Chemical mediated elasto-capillarity of elastic sheets

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Experimental setup for sheet adhesion. As shown in Figure S1, two slender sheets made of polyester are fixed on a digital calliper by the double faced adhesive tape, and the calliper is horizontally fixed on an iron support. The two sheets have the same sizes and physical parameters, and they are initially parallel, i.e. vertical to the calliper before adhered by liquid. The distance between the two fixed ends can be modulated by moving the sliding vernier horizontally. Each sheet can be divided into two segments, i.e. the adhesion or wetted part and the detachment or dry part.
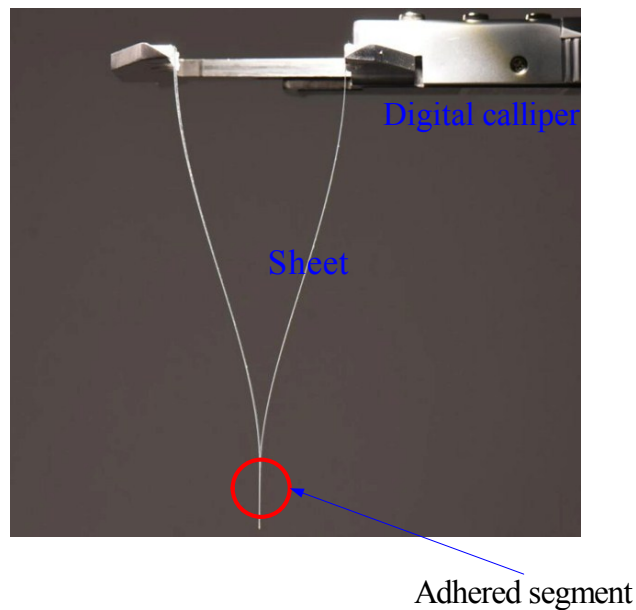

Digital calliper

Sheet

Adhered segment

Figure S1 Experimental setup for sheet adhesion.

Energy functional variation with movable boundary conditions. Let us consider a generalized elastic system represented as a continuous and smooth curve, where part of the curve is adhered at interfaces. As schematized by the anti-clockwise arc length *s* in Fig. S2, the total length of the curve is

denoted by L. Assume that the elastic deformation only occurs on the segment from $s=0$ to $s=a$. The total potential energy of the system encompasses two originations, namely, elastic strain energy and interfacial energy.
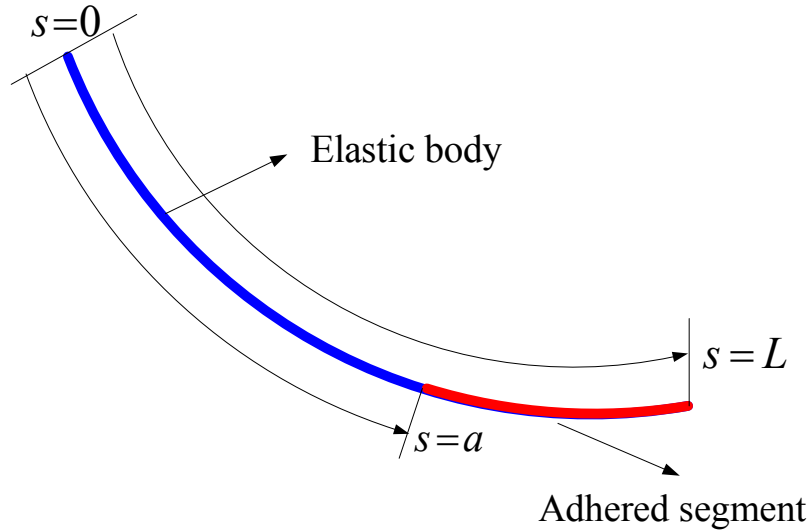

Figure S2 Schematics for a system incorporating two parts with elastic energy and interfacial energy, respectively.

