Supplementary Material

1. Derivation of linear elastic solution

The derivation of the linear elastic solution in the Sec.2 of the paper is given below. As is well known, the stresses are related to the Airy stress function ϕ by:

$$\sigma_{22} = \phi_{,11} \quad \sigma_{11} = \phi_{,22} \quad \sigma_{12} = -\phi_{,12} \tag{S1}$$

The stress-strain constitutive relation in plane strain is:

$$\begin{cases} \frac{E}{1+v}\varepsilon_{22} = (1-v)\sigma_{22} - v\sigma_{11} \\ \frac{E}{1+v}\varepsilon_{11} = (1-v)\sigma_{11} - v\sigma_{22} \\ \frac{E}{1+v}\varepsilon_{12} = \sigma_{12} \end{cases} \tag{S2}$$

The strains and displacements are related by

$$\varepsilon_{11} = u_{1,1} \quad \varepsilon_{22} = u_{2,2} \quad \varepsilon_{12} = \frac{1}{2}\left(u_{2,1} + u_{1,2}\right) \tag{S3}$$

where: u_1 and u_2 are horizontal and vertical displacements respectively and $u_{\alpha,\gamma} = \partial u_\alpha / \partial x_\gamma$. Substituting eqn (S1,S3) into eqn (S2), we obtain

$$\begin{cases} \frac{E}{1+v}u_{2,2} = (1-v)\phi_{,11} - v\phi_{,22} \\ \frac{E}{1+v}u_{1,1} = (1-v)\phi_{,22} - v\phi_{,11} \\ \frac{E}{2(1+v)}\left(u_{2,1} + u_{1,2}\right) = -\phi_{,12} \end{cases} \tag{S4}$$

Define the Fourier Transform of a function ϕ by:

$$\tilde{\phi} = \int_{-\infty}^{\infty} e^{i\lambda x_1}\phi(x_1)\,dx_1 \tag{S5}$$

The Fourier transform the stress components in eqn (S1) are:

$$\tilde{\sigma}_{22} = -\lambda^2\tilde{\phi} \quad \tilde{\sigma}_{11} = \tilde{\phi}_{,22} \quad \tilde{\sigma}_{12} = i\lambda\tilde{\phi}_{,2} \tag{S6}$$

The Airy stress function ϕ is biharmonic, that is,

$$\nabla^4 \phi = 0 \tag{S7}$$

Fourier transform the biharmonic equation in x_1 results in an ordinary differential equation in x_2 . The general solution of this differential equation that vanishes at infinity is

$$\tilde{\phi} = (A + Bx_2)e^{-|\lambda|x_2} \tag{S8}$$

Substituting eqn (S8) into eqn (S6), we obtain

$$\begin{aligned} \tilde{\sigma}_{22} &= -\lambda^2 (A + Bx_2)e^{-|\lambda|x_2} \\ \tilde{\sigma}_{11} &= \left[\lambda^2 (A + Bx_2) - 2|\lambda| B \right] e^{-|\lambda|x_2} \\ \tilde{\sigma}_{12} &= i\lambda \left[-|\lambda|(A + Bx_2) + B \right] e^{-|\lambda|x_2} \end{aligned} \tag{S9a-c}$$

The boundary conditions are:

$$\begin{aligned} &\sigma_{22}^{-} - \sigma_{22}^{+} = \sigma_0 u_{2,11} \quad for \quad x_2 = 0 \\ &\sigma_{22}^{-}\left(x_1, x_2 = 0^{-}\right) = N\delta(x_1) \\ &\sigma_{12}^{-} - \sigma_{12}^{+} = 0 \quad for \quad x_2 = 0 \\ &\sigma_{12}^{-}\left(x_1, x_2 = 0^{-}\right) = 0 \end{aligned} \tag{S10a-d}$$

In eqn (S10a), $\sigma_{22}^{-}\left(x_1, x_2 = 0^{-}\right)$ is the applied normal traction, which in our case is a line force given by eqn (S10b). For small deformation, the jump in vertical traction is balanced by the surface tension σ_0 multiply with the surfaces' curvature. Eqn (S10c,d) imply that the surface is not subjected to shear traction.

Fourier transforming eqn (S4) gives:

$$\begin{cases} \dfrac{E}{1+v}\tilde{u}_{2,2} = -(1-v)\lambda^2 \tilde{\phi} - v\tilde{\phi}_{,22} \\ \dfrac{E}{1+v}(-i\lambda \tilde{u}_1) = (1-v)\tilde{\phi}_{,22} + v\lambda^2 \tilde{\phi} \\ \dfrac{E}{2(1+v)}\left(-i\lambda \tilde{u}_2 + \tilde{u}_{1,2}\right) = i\lambda \tilde{\phi}_{,2} \end{cases} \tag{S11a-c}$$

