

Supplementary material for
“Inhomogeneous membrane receptor diffusion
explained by a fractional heteroscedastic time series model”
by
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1. Comparison of estimated ARFIMA parameters for wild and mutant cases

In Figures S1-S6 we present estimated parameters for different ARFIMA models for both mutant and wild cells cases. Figures S1-S3 were constructed for the 25 longest trajectories whereas Figures S4-S6 were created for the 100 longest trajectories. We can see that the differences are statistically significant only for the ARFIMA(0,d,1) processes for the smaller number of trajectories.

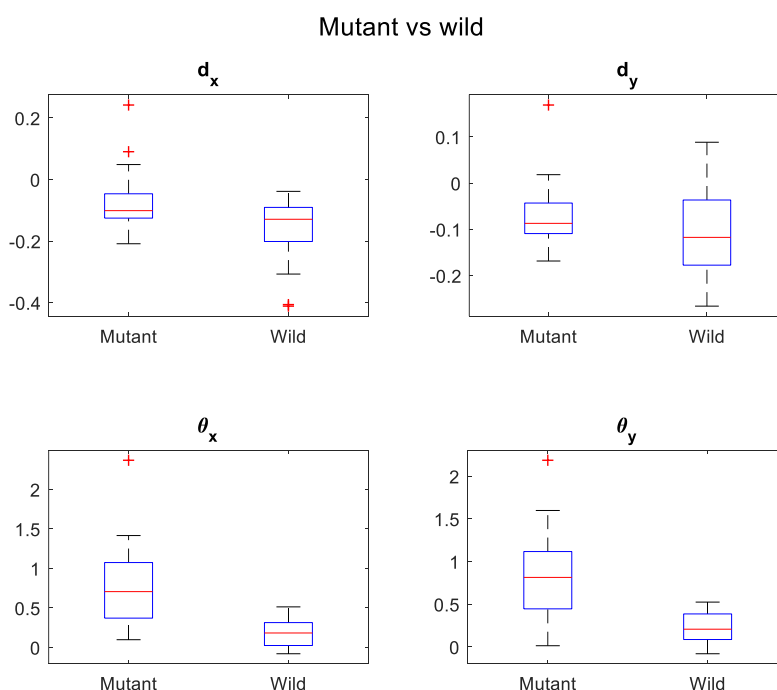


Figure S1. Fitted ARFIMA(0,d,1) parameters on the basis of 25 longest trajectories.

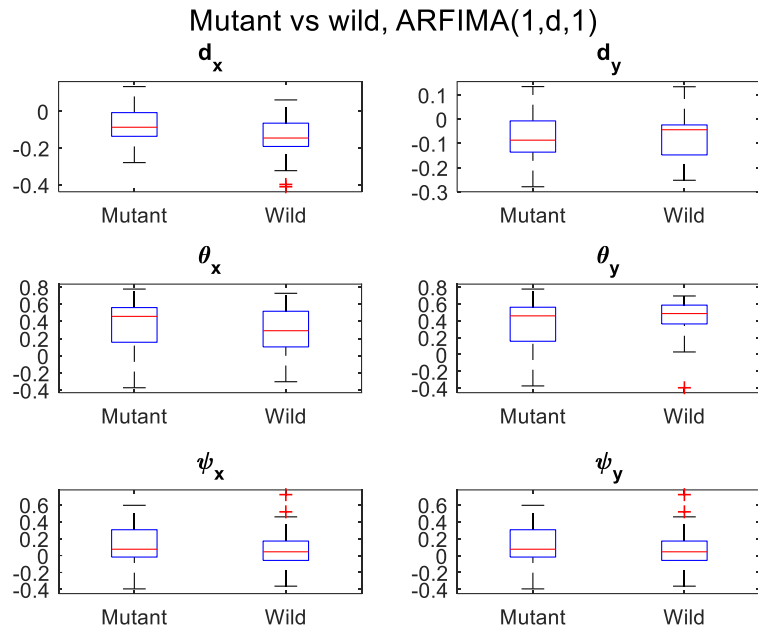


Figure S2. Fitted ARFIMA(1,d,1) parameters on the basis of 25 longest trajectories.

