stroboscopic imaging) to 100 s in Fig. 1(a). V reaches to a plateau of approximately 200 (in units of pixel gray levels squared) in roughly $10 - 20$ s. $1 - \frac{V}{V_\infty}$, where V_∞ is the average of fluctuations at the plateau, is plotted versus time in Fig. 1(b). This time dependence yields a characteristic time that is roughly comparable to that obtained from the orientational correlations (Fig.7 (a) of the manuscript).

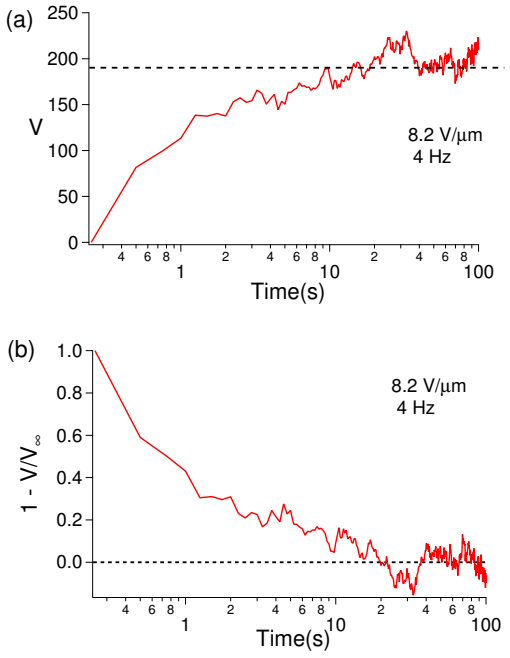

FIG. 1. a: Variance of intesity, V, for the same field and frequency as Fig.7 (a). After roughly $10 - 20$ s V reaches to a plateau of approximately 200. (b): $1 - \frac{V}{V_\infty}$, where V_∞ is the average of fluctuations at plateau. This characteristic time is comparable to the time required to lose orientational correlations (Fig.7 (a) of the manuscript).

II. SUPPLEMENTARY MOVIES

Supplementary Movie 1: **Phase diagram**
A small portion of entire field of view for each frequency and each amplitude is selected. Left to right corresponds to low to high amplitudes. A transition from disorder to order is seen by increasing frequency. Scale bar is 50 μm and fields are perpendicular to the page.

Supplementary Movie 2: **Dynamics at the disorder border.**
Five panels, from left to right, show all fields from low to high amplitudes at the threshold line corresponds to order to disorder transition, light grey region in Fig. 11 of manuscript. Drops stay circular with lowering frequency at the lowest field amplitude, $E = 3.5$ V/μm. The non-circular deformations are accompanied with breakup events at higher fields, $E = 5.8, 7.0$ and 8.2 V/μm. Scale bar is 50 μm and fields are perpendicular to the page.

Supplementary Movie 3: **Dynamics at two intermediate fields and lowest frequency.**
Left to right: At $f = 0.2$ Hz, the dynamics for two intermediate fields $E = 4.6$, and 5.8 V/μm is shown. The major breakup events, faster dynamics, and drops-free patterned spots are the major differences between the left and the right panels. Scale bar is 50 μm and fields are perpendicular to the page.

Supplementary Movie 4: **Effect of lowering frequency in the disordered regime.**
From left to right: Dynamics of drops at $E = 7.0$ V/μm and 0.8 and 1.6 Hz, both located on the green zone of disorder. Lowering frequency increases drop breakup, while the smaller drops are still stabilized and localized on the patterned ITO-free regions. Scale bar is 50 μm and field is perpendicular to the page.

Supplementary Movie 5: **Effect of lowering frequency in the disordered regime**
From left to right: Dynamics of drops at $E = 8.2$ V/μm and 1.8 and 3.5 Hz, both located on the green zone of disorder. Smaller, stabilized patterned drops are as a results of decreasing frequency in the left panel. The non-circular deformation, orientational deformation, is shown in the right panel. Scale bar is 50 μm and field is perpendicular to the page.

[1] R. Pastore, G. Pesce and M. Caggioni, *Scientific Reports*, 2017, **7**, 1–9.