

Electronic supplementary information (ESI): Density functional study of phase stabilities and Raman spectra of Yb_2O_3 , Yb_2SiO_5 and $\text{Yb}_2\text{Si}_2\text{O}_7$ under pressure

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1 Dependence of Raman spectra on hydrostatic pressure

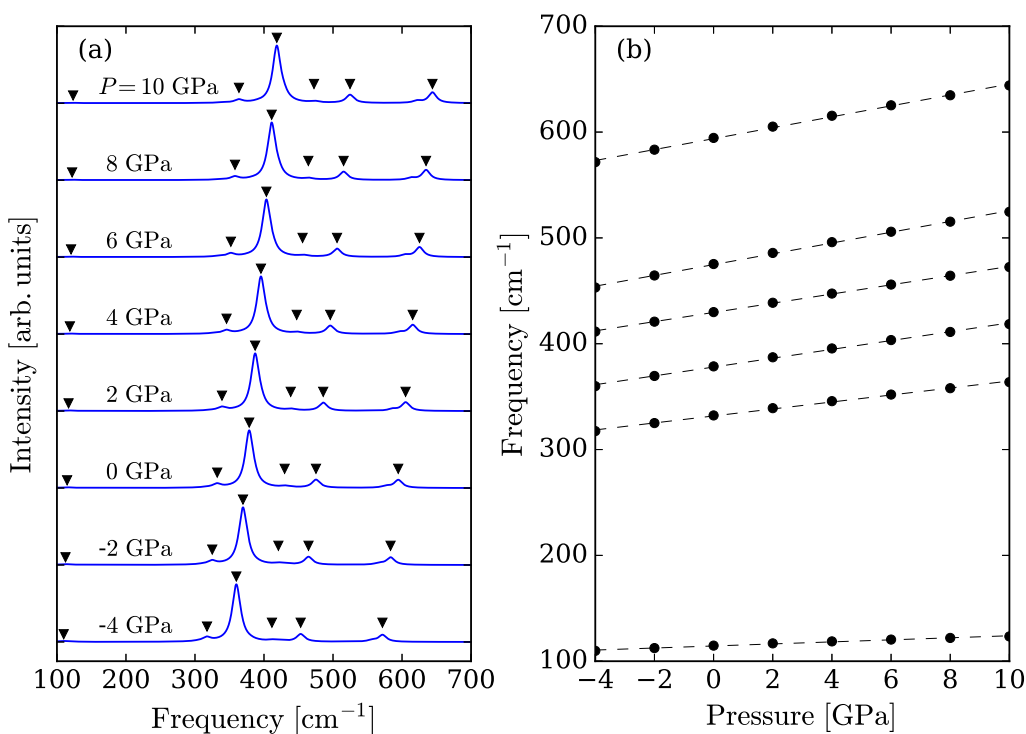


Fig. S1 Dependence of (a) Raman spectra and (b) peak positions of C-type Yb_2O_3 on hydrostatic pressure.

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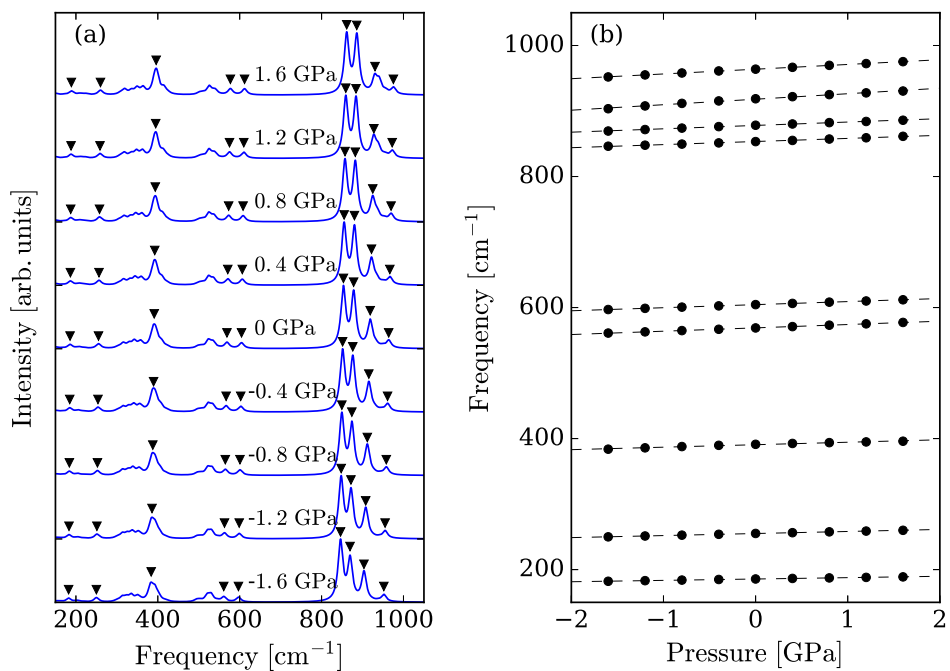


Fig. S2 Dependence of (a) Raman spectra and (b) peak positions of $X_2\text{-Yb}_2\text{SiO}_5$ on hydrostatic pressure.