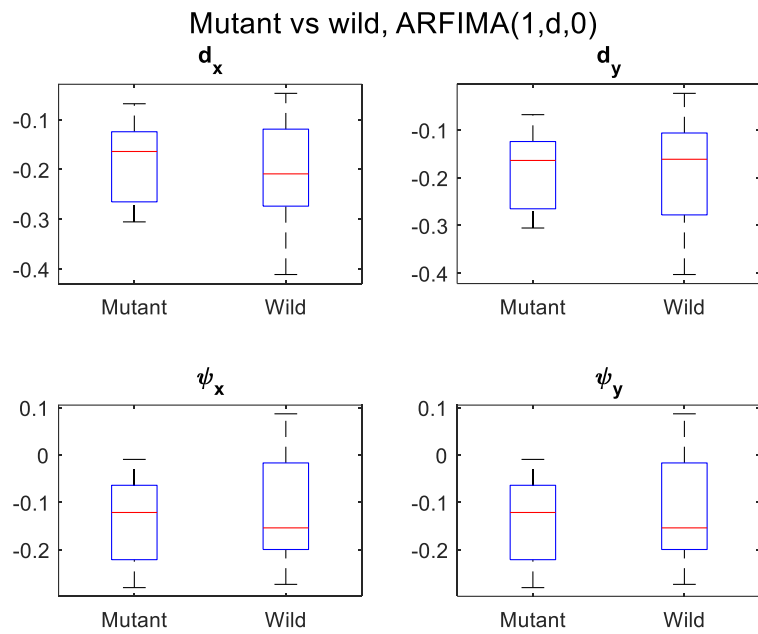


Figure S3. Fitted ARFIMA(1,d,0) parameters on the basis of 25 longest trajectories.

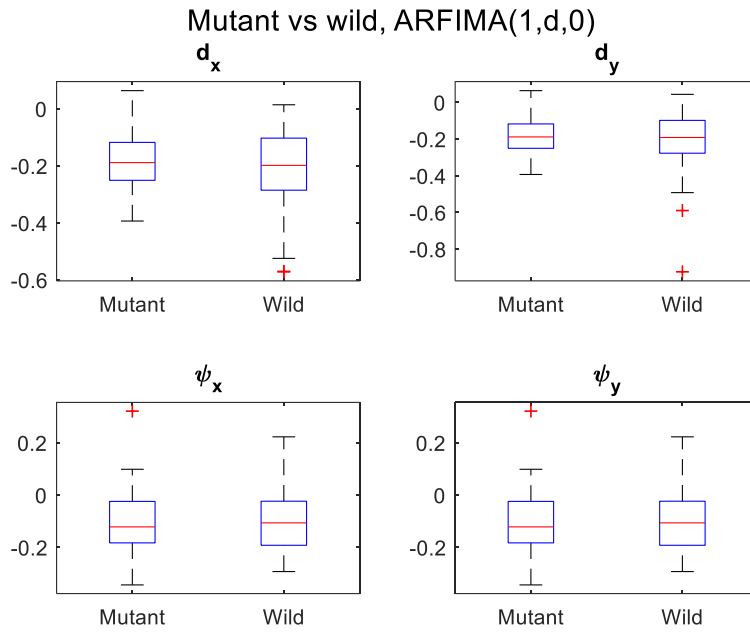


Figure S4. Fitted ARFIMA(1,d,0) parameters on the basis of 100 longest trajectories.

