## **Supporting Information**

Open-cell voltage and electrical conductivity of a protonic ceramic electrolyte under two chemical potential gradients

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## Calculation of proton and oxygen vacancy mobilities

The number of oxygen sites in BZY20 is restricted to 3 in total as follows:

$$[OH_o^{\bullet}] + [V_o^{\bullet\bullet}] + [O_o^{\times}] = 3$$
 (S1)

By combining Eq. (S1) with Eqs. (1) and (3) in the main text,  $K_w$  can be expressed as a function of proton concentration.

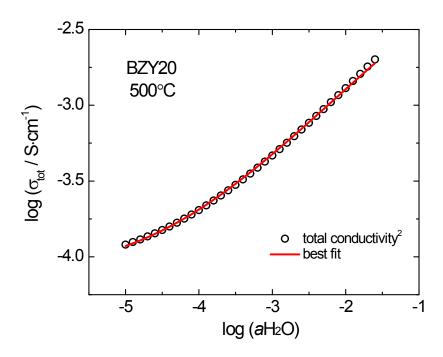
$$K_{w} = \frac{[OH_{O}^{\bullet}]^{2}}{aH_{2}O(3 - \frac{1}{2}[Y'_{Zr}] - \frac{1}{2}[OH_{O}^{\bullet}])(\frac{1}{2}[Y'_{Zr}] - \frac{1}{2}[OH_{O}^{\bullet}])}$$
(S2)

The  $K_{\rm w}$  for BZY20 at 500°C is reported by Yamazaki *et al.*<sup>1</sup>, and thus the proton and oxygen vacancy concentrations can be calculated at a given  $a{\rm H}_2{\rm O}$ .

The total electrical conductivity for BZY20 at 500°C can be obtained from the pre-factors and activation energies of the partial ionic conductivities reported by Nomura and Kageyama.<sup>2</sup> From the best fit of the general expression for the total conductivity expressed in Eq. (S3) to the actual values, the mobility of oxygen vacancies,  $u_{V_0^{\bullet\bullet}}$ , and that of protons,  $u_{OH_0^{\bullet}}$ , can be obtained.

$$\sigma_{tot} = \sigma_{OH_0^{\bullet}} + \sigma_{V_0^{\bullet\bullet}} = e[OH_0^{\bullet}]u_{OH_0^{\bullet}} + (2e)[V_0^{\bullet\bullet}]u_{V_0^{\bullet\bullet}}$$
(S3)

Figure S1 shows the fit result, giving  $u_{V_0^{\bullet}} = 1.81 \times 10^{-7}$  and  $u_{OH_0^{\bullet}} = 2.14 \times 10^{-5}$  (cm<sup>2</sup>V<sup>-1</sup>s<sup>-1</sup>).



**Figure S1.** Total electrical conductivity reported by Nomura and Kageyama<sup>2</sup> and the best fit for BZY20 at 500°C.

## Calculation of aO2 at 500°C

To calculate  $aO_2$  at fixed  $aH_2O$ , the reaction for thermolysis of  $H_2O(g)$  is considered with the assumption that thermolysis is negligible at low temperature. Hereafter, subscripts LT and HT indicate low and high temperatures, respectively, and  $p\Psi$  does partial pressure of species  $\Psi$ .

In the above,  $p_{\text{tot}}$  is total pressure considered as 1 atm at low temperature and even at 500°C after thermolysis, and  $\chi$  is the fractional concentration of H<sub>2</sub>O consumed by thermolysis. All activity values at 500°C are expressed in Eqs. (S5-S7), and the value of aH<sub>2</sub>O<sub>HT</sub> is fixed as A.

$$aH_{2}O_{HT} = \frac{pH_{2}O_{HT}}{p_{tot}} = \frac{(pH_{2}O_{LT}/p_{tot}) - \chi}{(pH_{2}O_{LT}/p_{tot}) + (pH_{2}I_{T}/p_{tot}) + (pO_{2}I_{T}/p_{tot}) + \frac{1}{2}\chi} = A$$
 (S5)

$$aH_{2,HT} = \frac{pH_{2,HT}}{p_{\text{tot}}} = \frac{(pH_{2,LT} / p_{\text{tot}}) + \chi}{(pH_2O_{LT} / p_{\text{tot}}) + (pH_{2,LT} / p_{\text{tot}}) + (pO_{2,LT} / p_{\text{tot}}) + \frac{1}{2}\chi}$$
(S6)

$$aO_{2,HT} = \frac{pO_{2,HT}}{p_{\text{tot}}} = \frac{(pO_{2,LT} / p_{\text{tot}}) + \frac{1}{2}\chi}{(pH_2O_{LT} / p_{\text{tot}}) + (pH_{2,LT} / p_{\text{tot}}) + (pO_{2,LT} / p_{\text{tot}}) + \frac{1}{2}\chi}$$
(S7)

