

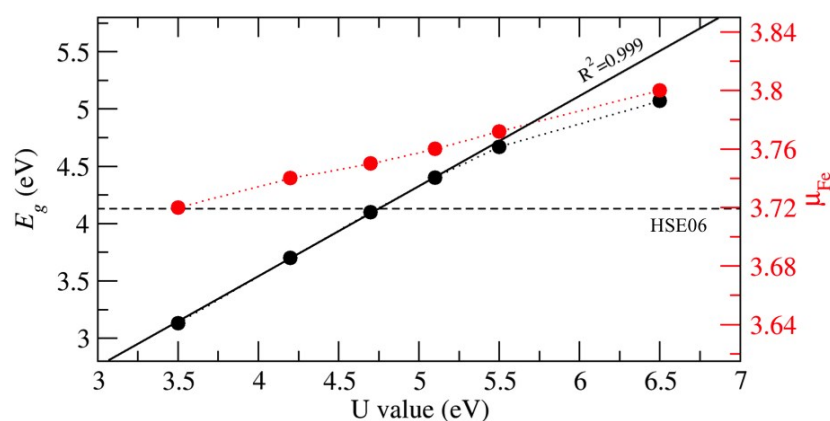
## Supporting Information

### Mechanistic study of Na-ion diffusion and small polaron formation in Kröhnkite $\text{Na}_2\text{Fe}(\text{SO}_4)_2 \cdot 2\text{H}_2\text{O}$ based cathode materials

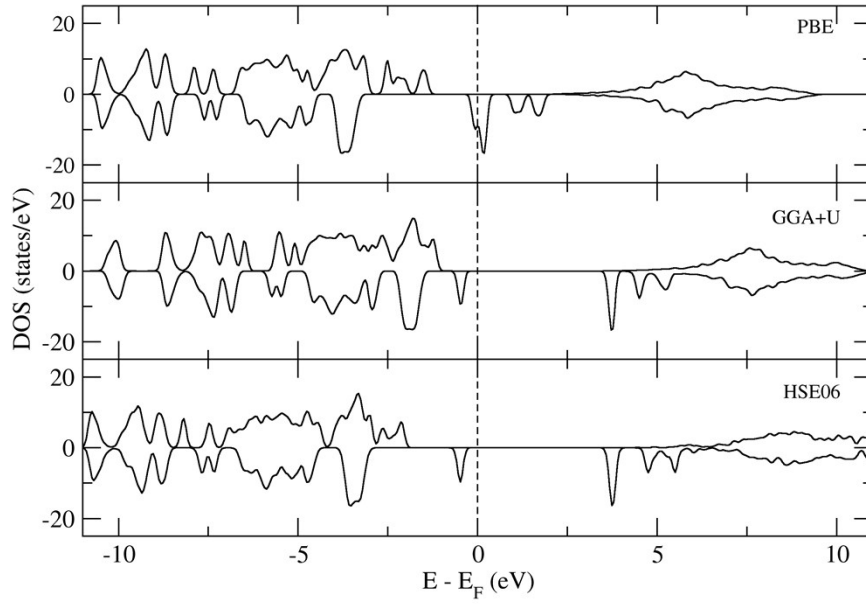
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#### *The effect of Hubbard $U$ parameters*

In this work, PBE+ $U$  exchange correlation functional is mainly employed as it can handle the self-interaction error of  $d$  orbitals and it requires reasonable computing cost compared to HSE06. It is crucial that the Hubbard  $U$  parameter is carefully chosen. Thus, we optimize the primitive structure with different  $U$  parameters from 3.5 to 6.5 eV, and then compare the resulting band gap ( $E_g$ ) and Fe magnetic moment ( $\mu_{\text{Fe}}$ ) with those calculated with HSE06. Based on the consistency of  $E_g$ , the  $U = 4.7$  eV is determined to give the similar value of  $E_g$  as shown in Fig.S1. Also, the density of states (DOS) calculated with this  $U$  value and with HSE06 are well corresponding as displayed in Fig.S2. Moreover, the DOS calculated with PBE clearly indicates that PBE cannot capture an electronic structure of this material.