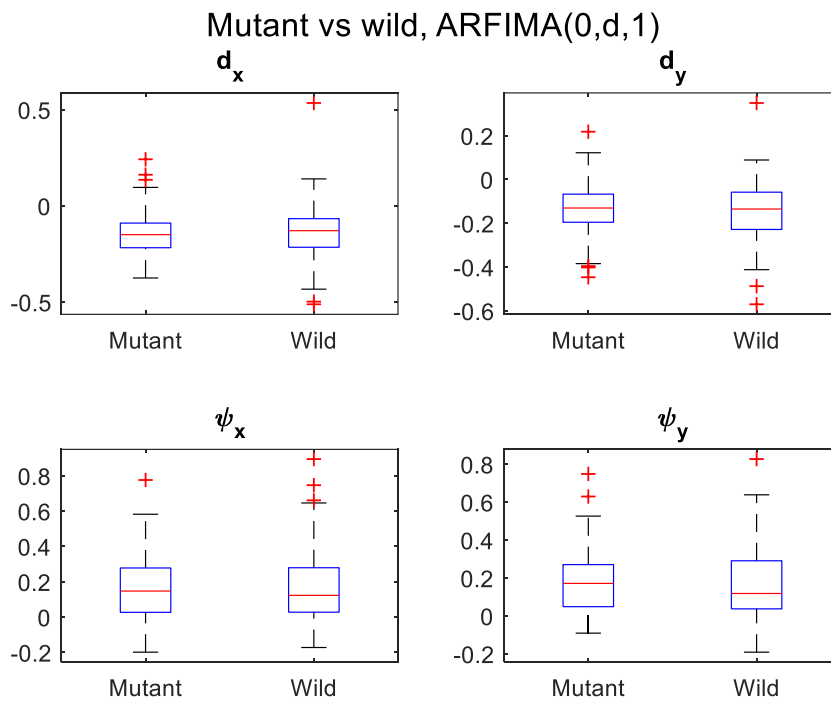


Figure S5. Fitted ARFIMA(0,d,1) parameters on the basis of 100 longest trajectories.

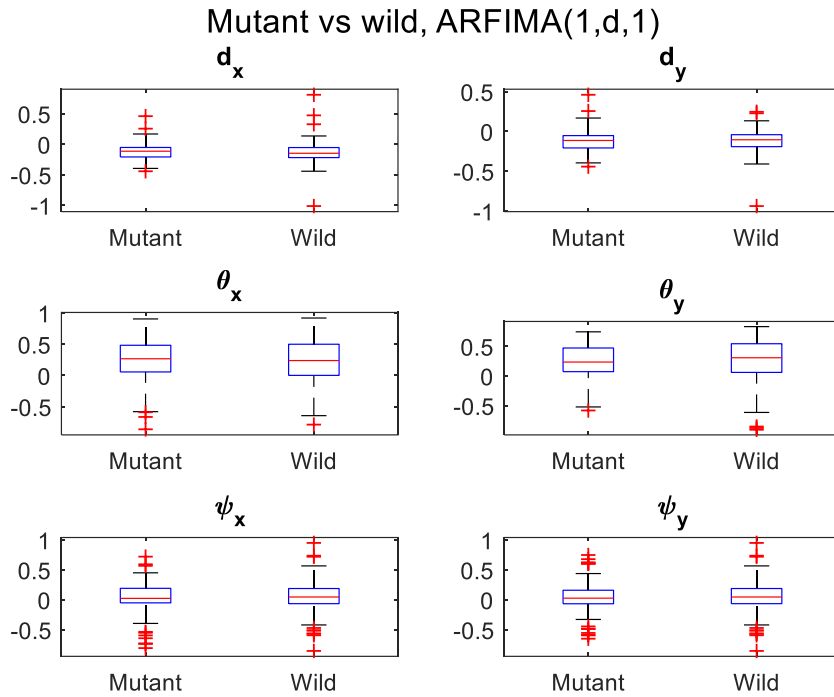


Figure S6. Fitted ARFIMA(1,d,1) parameters on the basis of 100 longest trajectories.

2. Validation of fitted ARFIMA-GARCH models

In Tables S1-S6 we present results of the validation procedure for different ARFIMA-GARCH and ARFIMA models for the 25 longest trajectories. We can clearly see that the added GARCH part significantly improves the fit. Moreover, the simple ARFIMA-GARCH(0,d,1)-(1,1) produces quite acceptable results. We also checked the simplest ARFIMA(0,d,0) and ARFIMA-GARCH(0,d,0)-(1,1) models but they were clearly rejected for almost all trajectories.