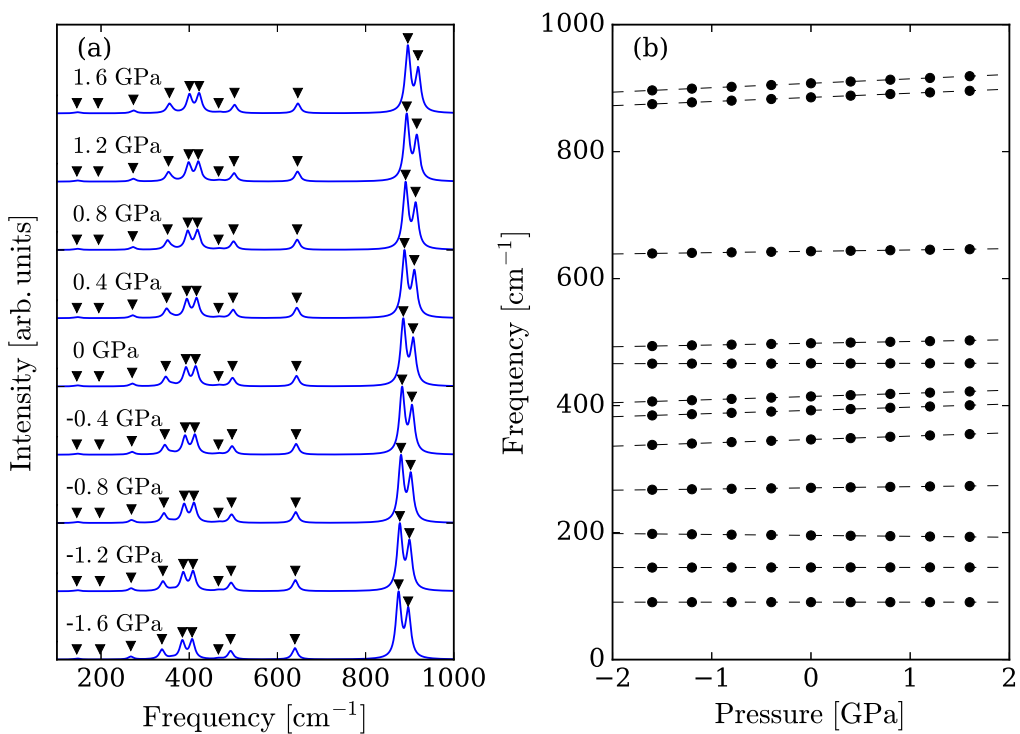


Fig. S3 Dependence of (a) Raman spectra and (b) peak positions of $\beta\text{-Yb}_2\text{Si}_2\text{O}_7$ on hydrostatic pressure.

2 Vibrations of Si_xO_y tetrahedral units in Yb_2SiO_5 and $\text{Yb}_2\text{Si}_2\text{O}_7$

The parameter used to measure maximum deviation in a bond length during vibration is defined in the manuscript as

$$\Delta l_{ij} = \frac{r_{ij}^2 - r_{ij}^0{}^2}{r_{ij}^0 A}. \quad (1)$$

Since expressing the interatomic vector by the atomic positions and vibrational displacement vectors leads to

$$\mathbf{r}_{ij} = (\mathbf{r}_j^0 + \mathbf{u}_j) - (\mathbf{r}_i^0 + \mathbf{u}_i) = \mathbf{r}_{ij}^0 + \mathbf{u}_j - \mathbf{u}_i, \quad (2)$$

the parameter, Δl_{ij} , can be rewritten as

$$\Delta l_{ij} = 2 \frac{\mathbf{r}_{ij}^0 \cdot (\mathbf{u}_j - \mathbf{u}_i)}{r_{ij}^0 A} + \frac{(\mathbf{u}_j - \mathbf{u}_i)^2}{r_{ij}^0 A}. \quad (3)$$

The second term on the right hand side of this equation vanishes when A is sufficiently smaller than r_{ij}^0 ($A/r_{ij}^0 \rightarrow 0$), leading to a converged value of Δl_{ij} in the limit of small A . To confirm this behaviour, the dependence of Δl_{ij} on A is examined for a mode of $\text{X}_2\text{-Yb}_2\text{SiO}_5$ as shown in Fig. S4. From the figure, we can see that the Δl_{ij} changes slightly for $A < 10^{-2}$. In this study, we used $A = 10^{-3}$ to calculate Δl_{ij} . The bond lengths between Si and O atoms and vibrational amplitudes, u_i^2/A^2 and Δl_{ij} , for characteristic modes with frequencies above 600 cm^{-1} are shown in Figs. S5 – S10, where the structural pictures of Si_xO_y were drawn using VESTA¹ and Raman spectra which were broadened by 7.0 and 0.05 cm^{-1} wide Lorentzians are shown by blue and red lines, respectively. The bond lengths in Si_3O_{10} and SiO_4 units in $\alpha\text{-Yb}_2\text{Si}_2\text{O}_7$ are summarised in Table S1.

