

**Cation-Selective and Anion-Controlled Benzothiazolyl-Attached
Macrocyclic as NLO-based Cation Detection: Variational First
Hyperpolarizabilities**

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The electronic density variation between the excited $\rho_{ES}(r)$ and ground states $\rho_{GS}(r)$ are defined as follows:

$$\Delta\rho(r) = \rho_{ES}(r) - \rho_{GS}(r) \quad (1)$$

Subsequently, the function is divided into two categories representing an increase (ρ_+) or a decrease (ρ_-) of the density in space resulting from the electronic transition.

Hence the q^{CT} is given as:

$$q^{CT} = \int \rho_+(r)dr = \int \rho_-(r)dr \quad (2)$$

The density distribution barycenters (r_+ and r_-), giving the starting and ending points of transfer, corresponding to ρ_+ and ρ_- are defined by:

$$\begin{aligned} r_+ &= (x_+, y_+, z_+) = \frac{1}{q^{CT}} \int r \rho_+(r) dr \\ r_- &= (x_-, y_-, z_-) = \frac{1}{q^{CT}} \int r \rho_-(r) dr \end{aligned} \quad (3)$$

And d^{CT} is thus defined as the distance separating between the two points (r_+ and r_-), which is an important factor to evaluate the CT length.

$$d^{CT} = |r_+ - r_-| \quad (4)$$

Eventually, an index H is defined as half of the sum of centroids axis (x axis) along the molecular structure direction.

$$H = \frac{\sigma_{+x} + \sigma_{-x}}{2} \quad (5)$$

If $H \cong d^{CT}$, there is a significant overlap between the centroids along this axis.

Finally, the t index is proposed to evaluate the degree of separation between $\rho_+(r)$ and $\rho_-(r)$ regions.

$$t = d^{CT} - H \quad (6)$$

Large t value indicates less overlap between the regions with an increment or a depletion of the density.

Table S1. Individual components of first hyperpolarizabilities and total first hyperpolarizabilities (10^{-30} esu) of **L** and **L*M** complexes computed at various levels of theory in acetonitrile solution.

Complex	Functionals	β_x	β_y	β_z	β_{tot}	$\beta_{\text{tot}}(\mathbf{L})/\beta_{\text{tot}}(\mathbf{L}^*\mathbf{M})$
L	CAM-B3LYP	125.7	-3.7	0.6	125.8	-
	ω B97XD	116.1	-3.1	1.0	116.1	-
	M06-2X	133.5	-3.8	0.5	133.5	-
L*Na⁺	CAM-B3LYP	97.8	1.0	6.0	98.0	1.3
	ω B97XD	90.7	1.5	6.4	91.0	1.3
	M06-2X	102.7	1.2	6.2	102.9	1.3
L*K⁺	CAM-B3LYP	102.2	0.7	7.3	102.4	1.2
	ω B97XD	94.6	1.2	7.5	95.0	1.2
	M06-2X	107.5	0.8	7.5	107.8	1.2
L*Mg²⁺	CAM-B3LYP	49.0	-2.8	-1.6	49.1	2.6
	ω B97XD	46.1	-2.3	-0.9	46.1	2.5
	M06-2X	48.9	-2.5	-2.2	49.0	2.7
L*Ca²⁺	CAM-B3LYP	77.6	0.9	5.4	77.8	1.6
	ω B97XD	72.6	1.6	5.5	72.9	1.6
	M06-2X	81.0	0.9	4.9	81.2	1.6
L*Zn²⁺	CAM-B3LYP	46.5	-3.9	-3.6	46.8	2.7
	ω B97XD	43.6	-3.3	-2.9	43.8	2.7
	M06-2X	47.7	-3.4	-3.0	48.0	2.8
L*Cd²⁺	CAM-B3LYP	58.9	-3.7	1.9	59.0	2.1
	ω B97XD	54.8	-3.0	2.5	54.9	2.1
	M06-2X	60.9	-3.4	2.3	61.0	2.2
L*Hg²⁺	CAM-B3LYP	-2.6	6.4	-4.5	8.2	15.3
	ω B97XD	-3.9	7.6	-5.4	10.1	11.5
	M06-2X	-3.0	6.7	-4.4	8.6	15.5