Table S1. P-values of the Ljung-Box test for the residuals and squared residuals of the fitted ARFIMA-GARCH(2,d,2)-(1,1) model calculated for different lags.

ARFIMA(2,d,2)-GARCH(1,1)		Weighted Ljung-Box for standardized residuals			Weighted Ljung-Box for standardized squared residuals		
Trajectory number		lag 1	lag 5	lag 9	lag 1	lag 5	lag 9
1		0.84	1.00	0.97	0.43	0.72	0.44
2		0.91	0.70	0.72	0.90	0.76	0.78
3		0.74	0.83	0.46	0.00	0.01	0.05
4		0.96	0.95	0.94	0.00	0.00	0.00
5		0.13	0.20	0.21	0.02	0.04	0.04
6		0.82	0.63	0.61	0.95	0.96	0.98
7		0.23	0.21	0.09	0.44	0.14	0.14
8		0.57	0.82	0.71	0.01	0.02	0.07
9		0.95	0.06	0.03	0.78	0.25	0.16
10		0.88	0.78	0.81	0.75	0.77	0.77
11		0.52	0.80	0.75	0.04	0.09	0.12
12		0.91	0.90	0.71	0.00	0.00	0.00
13		0.93	0.78	0.43	0.16	0.21	0.17
14		0.98	0.99	0.98	0.83	0.55	0.49
15		0.50	0.21	0.28	0.94	0.87	0.91
16		0.65	0.73	0.81	0.22	0.29	0.20
17		0.85	0.70	0.77	0.13	0.40	0.47
18		0.58	0.87	0.63	0.03	0.14	0.19
19		0.94	0.90	0.52	0.00	0.00	0.00
20		0.89	0.50	0.28	0.56	0.76	0.88
21		0.60	0.81	0.92	0.12	0.45	0.55
22		0.83	0.97	0.97	0.01	0.06	0.08
23		0.63	0.93	0.98	0.93	0.29	0.18
24		0.98	0.95	0.92	0.00	0.01	0.03
25		0.66	0.93	0.58	0.14	0.35	0.18

Table S2. P-values of the Ljung-Box test for the residuals and squared residuals of the fitted ARFIMA(2,d,2) model calculated for different lags.

ARFIMA(2,d,2)		Weighted Ljung-Box for standardized residuals			Weighted Ljung-Box for standardized squared residuals		
Trajectory number		lag 1	lag 5	lag 9	lag 1	lag 5	lag 9
1		0.97	1.00	0.97	0.47	0.00	0.00
2		0.97	1.00	0.98	0.63	0.75	0.75
3		0.99	0.81	0.57	0.00	0.01	0.06
4		0.98	0.96	0.94	0.00	0.00	0.00
5		0.97	0.53	0.53	0.00	0.00	0.00
6		0.74	0.86	0.79	0.00	0.00	0.00
7		0.96	0.98	0.84	0.00	0.00	0.00
8		0.76	0.89	0.77	0.01	0.02	0.07
9		0.55	0.14	0.07	0.58	0.32	0.23
10		0.99	0.79	0.78	0.33	0.40	0.46
11		0.96	1.00	0.88	0.03	0.07	0.07
12		0.93	0.93	0.81	0.00	0.00	0.00
13		0.93	0.72	0.20	0.00	0.00	0.00
14		0.89	1.00	0.99	0.86	0.43	0.32
15		0.97	0.86	0.89	0.20	0.27	0.16
16		0.73	0.71	0.78	0.18	0.28	0.18
17		0.99	0.96	0.79	0.00	0.00	0.00
18		0.45	0.87	0.63	0.00	0.00	0.00
19		0.93	0.91	0.51	0.00	0.00	0.00
20		0.98	1.00	0.80	0.00	0.00	0.00
21		0.68	0.78	0.88	0.00	0.00	0.01
22		0.98	0.99	0.98	0.02	0.12	0.15
23		0.97	1.00	1.00	0.03	0.02	0.02
24		0.90	0.94	0.92	0.01	0.02	0.05
25		0.96	0.98	0.63	0.17	0.38	0.17

Table S3. P-values of the Ljung-Box test for the residuals and squared residuals of the fitted ARFIMA-GARCH(1,d,1)-(1,1) model calculated for different lags.