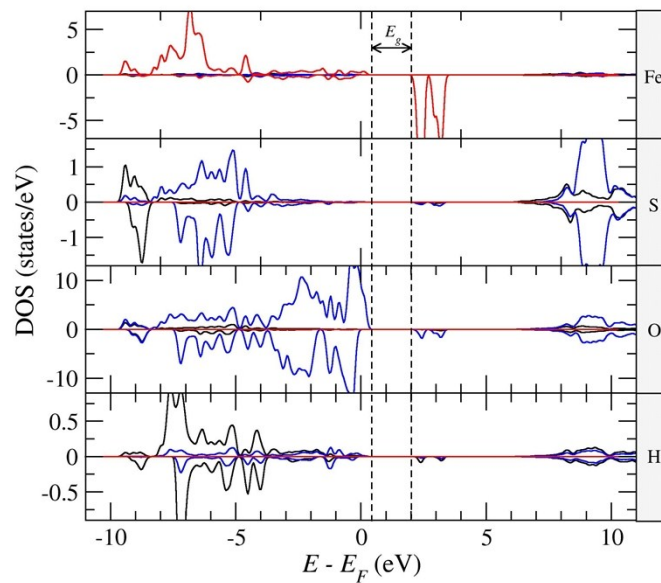
**Fig. S1** The effect of Hubbard  $U$  parameter toward electronic band gap of  $\text{Na}_2\text{Fe}(\text{SO}_4)_2 \cdot 2\text{H}_2\text{O}$ . Dashed-lines refer to those parameters calculated with HSE06.



**Fig. S2** Total-DOS of fully-sodiated structure calculated with PBE, PBE+ $U$ , and HSE06 exchange correlation functionals. The energies are referenced with the Fermi level ( $E_F$ ).

### *The second sodium de-intercalation*

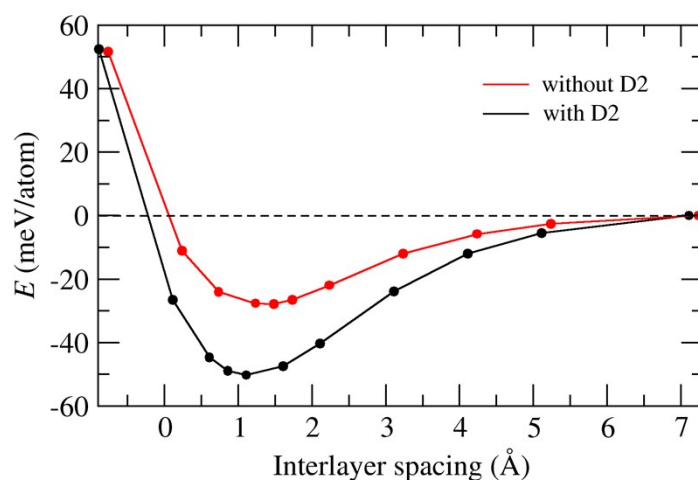
After first sodium de-intercalation is completed, we found that the redox potential will occur via the oxidation transition of oxygen, whereas there is no significant change on  $H^+$  ions. The DOS of fully-desodiated structure is displayed in Fig.S3. All PDOS are corresponding to those obtained for half-desodiated structure, except O-DOS. Thus, it implies the possibility of the evolution of oxygen in second sodium de-intercalation.



**Fig. S3** The density of states of fully-desodiated  $Fe(SO_4)_2 \cdot 2H_2O$ . Black, blue, and red denote the electron distribution associated with  $s$ -,  $p$ -, and  $d$ -orbitals.

### *The effect of D2 correction on energy*

The influence of vdW correction (D2) is presented in the view of energy variation and interlayer spacing as illustrated in Fig.S4. We find that there is a binding between each pseudolayer (27.8 meV/atom) for the calculations with and without D2 correction. Using the correction leads to an increase in the binding energy  $\sim 22.4$  meV/atom (higher than that in non-vdW case) and a shorter equilibrium interlayer spacing. Thus, the stronger layered-binding is relevant to the higher intercalation voltage when PBE+ $U$ +D2 is employed.



**Fig. S4** Energy profile of Na<sub>2</sub>Fe(SO<sub>4</sub>)<sub>2</sub>·2H<sub>2</sub>O as a function of interlayer spacing calculated using PBE+ $U$  with and without D2 correction. The given energy is referenced with infinite separation ( $E=0$ eV).