From eqn (S11b) and (S8) ,

$$\tilde{u}_1 = -\frac{1+v}{i \cdot E\lambda}\left[\lambda^2 (A + Bx_2) - 2(1-v)B|\lambda| \right] e^{-|\lambda|x_2} \tag{S12a}$$

where: $i = \sqrt{-1}$. Differentiate (S12a) by x_2 , we obtain

$$\tilde{u}_{1,2} = -\frac{1+v}{i \cdot E\lambda}\left[(1-v)\tilde{\phi}_{,222} + \lambda^2 v \tilde{\phi}_{,2}\right] \tag{S12b}$$

Substituting eqn (S12b) into eqn (S11c) and after some algebra, we found

$$\tilde{u}_2 = \frac{1+v}{E}\left[|\lambda|(A + Bx_2) + (1-2v)B\right]e^{-|\lambda|x_2} \tag{S13}$$

The boundary condition eqn (S10d) and eqn (S9b) imply

$$B = |\lambda| A \tag{S14}$$

The boundary conditions eqn (S10a,b) and (S9a) imply

$$A = -\frac{N}{\lambda^2\left(1+\beta|\lambda|\right)}, \quad B = -\frac{N}{|\lambda|\left(1+\beta|\lambda|\right)} \tag{S15a,b}$$

where:

$$\beta = \frac{2\left(1-v^2\right)\sigma}{E} \tag{S16}$$

With the coefficients of A and B, we can calculate the displacements u_1 and u_2 by inverting the Fourier transform given by eqn (S12a) and eqn (S13). However, the inverse Fourier transform of $\tilde{u}_2$ does not exist since the displacement at infinity has a logarithmic singularity. We therefore consider

$$\Delta\tilde{u}_2 \equiv \tilde{u}_2 - \tilde{u}_2^E \tag{S17}$$

where $\tilde{u}_2^E$ is the transform of the elastic displacement, which is obtained by setting $\sigma_0 = 0$ in *A* and *B*, this results in

$$\tilde{u}_2^E = -\frac{(1+v)N}{E}\left[(2-2v)\frac{1}{|\lambda|} + x_2\right]e^{-|\lambda|x_2} \tag{s18}$$

The elastic displacement field corresponding to eqn (S18) can be found in the literature, it is

$$u_2^E = \frac{(1+v)N}{\pi E}\left[2(1-v)\ln\sqrt{\overline{x}_1^2 + \overline{x}_2^2} - \frac{\overline{x}_2^2}{\overline{x}_1^2 + \overline{x}_2^2}\right] + Const \tag{S19}$$

where we have used the following normalization:

$$\eta = \beta\lambda, \quad x_1 = \beta\overline{x}_1, \quad x_2 = \beta\overline{x}_2, z = \overline{x}_1 + i \cdot \overline{x}_2 \tag{S20}$$

Combining eqn (S13),(S17) and (S18),

$$\Delta \tilde{u}_2 = \frac{(1+v)\beta N}{E}\left\{\frac{|\eta|}{(1+|\eta|)}\bar{x}_2 + \frac{2(1-v)}{(1+|\eta|)}\right\}e^{-|\eta|\bar{x}_2} \tag{S21}$$

The inverse transform of eqn (S21) is

$$\Delta u_2 = \frac{(1+v)N}{\pi E}\left[(2-2v-\bar{x}_2)\mathrm{Re}\left[g(z)+i\cdot f(z)\right]+\frac{\bar{x}_2^2}{\bar{x}_1^2+\bar{x}_2^2}\right] \tag{S22}$$

where the dimensionless functions f and g are defined in the paper.

The transform of the horizontal displacement is obtained using eqn (S12a) and (S15),

$$\tilde{u}_1 = \frac{(1+v)N\beta}{i\cdot E}\left[\frac{(1-2v)}{\eta(1+|\eta|)}-\frac{|\eta|\bar{x}_2}{\eta(1+|\eta|)}\right]e^{-|\eta|\bar{x}_2} \tag{S23}$$

Inverting eqn (S23), we have

$$u_1 = \frac{(1+v)N}{\pi E}\left\{(1-2v+\bar{x}_2)\mathrm{Im}\left[if(z)+g(z)\right]-(1-2v)\tan^{-1}\left(\frac{\bar{x}_1}{\bar{x}_2}\right)\right\} \tag{S24}$$

The strains can be obtained using eqn (S3) using (S19), (S22) and (S24). After some calculations, they are

$$\varepsilon_{11} = \frac{(1+v)N}{\pi E\beta}\left\{(1-2v+\bar{x}_2)\mathrm{Re}\left[g(z)+i\cdot f(z)\right]-\frac{\bar{x}_2^2}{\bar{x}_1^2+\bar{x}_2^2}\right\} \tag{S25}$$

$$\varepsilon_{22} = \frac{(1+v)N}{\pi E\beta}\left\{(1-2v-\bar{x}_2)\mathrm{Re}\left[g(z)+i\cdot f(z)\right]+\frac{\bar{x}_2^2}{\bar{x}_1^2+\bar{x}_2^2}\right\} \tag{S26}$$

$$\varepsilon_{12} = \frac{(1+v)N}{\pi E\beta}\left\{\frac{\bar{x}_1\bar{x}_2}{\bar{x}_1^2+\bar{x}_2^2}-\bar{x}_2\,\mathrm{Im}\left[g(z)+i\cdot f(z)\right]\right\} \tag{S27}$$

2. Kink angle versus applied load in the cases of large stiffening surfaces

Fig. S1 plots the kink angle versus the normalized load for surfaces with large stiffening moduli. We include these results in the SI because these stiffening moduli may be too large to be realistic. The purpose of these calculations is to show that for very large surface modulus, the kink angle eventually exceeds the prediction of linearized theory.

