# Supplementary Information 

# Nodal chain network, intersecting nodal rings and triple points coexisting in $\mathrm{Ba}_{3} \mathrm{Si}_{4}$ 

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FIG. S1. (a) and (b) A whole topological phase for the structure without strain and under strain, respectively.


FIG. S2. ( $\left.a_{1}-a_{3}\right)$ Band structures along three different directions parallel to $k_{x y / z}$ for the point $\mathrm{N}_{1}$ on the nodal ring (on the plane $k_{z}=0$ ) in ( $\mathrm{a}_{4}$ ), which belongs to type-I nodal ring. ( $\mathrm{b}_{1}-\mathrm{b}_{3}$ ) Band structures along three different directions parallel to $k_{x y / y}$ for the point $\mathrm{N}_{2}$ on the nodal ring (on the plane $k_{x}=k_{y}$ ) in ( $\mathrm{b}_{4}$ ), which belongs to type-II nodal ring. ( $\mathrm{c}_{1}-\mathrm{c}_{3}$ ) Band structures along three different directions parallel to $k_{x / y / z}$ for the point $\mathrm{N}_{3}$ on the nodal ring (on the plane $k_{y}=0$ ) in ( $\mathrm{c}_{4}$ ), which belongs to type-III nodal ring.


FIG. S3. Bulk band structure along A-B (a) and surface band structure along $\bar{A}-\bar{B}$ on [001] surface (b) for $\mathrm{Ba}_{3} \mathrm{Si}_{4}$ in the case of no strain. Bulk band structure along A-B (c) and surface band structure along $\bar{A}-\bar{B}$ on [001] surface (d) for $\mathrm{Ba}_{3} \mathrm{Si}_{4}$ under strain. (e) Bulk BZ and corresponding surface BZ , where the high-symmetry points and k paths are labeled.

|  | Without strain | With strain |
| :---: | :---: | :---: |
| Name | P42/MNM | PNNM |
| International Tables | 136 | 58 |
| Long Name | P42/M21/N2/M | P21/N21/N2/M |
| Identity | x y z | x y z |
| Rotation Axis | -x -y z | -x -y z |
|  | y $\mathrm{x}-\mathrm{z}$ |  |
|  | -y -x -z |  |
| Mirror plane | $x$ y -z | $x$ y -z |
|  | -y -x z |  |
|  | y x z |  |
| Glide plane | $\begin{array}{lll}x+1 / 2 & -y+1 / 2 & z+1 / 2\end{array}$ | $\begin{array}{lll}x+1 / 2 & -y+1 / 2 & z+1 / 2\end{array}$ |
|  | $\begin{array}{llll}-x+1 / 2 & y+1 / 2 & z+1 / 2\end{array}$ | $\begin{array}{llll}-x+1 / 2 & y+1 / 2 & z+1 / 2\end{array}$ |

Table S1 Symmetry elements of $\mathrm{Ba}_{3} \mathrm{Si}_{4}$ without/with strain.