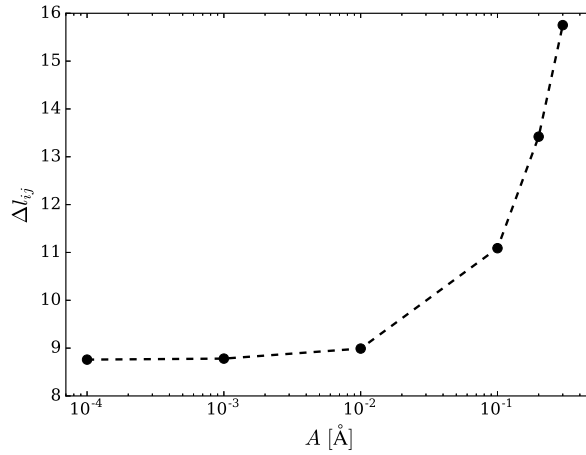


Fig. S4 Dependence of maximum deviations in a bond length, Δl_{ij} , on an amplitude, A , for the Si-O4 bond in $\text{X}_2\text{-Yb}_2\text{SiO}_5$ for the 853 cm^{-1} peak, shown in Fig. S5 (f).

References

- 1 K. Momma, F. Izumi, *J. Appl. Crystallogr.*, 2011, **44**, 1272–1276.

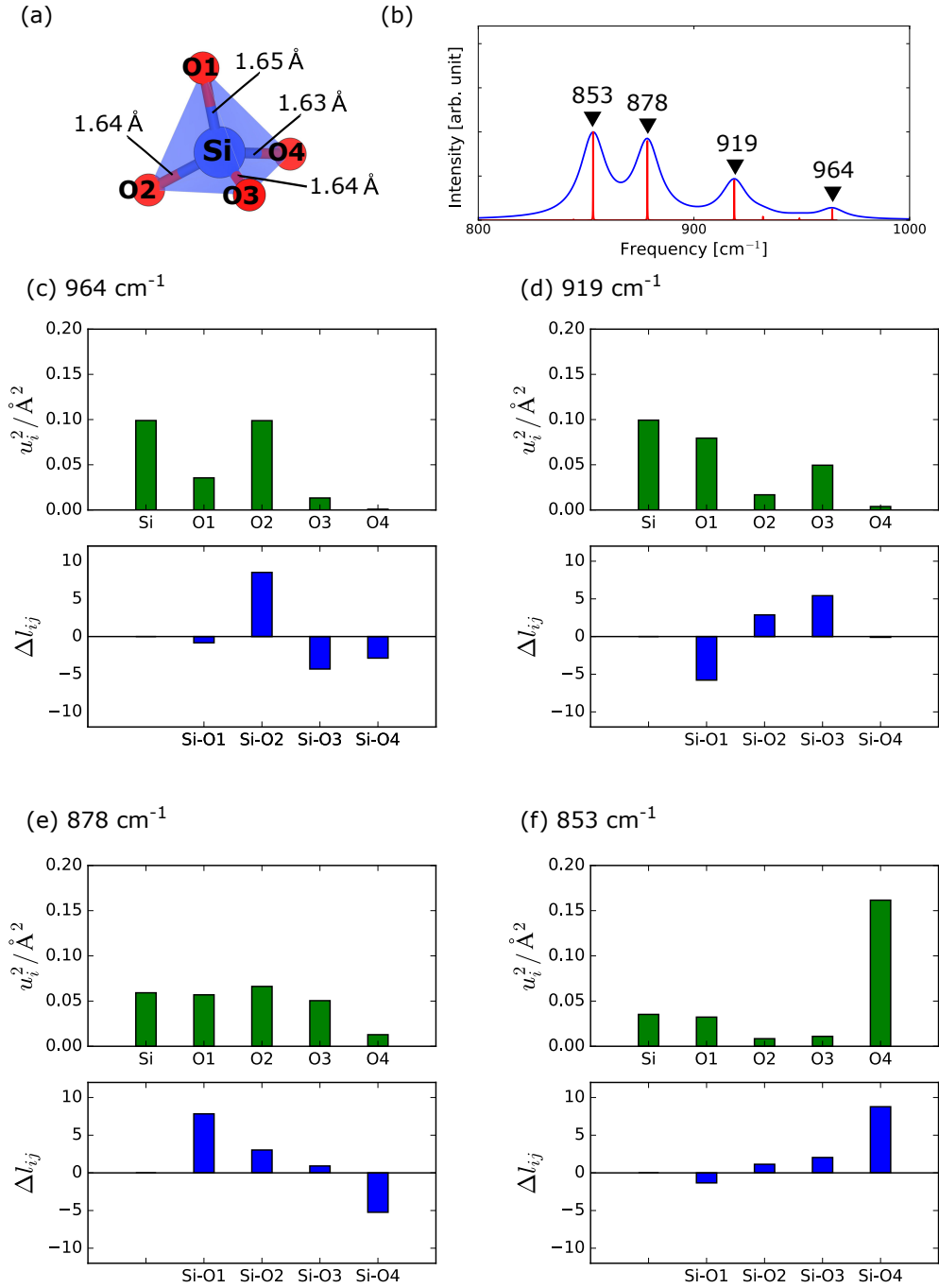


Fig. S5 (a) Structure of the SiO₄ unit in X₂-Yb₂SiO₅, (b) high frequency range of the calculated Raman spectrum of X₂-Yb₂SiO₅, and (c-f) vibrational amplitudes and Δl_{ij} for the normal modes at 964, 919, 878, and 853 cm⁻¹, respectively.