The value of a is an unknown and should be determined in calculation, so the total potential energy of the system is viewed as a function of the parameter a:

$$\Pi[a]=U(a)+\Gamma(a), \tag{S1}$$

where $\Gamma(a)$ is the interfacial energy, $U(a)$ is the strain energy stored in the system which is often expressed as $U=\int_0^a F\left[s,y(s,a),y'(s,a),y''(s,a)\right]ds$, and y is a function with two variables, i.e. $y=y(s,a)$.

The parameter W is designated as the surface energy density or the work of adhesion at the interface. The work of adhesion between two surfaces is normally expressed as

$$W = \gamma_1 + \gamma_2 - \gamma_{12}, \tag{S2}$$

where γ_1 and γ_2 are the surface energy densities of the two different phases, and γ_{12} is the interfacial energy density. For the two phases being of the same material, the work of adhesion degenerates to the cohesive work

$$W = 2\gamma_1. \tag{S3}$$

For a droplet on a substrate, the work of adhesion becomes

$$W = \gamma_{\mathrm{SV}} + \gamma - \gamma_{\mathrm{SL}} = \gamma(1 + \cos\theta_{\mathrm{Y}}). \tag{S4}$$

where γ_{SV}, γ_{SL} and γ are the interfacial tensions of the solid/vapor, solid/liquid and liquid/vapor interfaces, respectively, with θ_{Y} being the Young's contact angle of the liquid. In the above derivation, the Young's equation $\gamma_{\mathrm{SV}} - \gamma_{\mathrm{SL}} = \gamma\cos\theta_{\mathrm{Y}}$ is utilized.

The functional of the total potential energy about the system is normally written as:

$$\Pi\big[y(s,a)\big] = \int_0^a F\big[s, y(s,a), y'(s,a), y''(s,a)\big]\mathrm{d}s - \int_a^L W\mathrm{d}s. \tag{S5}$$

In fact, the energy functional of Eq. (S5) is special in that it deals with two variables, i.e. the function y and the length a. This fact yields an intractable problem, because the undetermined variable a causes the boundary movement of the system, which should create an additional term during the variation process. By using the principle of least potential energy and considering the movable boundary, one obtains the following variational result

$$\delta\Pi\big[y(s,a)\big] = \delta\Pi_1 + \delta\Pi_2 = 0, \tag{S6}$$

where

$$\delta\Pi_1 = \int_0^a \big(F_y\delta y + F_{y'}\delta y' + F_{y''}\delta y''\big)\mathrm{d}s \tag{S7}$$

$$= \left[F_{y'}\delta y + F_{y''}\delta y' - \frac{\partial}{\partial x}F_{y''}\delta y \right]_0^a + \int_0^a \left(F_y - \frac{\partial}{\partial x}F_{y'} + \frac{\partial^2}{\partial x^2}F_{y''} \right)\delta y \mathrm{d}s ,$$

and

$$\delta \Pi_2 = \left[F - y'F_{y'} - y''F_{y''} + y'\frac{\mathrm{d}F_{y''}}{\mathrm{d}s} + W \right]_{s=a} \delta a . \tag{S8}$$

The fixed boundary conditions are prescribed as

$$y(0)=y_0, \ y'(0)= y_0' ; y(a)=y_a, \ y'(a)= y_a' . \tag{S9}$$

According to the arbitrariness of the variation, one can get the governing differential

equation, i.e. the Euler-Poisson equation:

$$F_y - \frac{\partial}{\partial s}F_{y'} + \frac{\partial^2}{\partial s^2}F_{y''} = 0 , \tag{S10}$$

and the arbitrariness of variation about the point a leads to the transversality condition

$$W = \left[y'F_{y'} + y''F_{y''} - y'\frac{\mathrm{d}F_{y''}}{\mathrm{d}s} - F \right]_{s=a} . \tag{S11}$$

Actually, the transversality condition stands for the equilibrium state originating from

the competition between surface energy and elastic energy at the critical point.