ARFIMA(1,d,1)-GARCH(1,1)		Weighted Ljung-Box for standardized residuals			Weighted Ljung-Box for standardized squared residuals		
Trajectory number		lag 1	lag 5	lag 9	lag 1	lag 5	lag 9
1		0.89	0.81	0.82	0.41	0.73	0.47
2		0.57	0.77	0.74	0.52	0.67	0.74
3		0.92	0.62	0.39	0.01	0.03	0.11
4		0.96	0.89	0.84	0.00	0.00	0.00
5		0.85	0.60	0.65	0.49	0.35	0.24
6		0.69	0.69	0.53	0.98	0.93	0.99
7		0.22	0.19	0.07	0.51	0.15	0.14
8		0.77	0.65	0.41	0.02	0.09	0.24
9		0.86	0.42	0.24	0.89	0.19	0.02
10		0.67	0.15	0.22	0.61	0.86	0.78
11		0.43	0.17	0.17	0.02	0.06	0.11
12		0.79	0.54	0.25	0.00	0.00	0.00
13		0.89	0.52	0.25	0.15	0.25	0.23
14		0.73	0.35	0.43	0.78	0.39	0.36
15		0.59	0.16	0.14	0.36	0.27	0.38
16		0.94	0.58	0.38	0.15	0.26	0.24
17		0.58	0.74	0.80	0.03	0.13	0.21
18		0.74	0.82	0.56	0.00	0.00	0.00
19		0.79	0.89	0.45	0.00	0.00	0.00
20		0.90	0.89	0.42	0.31	0.54	0.72
21		0.82	0.50	0.65	0.06	0.28	0.40
22		0.85	0.88	0.90	0.01	0.05	0.06
23		0.66	0.97	0.99	0.94	0.33	0.20
24		0.93	0.85	0.81	0.01	0.02	0.05
25		0.95	0.73	0.30	0.11	0.38	0.16

Table S4. P-values of the Ljung-Box test for the residuals and squared residuals of the fitted ARFIMA(1,d,1) model calculated for different lags.

ARFIMA(1,d,1)		Weighted Ljung-Box for standardized residuals			Weighted Ljung-Box for standardized squared residuals		
Trajectory number		lag 1	lag 5	lag 9	lag 1	lag 5	lag 9
1		0.90	0.82	0.76	0.47	0.00	0.00
2		0.97	0.95	0.88	0.84	0.80	0.79
3		0.95	0.60	0.38	0.00	0.01	0.04
4		1.00	0.90	0.84	0.00	0.00	0.00
5		0.77	0.41	0.44	0.00	0.00	0.00
6		0.90	0.75	0.67	0.00	0.00	0.00
7		0.96	0.85	0.62	0.00	0.00	0.00
8		0.73	0.69	0.42	0.02	0.08	0.21
9		0.98	0.39	0.25	0.90	0.61	0.05
10		0.43	0.11	0.18	0.34	0.56	0.53
11		0.67	0.23	0.22	0.01	0.03	0.08
12		0.89	0.56	0.24	0.00	0.00	0.00
13		0.96	0.45	0.07	0.00	0.00	0.00
14		0.90	0.38	0.46	0.94	0.25	0.19
15		0.78	0.19	0.17	0.29	0.25	0.31
16		0.96	0.58	0.37	0.10	0.20	0.19
17		0.99	0.70	0.57	0.00	0.00	0.00
18		0.92	0.85	0.63	0.00	0.00	0.00
19		0.79	0.90	0.45	0.00	0.00	0.00
20		0.99	0.99	0.62	0.00	0.00	0.00
21		0.83	0.29	0.40	0.00	0.00	0.00
22		0.96	0.90	0.92	0.02	0.11	0.13
23		0.97	1.00	1.00	0.03	0.02	0.02
24		0.98	0.86	0.82	0.01	0.03	0.07
25		0.94	0.73	0.31	0.09	0.30	0.14

Table S5. P-values of the Ljung-Box test for the residuals and squared residuals of the fitted ARFIMA-GARCH(0,d,1)-(1,1) model calculated for different lags.