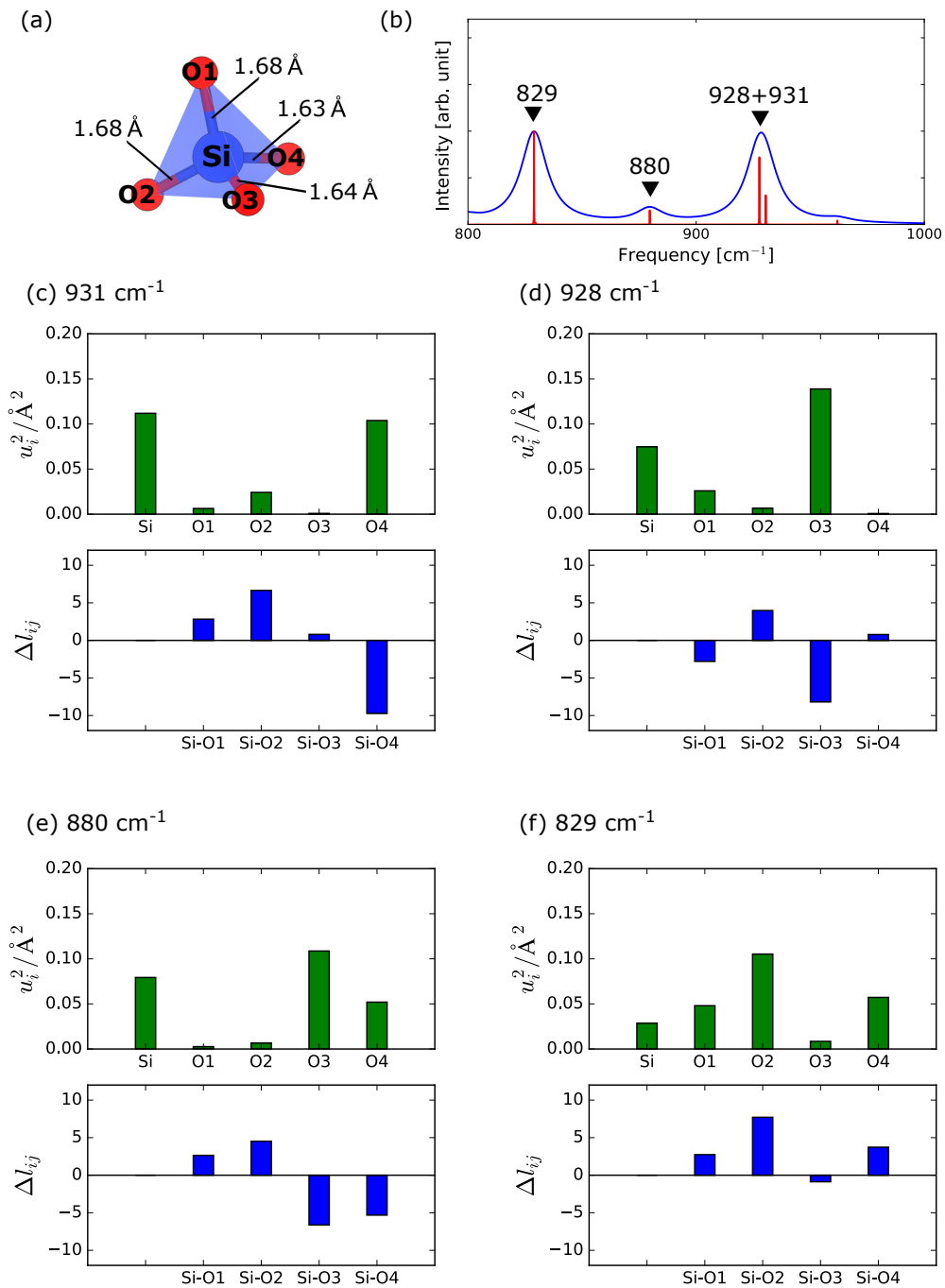


Fig. S6 (a) Structure of the SiO_4 unit in $\text{X}_1\text{-Yb}_2\text{SiO}_5$, (b) high frequency range of the calculated Raman spectrum of $\text{X}_1\text{-Yb}_2\text{SiO}_5$, and (c-f) vibrational amplitudes and Δl_{ij} for the normal modes at 931, 928, 880, and 829 cm^{-1} , respectively.