The expression for the thermodynamic equilibrium constant at 500 °C,  $K_{\rm H,O}$ , is given by

$$K_{\rm H_2O}(500 \, {}^{\rm o}{\rm C}) = \frac{p{\rm H}_{2,HT} \cdot (p{\rm O}_{2,HT})^{\frac{1}{2}}}{p{\rm H}_2{\rm O}_{HT} \cdot (p_{\rm tot})^{\frac{1}{2}}}$$
(S8)

For the anode side, since water-saturated hydrogen is normally used,  $pO_{2,LT}$  can be considered to be 0, and thus,  $pH_2O_{LT} + pH_{2,LT} = 1$ . Overall, we have three unknown, x,  $pH_2O_{LT}$ , and  $pH_{2,LT}$ ,

and three equations, Eqs. (S5), (S8), and  $pH_2O_{LT} + pH_{2,LT} = 1$ . Thus,  $aO_{2,HT} (= pO_{2,HT}/p_{tot})$  can be obtained by solving these simultaneous equations.

For the cathode side,  $pH_{2,LT}$  can be considered to be 0, and thus,  $pO_{2,LT} = 0.21 \times (1 - pH_2O_{LT})$ . Overall, we have three unknowns, x,  $pH_2O_{LT}$ , and  $pO_{2,LT}$ , and three equations. Thus,  $pO_{2,HT}$  can be obtained by solving the simultaneous equations in the same way. The results of  $aO_{2,HT}$  (=  $pO_{2,HT}/p_{tot}$ ) for given  $aH_2O_{HT}$  (=  $pH_2O_{HT}/p_{tot}$ ) are summarized in Table S1.

Table S1. Thermodynamic calculation of aO<sub>2</sub> at 500 °C

Anode		Cathode	
aH₂O at 500 °C	aO <sub>2</sub> at 500 °C (calculation)	aH₂O at 500 °C	aO <sub>2</sub> at 500 °C (calculation)
1.00 × 10 <sup>-5</sup>	$1.98 \times 10^{-38}$	1.00 × 10 <sup>-5</sup>	2.10 × 10 <sup>-1</sup>
1.00 × 10 <sup>-4</sup>	$1.98 \times 10^{-36}$	1.00 × 10 <sup>-4</sup>	2.10 × 10 <sup>-1</sup>
$1.00 \times 10^{-3}$	1.98 × 10 <sup>-34</sup>	$1.00 \times 10^{-3}$	2.10 × 10 <sup>-1</sup>
1.00 × 10 <sup>-2</sup>	$2.02 \times 10^{-32}$	1.00 × 10 <sup>-2</sup>	2.08 × 10 <sup>-1</sup>
3.00 × 10 <sup>-2</sup>	1.89 × 10 <sup>-31</sup>	3.00 × 10 <sup>-2</sup>	$2.04 \times 10^{-1}$
5.00 × 10 <sup>-2</sup>	$5.48 \times 10^{-31}$	5.00 × 10 <sup>-2</sup>	2.00 × 10 <sup>-1</sup>
7.00 × 10 <sup>-2</sup>	$1.12 \times 10^{-30}$	7.00 × 10 <sup>-2</sup>	1.96 × 10 <sup>-1</sup>
$1.00 \times 10^{-1}$	$2.44 \times 10^{-30}$	$1.00 \times 10^{-1}$	1.91 × 10 <sup>-1</sup>

## References

- 1. Y. Yamazaki, P. Babilo and S. M. Haile, *Chem. Mat.*, 2008, **20**, 6352-6357.
- 2. K. Nomura and H. Kageyama, *Solid State Ionics*, 2007, **178**, 661-665.