Hg(L)(Cl)₂	CAM-B3LYP	119.5	-2.0	12.1	120.1	1.0
	ωB97XD	110.9	-0.5	12.0	115.7	1.0
	M06-2X	125.1	-0.9	11.1	125.6	1.1
Hg(L)(ClO₄)₂	CAM-B3LYP	38.0	2.0	5.7	38.6	3.3
	ωB97XD	34.3	2.4	6.0	34.9	3.3
	M06-2X	37.5	1.5	5.2	37.9	3.5

Table S2. The β_{HRS} , β_{J} and $\Phi(\beta_{\text{J}})$ values, as well as ρ and DR of complexes **L** and **L*M** calculated at the $\omega\text{B97XD}/6\text{-311+G(d,p)}/\text{SDD}$ level in acetonitrile, using wavelengths of 1064, 1340, 1460, 1907 nm and in the static limit ($\lambda=\infty$).

Complex	β_{HRS}	DR	$ \beta_{\text{J}=1} $	$ \beta_{\text{J}=3} $	ρ	$\Phi(\beta_{\text{J}=1})$	$\Phi(\beta_{\text{J}=3})$
$\lambda=1064\text{nm}$							
L	62.7	4.8	115.8	100.1	0.87	0.54	0.46
L*Na⁺	51.3	4.8	94.4	82.3	0.87	0.53	0.47
L*K⁺	53.6	4.7	98.7	86.3	0.87	0.53	0.47
L*Mg²⁺	27.7	4.46	50.0	47.2	0.94	0.51	0.49
L*Ca²⁺	41.7	4.7	76.4	68.0	0.89	0.53	0.47
L*Zn²⁺	27.0	4.4	48.6	46.4	0.95	0.51	0.49
L*Cd²⁺	33.6	4.5	61.0	56.4	0.92	0.52	0.48
L*Hg²⁺	2.5	2.6	3.4	6.3	1.85	0.35	0.65
$\lambda=1340\text{nm}$							
L	47.8	4.7	87.9	77.3	0.88	0.53	0.47
L*Na⁺	39.8	4.7	73.0	64.5	0.88	0.53	0.47
L*K⁺	41.5	4.7	76.1	75.6	0.89	0.53	0.47
L*Mg²⁺	21.8	4.4	39.0	37.9	0.97	0.51	0.49
L*Ca²⁺	32.6	4.6	59.5	53.9	0.91	0.52	0.48
L*Zn²⁺	21.1	4.3	37.7	37.0	0.98	0.50	0.50
L*Cd²⁺	26.2	4.5	47.3	44.7	0.94	0.51	0.49
L*Hg²⁺	2.3	2.6	3.1	5.7	1.86	0.35	0.65
$\lambda=1460\text{nm}$							
L	44.8	4.7	82.3	72.6	0.88	0.53	0.47
L*Na⁺	37.4	4.7	68.6	60.9	0.89	0.53	0.47
L*K⁺	39.0	4.7	71.4	63.7	0.89	0.53	0.47
L*Mg²⁺	20.5	4.3	36.7	35.8	0.98	0.51	0.49
L*Ca²⁺	30.7	4.6	55.9	55.9	0.91	0.52	0.48
L*Zn²⁺	19.9	4.3	35.5	35.0	0.99	0.50	0.50
L*Cd²⁺	24.7	4.4	44.4	42.2	0.51	0.49	0.51
L*Hg²⁺	2.2	2.6	3.0	5.6	1.87	0.34	0.66
$\lambda=1907\text{nm}$							
L	39.1	4.7	71.6	63.8	0.89	0.53	0.47
L*Na⁺	32.9	4.7	60.1	53.8	0.89	0.53	0.47
L*K⁺	34.2	4.6	62.5	56.3	0.90	0.53	0.47
L*Mg²⁺	18.1	4.3	32.2	32.0	0.99	0.50	0.50

L*Ca²⁺	27.1	4.6	49.2	45.2	0.92	0.52	0.48
L*Zn²⁺	17.5	4.3	31.1	31.2	1.0	0.50	0.50
L*Cd²⁺	21.7	4.4	38.9	37.5	0.96	0.51	0.49
L*Hg²⁺	2.1	0.6	2.8	5.3	1.89	0.35	0.65
			$\lambda=\infty$				
L	49.7	4.5	89.9	84.1	0.94	0.52	0.48
L*Na⁺	39.2	4.4	70.4	67.8	0.96	0.50	0.50
L*K⁺	41.0	4.4	73.6	71.0	0.97	0.51	0.49
L*Mg²⁺	21.1	3.8	35.7	41.0	1.15	0.47	0.53
L*Ca²⁺	31.9	4.2	56.4	56.9	1.00	0.50	0.50
L*Zn²⁺	20.3	3.7	33.9	40.4	1.19	0.46	0.54
L*Cd²⁺	24.9	3.9	42.5	48.0	1.12	0.47	0.53
L*Hg²⁺	4.9	3.3	7.8	9.7	1.34	0.42	0.58