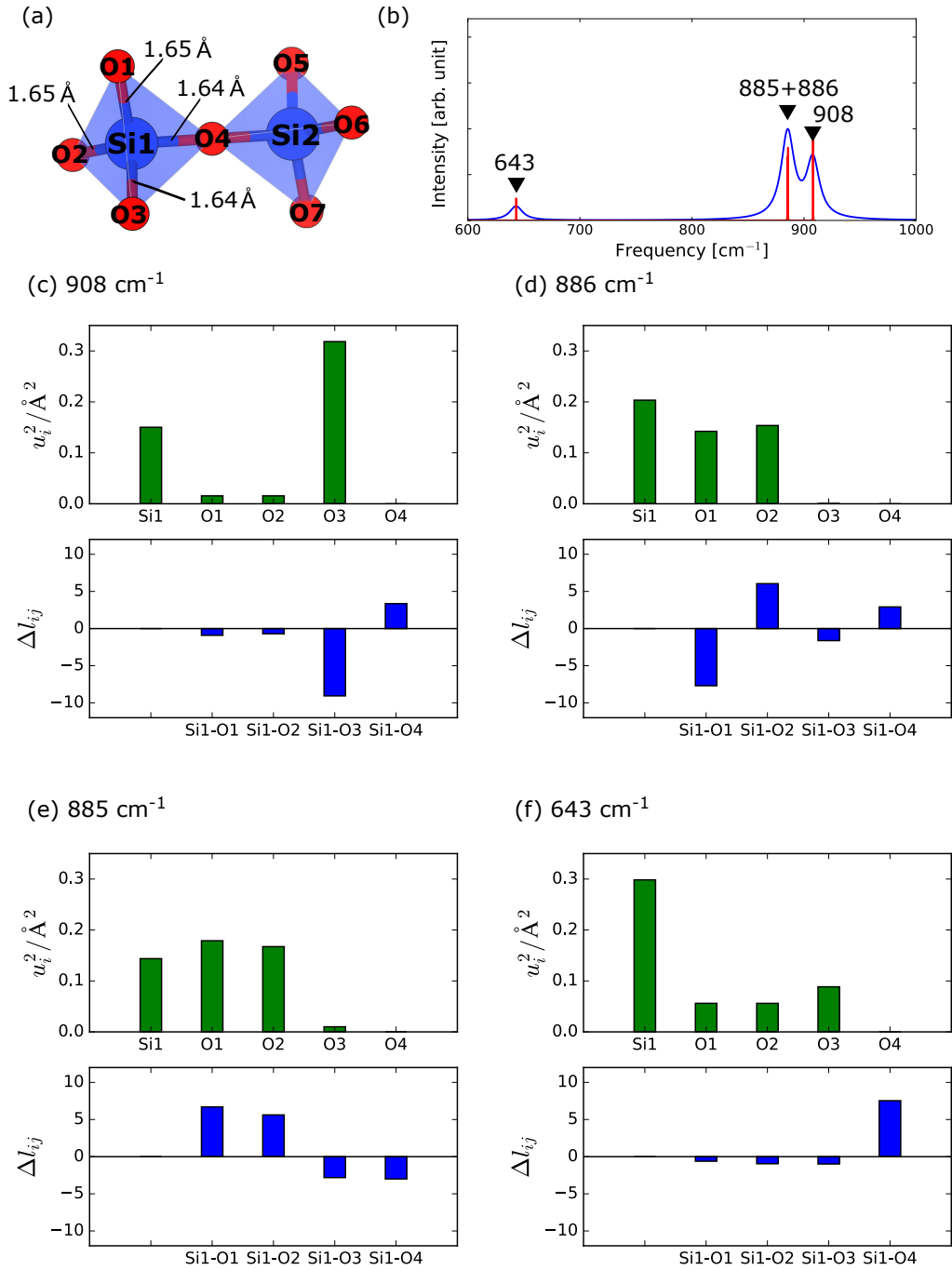


Fig. S7 (a) Structure of the Si_2O_7 unit in $\beta\text{-Yb}_2\text{Si}_2\text{O}_7$, (b) high frequency range of the calculated Raman spectrum of $\beta\text{-Yb}_2\text{Si}_2\text{O}_7$, and (c-f) vibrational amplitudes and Δl_{ij} for the normal modes at 908, 886, 885, and 643 cm^{-1} , respectively.

