

Electronic Supplementary Information for *Soft
Matter* manuscript:

**Non-equilibrium phase transition in a
model for supercoiling-dependent DNA
transcription**

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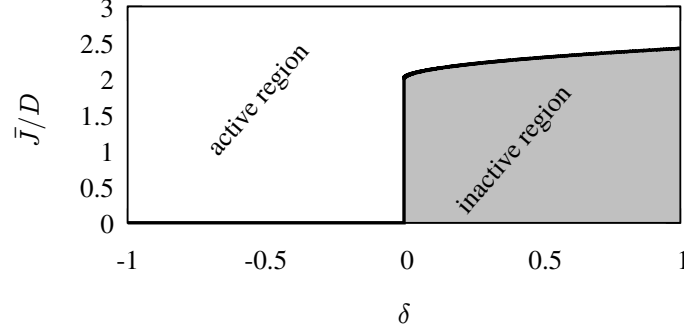


Figure S1: Phase diagram in \bar{J}/D vs δ plane as predicted by the mean field theory. For $\delta > 0$ the mean field predicts a discontinuous (first order) transition in accordance with our numerical simulations. The line of transitions at $\delta = 0$ is continuous (second order) within the mean field theory.

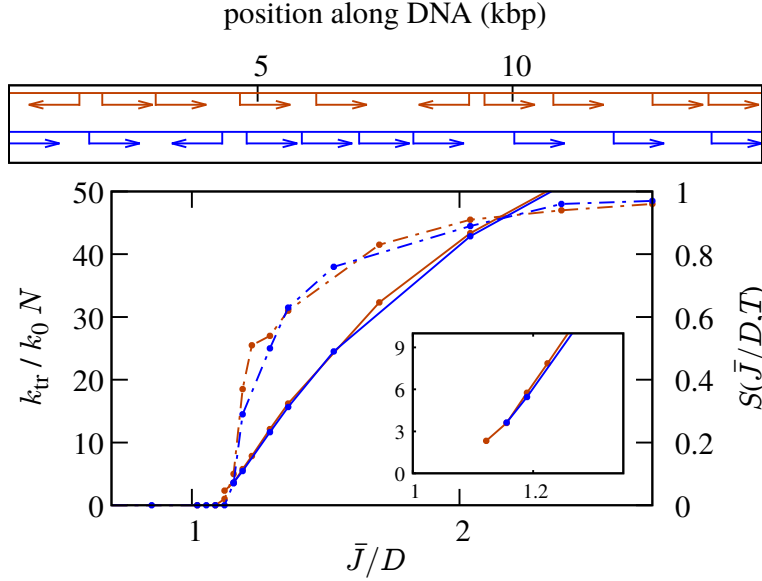


Figure S2: Bidirectional genes. Transcription rates normalised by $k_0 N$ (solid curves) and survival probability S (dashed curves) for two different configurations (red and blue) of bidirectional genes as shown on the top of the figure. Data were obtained from 100 runs, $10^8 \Delta t$ long, for every \bar{J} value. Our simulations suggest that the nature of the non-equilibrium transition does not change qualitatively. The trend of transcription rate is more similar to that one for a single gene. This is not surprising, indeed the pair of divergent genes picked stochastically by the system (see also Supplemental Material in [1]) is highly up regulated and their behaviour is more similar to that one for a single gene.

References

- [1] C. A. Brackley, J. Johnson, A. Bentivoglio, S. Corless, N. Gilbert, G. Gonnella, and D. Marenduzzo. Stochastic model of supercoiling-dependent transcription. *Phys. Rev. Lett.*, 117:018101, 2016.