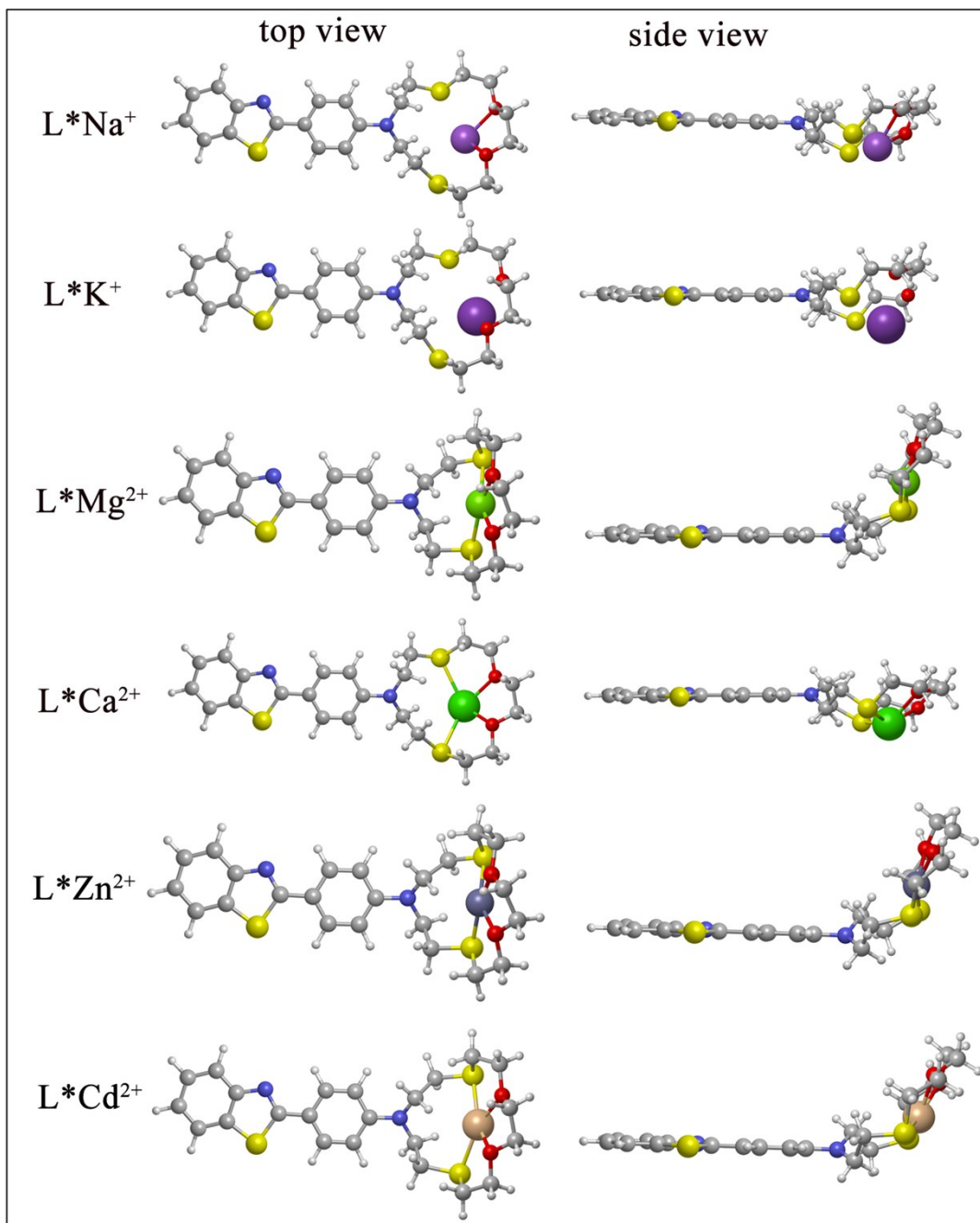


Figure S1. Optimized structures of L^*M ($M = Na^+, K^+, Mg^{2+}, Ca^{2+}, Zn^{2+},$ and Cd^{2+}).