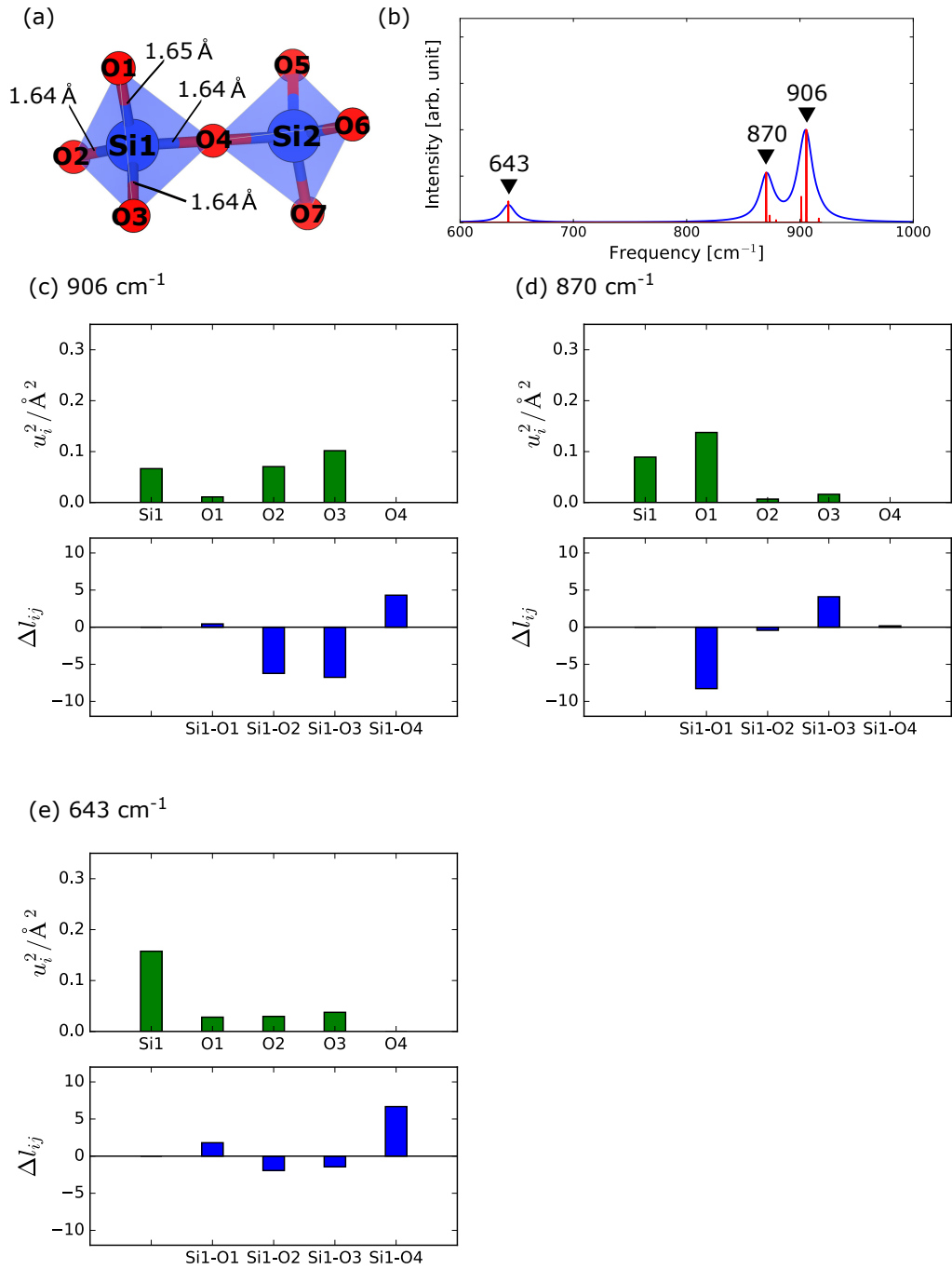


Fig. S8 (a) Structure of the Si_2O_7 unit in $\gamma\text{-Yb}_2\text{Si}_2\text{O}_7$, (b) high frequency range of the calculated Raman spectrum of $\gamma\text{-Yb}_2\text{Si}_2\text{O}_7$, and (c-e) vibrational amplitudes and Δl_{ij} for the normal modes at 906, 870, and 643 cm^{-1} , respectively.