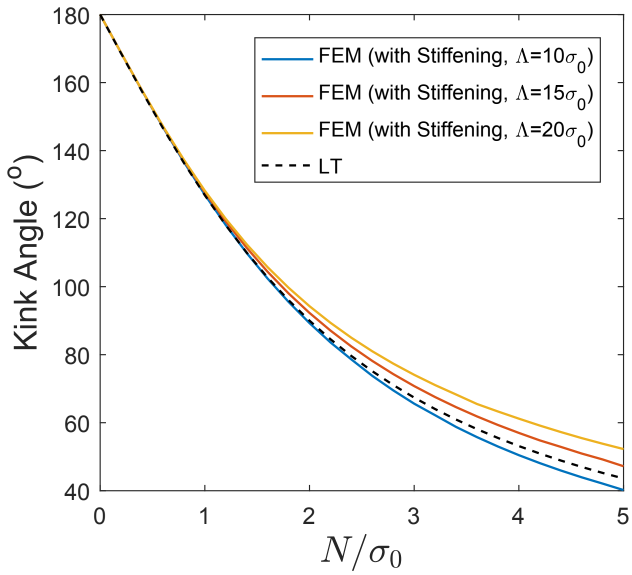

Fig. S1 Kink angle versus applied normalized applied load in large surface stiffening cases. The dashed line represents the kink angle predicted by linearized elastic solution, i.e. Eq. (7a) in the paper. The colored lines represent the FEM results of different large stiffening parameters.

3. Contour plots of kink angle and maximum vertical displacement versus load and surface stiffening parameters

Contour plots of kink angle θ_k (in degrees) and maximum vertical displacement $\bar{u}_{2(kink)}^{FEM}$ plotted against normalized load N/σ_0 and dimensionless surface stiffening modulus Λ/σ_0. It shows that for N/σ_0 smaller than 1.5, both the kink angle and the maximum vertical displacement is insensitive to the surface stiffening modulus ($\Lambda \leq 8\sigma_0$). For larger values of N/σ_0, surface stiffening effect becomes increasingly significant in reducing the kink angle and kink vertical displacement.

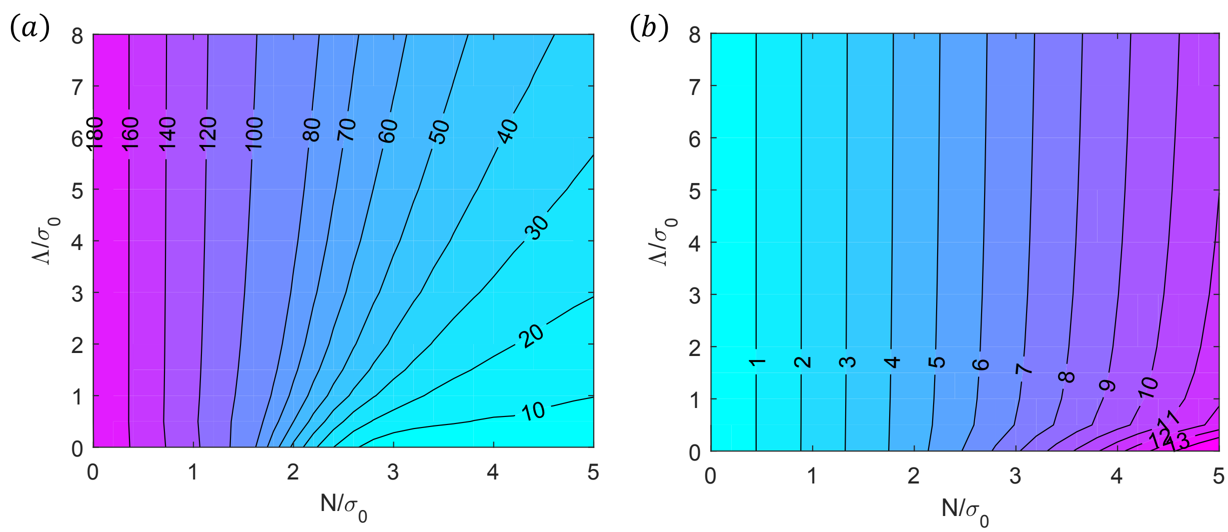

Fig.S2 (a) Contour plot of kink angle θ_k in degrees and (b) normalized kink maximum vertical displacement $\bar{u}_{2(kink)}^{FEM}$ versus different normalized applied load N / σ_0 and the dimensionless surface stiffening parameter Λ / σ_0 .