## Supplementary Information

## Nodal chain network, intersecting nodal rings and triple points

## coexisting in Ba<sub>3</sub>Si<sub>4</sub>

Jin Cai, <sup>a</sup> Yuee Xie, <sup>\*a</sup> Po-yao Chang, <sup>b</sup> Heung-Sik Kim, <sup>b</sup> Yuanping Chen<sup>\*a</sup>

<sup>a</sup> School of Physics and Optoelectronics, Xiangtan University, Xiangtan, Hunan, 411105, China <sup>b</sup> Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854-8019, USA



FIG. S1. (a) and (b) A whole topological phase for the structure without strain and under strain, respectively.



FIG. S2. (a<sub>1</sub>-a<sub>3</sub>) Band structures along three different directions parallel to  $k_{x/y/z}$  for the point N<sub>1</sub> on the nodal ring (on the plane  $k_z = 0$ ) in (a<sub>4</sub>), which belongs to type-I nodal ring. (b<sub>1</sub>-b<sub>3</sub>) Band structures along three different directions parallel to  $k_{x/y/z}$  for the point N<sub>2</sub> on the nodal ring (on the plane  $k_x = k_y$ ) in (b<sub>4</sub>), which belongs to type-II nodal ring. (c<sub>1</sub>-c<sub>3</sub>) Band structures along three different directions parallel to  $k_{x/y/z}$  for the point N<sub>3</sub> on the nodal ring (on the plane  $k_y = 0$ ) in (c<sub>4</sub>), which belongs to type-III nodal ring.



FIG. S3. Bulk band structure along A-B (a) and surface band structure along  $\overline{A} - \overline{B}$  on [001] surface (b) for Ba<sub>3</sub>Si<sub>4</sub> in the case of no strain. Bulk band structure along A-B (c) and surface band structure along  $\overline{A} - \overline{B}$  on [001] surface (d) for Ba<sub>3</sub>Si<sub>4</sub> under strain. (e) Bulk BZ and corresponding surface BZ, where the high-symmetry points and k paths are labeled.

	Without strain	With strain
Name	P42/MNM	PNNM
International Tables	136	58
Long Name	P42/M21/N2/M	P21/N21/N2/M
Identity	x y z	x y z
Rotation Axis	-x -y z	
	ух-г	-х -у z
	-y -x -z	
	х у - z	
Mirror plane	-y -x z	х у - г
	y x z	
Glide plane	x+1/2 -y+1/2 z+1/2	x+1/2 -y+1/2 z+1/2
	-x+1/2 y+1/2 z+1/2	-x+1/2 y+1/2 z+1/2

Table S1 Symmetry elements of  $Ba_3Si_4$  without/with strain.