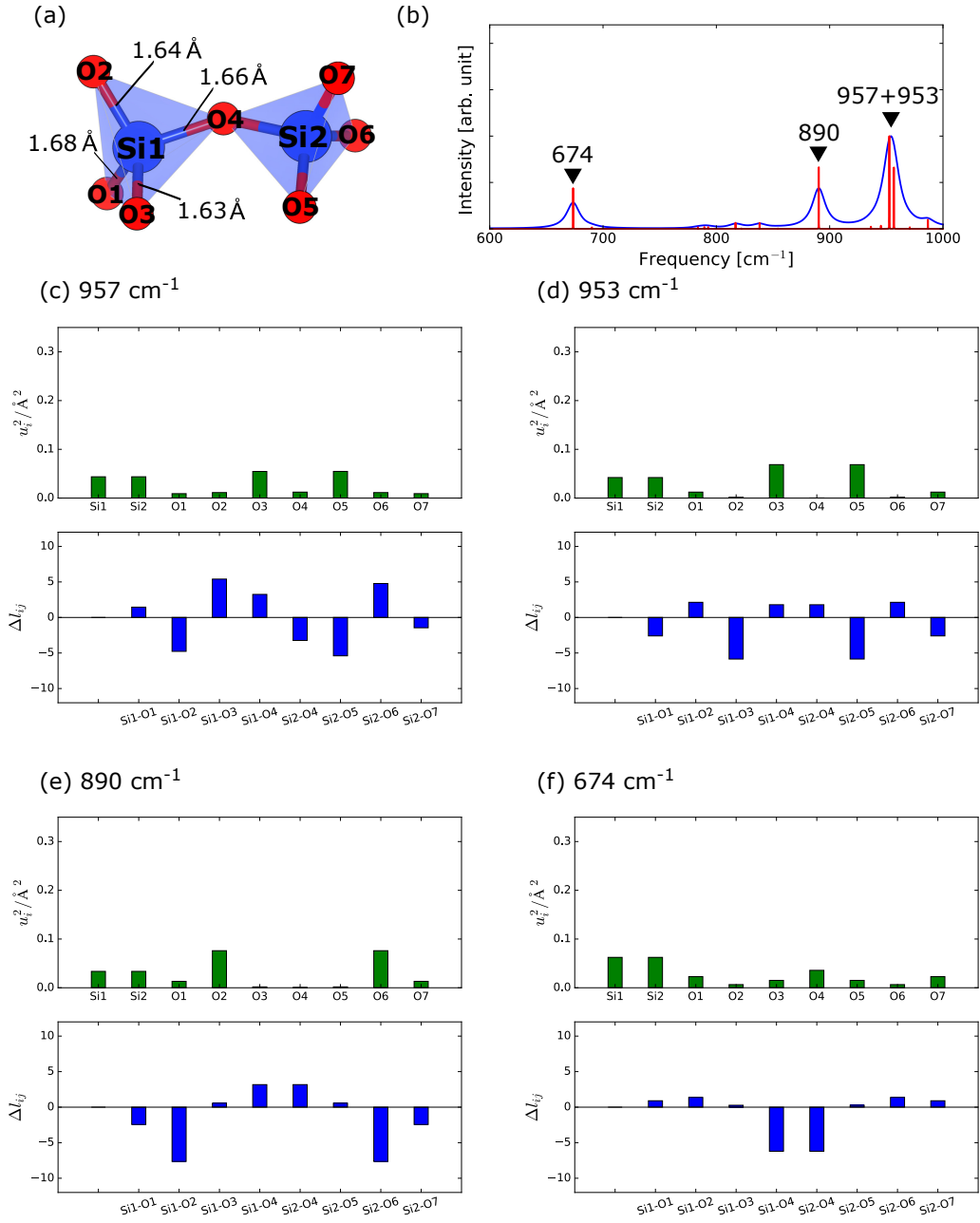


Fig. S9 (a) Structure of the Si_2O_7 unit in $\text{X-Yb}_2\text{Si}_2\text{O}_7$, (b) high frequency range of the calculated Raman spectrum of $\text{X-Yb}_2\text{Si}_2\text{O}_7$, and (c-f) vibrational amplitudes and Δl_{ij} for the normal modes at 957, 953, 890, and 674 cm^{-1} , respectively.

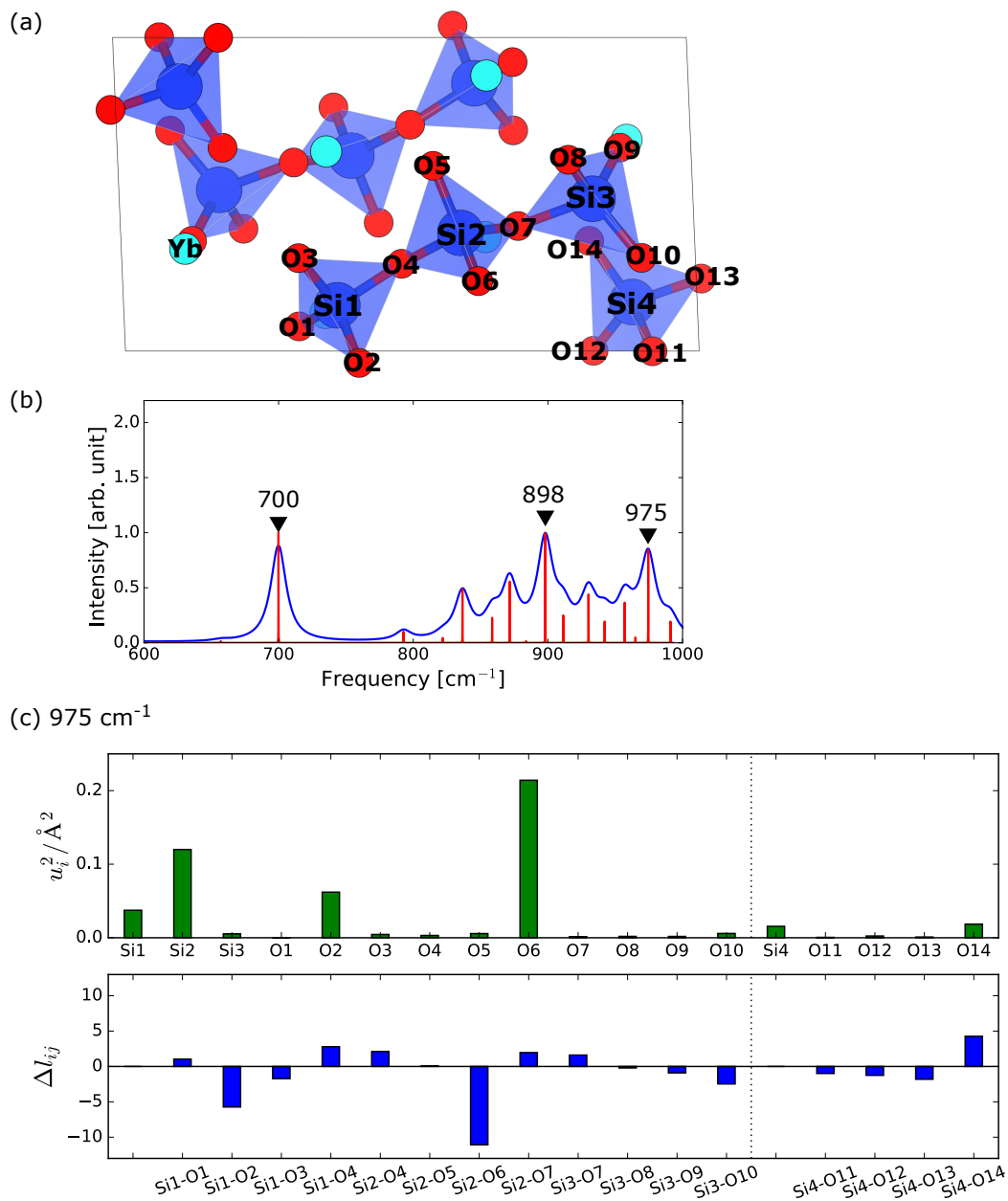
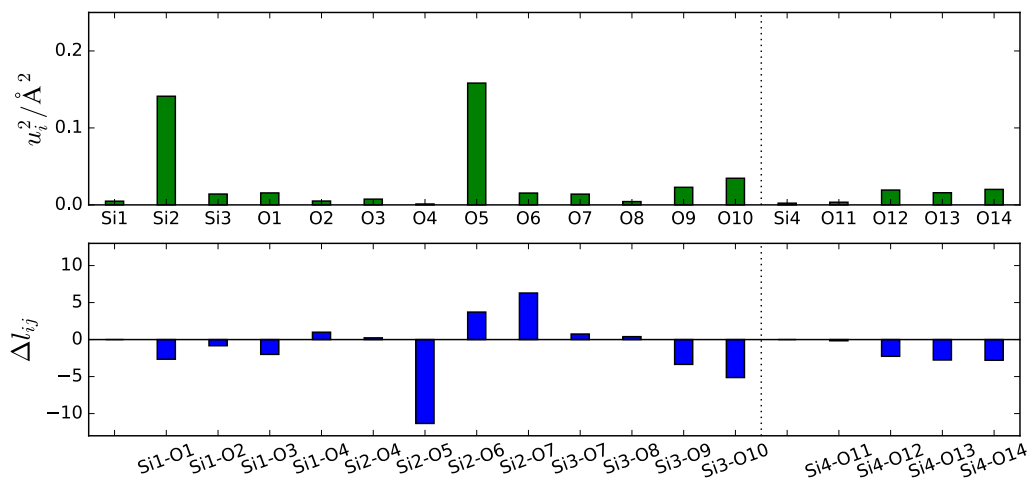


