```
r1[ϕ_, θ_] = Cos[ϕ] * Sin[θ];
r2[ϕ_, θ_] = Sin[ϕ] * Sin[θ];
r3[ϕ_, θ_] = Cos[θ];

(*ki KF1i term:*)

Sum[Qi,j Integrate[
    Sin[θ] * k * Cos[θ] *
     ri[ϕ, θ] * (ri[ϕ, θ] * rj[ϕ, θ] - 1 / 3 * KroneckerDelta[i, j])
     * Exp[-I * k * Cos[θ] * ρ], {ρ, 0, Infinity}, {θ, 0, Pi},
    {ϕ, 0, 2 * Pi}, Assumptions → {k ∈ Reals}], {j, 1, 3}, {i, 1, 3}]
```

0

```
(*ki KF2i term:*)

Sum[Pj Integrate[
    Sin[θ] * k * (Cos[θ] * rj[ϕ, θ] - 1 / 3 * KroneckerDelta[3, j])
     * Exp[-I * k * Cos[θ] * ρ] / ρ, {ρ, 0, Infinity},
    {θ, 0, Pi}, {ϕ, 0, 2 * Pi}, Assumptions → {k ∈ Reals}], {j, 1, 3}]
```

$$-\frac{8}{9} k \pi P_3$$

```
(*[Kτij]ST term:*)

K = Table[Sum[Qn,m Integrate[Sin[θ] *
        (
         ri[ϕ, θ] * (5 * rn[ϕ, θ] * rj[ϕ, θ] * rm[ϕ, θ] -
             ( KroneckerDelta[j, m] * rn[ϕ, θ] +
               KroneckerDelta[n, m] * rj[ϕ, θ] + KroneckerDelta[n, j] * rm[ϕ, θ]) )
        )
        * Exp[-I * k * Cos[θ] * ρ] / ρ, {ρ, 0, Infinity}, {θ, 0, Pi}, {ϕ, 0, 2 * Pi},
       Assumptions → {k ∈ Reals}], {m, 1, 3}, {n, 1, 3}], {i, 1, 3}, {j, 1, 3}];

M = Table[ Simplify[1 / 2 * (K[[i, j]] + K[[j, i]]) -
      1 / 3 * Sum[K[[k, k]], {k, 1, 3}] * KroneckerDelta[i, j]], {j, 1, 3}, {i, 1, 3}];

MatrixForm[Table[ Simplify[M[[i, j]],
    Assumptions → {Qi,j == Qj,i, Q1,1 + Q2,2 + Q3,3 == 0}], {j, 1, 3}, {i, 1, 3}]]
```

$$\begin{pmatrix} -\frac{8}{15}\pi\,(Q_{2,2}+Q_{3,3}) & \frac{8}{15}\pi\,Q_{2,1} & -\frac{4}{5}\pi\,Q_{3,1} \\ \frac{8}{15}\pi\,Q_{2,1} & \frac{8}{15}\pi\,Q_{2,2} & -\frac{4}{5}\pi\,Q_{3,2} \\ -\frac{4}{5}\pi\,Q_{3,1} & -\frac{4}{5}\pi\,Q_{3,2} & \frac{8}{15}\pi\,Q_{3,3} \end{pmatrix}$$

```
(* assume the wavevector k is in z direction *)

(*compute splay contribution of M*)

Sum[ Simplify[M[[i, j]], Assumptions → {Qi,j == Qj,i, Q1,1 + Q2,2 + Q3,3 == 0}] *
  KroneckerDelta[i, 3] * KroneckerDelta[3, j], {j, 1, 3}, {i, 1, 3}]
```

$$\frac{8}{15}\pi\,Q_{3,3}$$

```
(*compute bend contribution of M*)

Table[ Sum[ Simplify[M[[i, j]], Assumptions → {Q_{i,j} == Q_{j,i}, Q_{1,1} + Q_{2,2} + Q_{3,3} == 0}] *
    KroneckerDelta[i, 3] * (KroneckerDelta[j, l] -
      KroneckerDelta[3, j] * KroneckerDelta[3, l]), {j, 1, 3}, {i, 1, 3}], {l, 1, 3}]
```

$$\left\{-\frac{4}{5}\pi Q_{3,1},\ -\frac{4}{5}\pi Q_{3,2},\ 0\right\}$$