ARFIMA(0,d,1)-GARCH(1,1)	Weighted Ljung-Box for standardized residuals			Weighted Ljung-Box for standardized squared residuals		
	lag 1	lag 5	lag 9	lag 1	lag 5	lag 9
Trajectory number						
1	0.47	0.27	0.35	0.44	0.75	0.51
2	0.67	0.11	0.11	0.12	0.17	0.27
3	0.70	0.18	0.07	0.00	0.01	0.06
4	0.80	0.63	0.63	0.00	0.00	0.00
5	0.90	0.52	0.55	0.60	0.56	0.44
6	0.46	0.47	0.28	0.94	0.97	0.99
7	0.03	0.05	0.03	0.38	0.18	0.19
8	0.00	0.00	0.00	0.11	0.34	0.59
9	0.00	0.00	0.00	0.75	0.64	0.24
10	0.32	0.03	0.03	0.74	0.91	0.86
11	0.38	0.05	0.06	0.01	0.05	0.10
12	0.09	0.00	0.00	0.00	0.00	0.01
13	0.87	0.53	0.25	0.15	0.25	0.23
14	0.75	0.10	0.14	0.69	0.60	0.58
15	0.73	0.14	0.13	0.32	0.16	0.22
16	0.73	0.52	0.36	0.10	0.19	0.20
17	0.73	0.73	0.80	0.07	0.21	0.33
18	0.84	0.82	0.58	0.00	0.00	0.01
19	0.93	0.72	0.28	0.00	0.00	0.00
20	0.50	0.63	0.28	0.00	0.00	0.01
21	0.88	0.49	0.61	0.04	0.19	0.31
22	0.33	0.05	0.10	0.01	0.02	0.02
23	0.71	0.96	0.99	0.87	0.40	0.24
24	0.76	0.58	0.64	0.00	0.02	0.05
25	0.82	0.63	0.28	0.10	0.35	0.15

Table S6. P-values of the Ljung-Box test for the residuals and squared residuals of the fitted ARFIMA(0,d,1) model calculated for different lags.

ARFIMA(0,d,1)	Weighted Ljung-Box for standardized residuals			Weighted Ljung-Box for standardized squared residuals		
	lag 1	lag 5	lag 9	lag 1	lag 5	lag 9
Trajectory number						
1	0.19	0.07	0.09	0.28	0.00	0.00
2	0.34	0.12	0.13	0.92	0.67	0.68
3	0.79	0.16	0.06	0.00	0.00	0.02
4	0.76	0.60	0.62	0.00	0.00	0.00
5	0.99	0.24	0.28	0.00	0.00	0.00
6	0.86	0.69	0.53	0.00	0.00	0.00
7	0.82	0.81	0.58	0.00	0.00	0.00
8	0.00	0.00	0.00	0.11	0.30	0.53
9	0.00	0.00	0.00	0.94	0.05	0.00
10	0.14	0.02	0.02	0.92	0.94	0.90
11	0.63	0.06	0.07	0.01	0.03	0.08
12	0.05	0.00	0.00	0.00	0.00	0.00
13	0.82	0.41	0.07	0.00	0.00	0.00
14	0.58	0.10	0.15	0.89	0.35	0.28
15	0.99	0.18	0.17	0.27	0.22	0.27
16	0.76	0.52	0.35	0.06	0.14	0.15
17	0.28	0.01	0.01	0.00	0.00	0.00
18	1.00	0.83	0.60	0.00	0.00	0.00
19	0.94	0.72	0.28	0.00	0.00	0.00
20	0.43	0.37	0.17	0.00	0.00	0.00
21	0.88	0.23	0.31	0.00	0.00	0.00
22	0.25	0.05	0.10	0.02	0.07	0.06
23	0.99	0.99	1.00	0.03	0.02	0.02
24	0.69	0.58	0.64	0.01	0.03	0.09
25	0.80	0.62	0.28	0.08	0.28	0.13