Fig. S10 (a) Structure of the Si_2O_7 unit in $\alpha\text{-Yb}_2\text{Si}_2\text{O}_7$, (b) high frequency range of the calculated Raman spectrum of $\alpha\text{-Yb}_2\text{Si}_2\text{O}_7$, and (c) vibrational amplitudes and Δl_{ij} for the normal mode at 975 cm^{-1} .

(e) 898 cm^{-1}



(f) 700 cm^{-1}

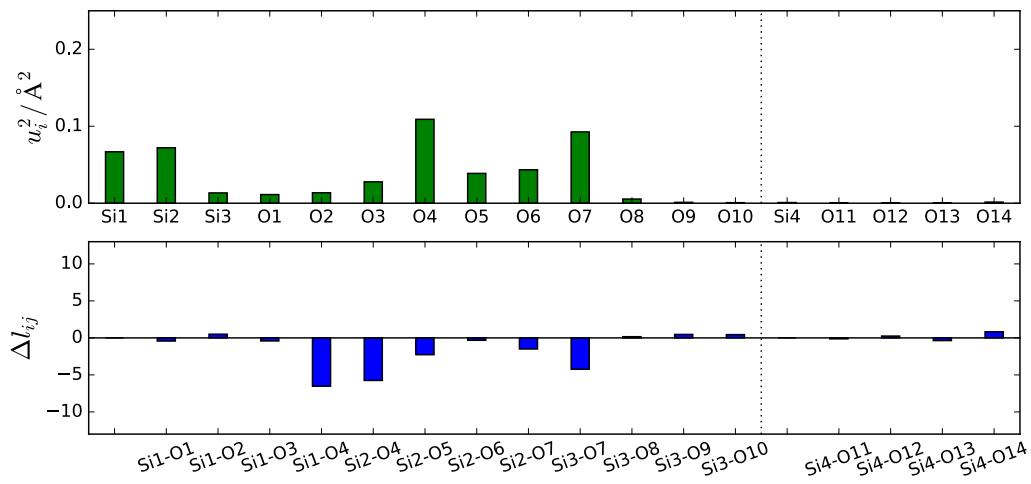


Fig. S10 (Continued) (d, e) normalized amplitudes and bond deviations for the normal modes at 898, and 700 cm^{-1} , respectively.

Table S1 Bond lengths between Si and O atoms in α -Yb₂Si₂O₇.

Bonds	Bond length [Å]
Si1-O1	1.62
Si1-O2	1.64
Si1-O3	1.66
Si1-O4	1.66
Si2-O4	1.66
Si2-O5	1.64
Si2-O6	1.62
Si2-O7	1.68
Si3-O7	1.75
Si3-O8	1.65
Si3-O9	1.63
Si3-O10	1.63
Si4-O11	1.67
Si4-O12	1.64
Si4-O13	1.62
Si4-O14	1.62