Supplementary Information

Sub-diffractional waveguiding by mid-infrared plasmonic resonators in semiconductor nanowires

Eric J. Tervo,[a] *Dmitriy S. Boyuk,*[b] *Baratunde A. Cola,*[a,c] *Zhuomin M. Zhang,**[a] *and Michael A. Filler**[b]

[a]George W. Woodruff School of Mechanical Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, United States. Email: zhuomin.zhang@me.gatech.edu

[b]School of Chemical and Biomolecular Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, United States. Email: mfiller@gatech.edu

[c]School of Materials Science and Engineering, Georgia Institute of Technology, Atlanta, Georgia 30332, United States

S1: Derivation of Dispersion Relation

For N identical plasmonic resonators in a chain modeled as harmonic oscillators[1, 2] with center-to-center spacing d, displacement $\mathbf{x}$, restoring force spring constant K, effective mass m, and damping coefficient Γ, a force balance on the nth resonator leads to the equation of motion

$$\sum_{l=1}^{N} B_l(x_{n-l} + x_{n+l}) = m\frac{d^2x_n}{dt^2} + \Gamma\frac{dx_n}{dt} + Kx_n \tag{S1}$$

where N is the number of neighboring resonators to consider, $B_l x_{n\pm l}$ is the force exerted by the lth neighbor on the nth resonator, and we have removed the vector notation in $\mathbf{x}$ because we consider only longitudinal polarization. Introducing the coupling strength of the lth neighbor with the nth resonator $\omega_{c,l} = \sqrt{B_l/m}$, the dipole moment $p = qx$ where q is the charge, the damping ratio $\xi = \Gamma/m$, and the natural frequency $\omega_0 = \sqrt{K/m}$, we obtain eqn (1) of the main text:

$$\sum_{l=1}^{N} \omega_{c,l}^2(p_{n-l} + p_{n+l}) = \frac{d^2p_n}{dt^2} + \xi\frac{dp_n}{dt} + \omega_0^2 p_n \tag{S2}$$

We search for propagating wave solutions to this equation of the form $p_{n\pm l} = P\mathrm{e}^{-\zeta t - i[k(n\pm l)d - \omega t]}$, where P is the amplitude, k is the wavevector, ω is the real part of the complex frequency, and ζ is the imaginary part of the complex frequency corresponding to the spectral damping rate. Inserting this into eqn (S2), evaluating the time derivatives, and simplifying results in

$$\sum_{l=1}^{N} \omega_{c,l}^2\left(\mathrm{e}^{ikld} + \mathrm{e}^{-ikld}\right) = (-\zeta + i\omega)^2 + (-\zeta + i\omega)\xi + \omega_0^2 \tag{S3}$$

Using the trigonometric identity $\cos y = \left(\mathrm{e}^{ix} + \mathrm{e}^{-ix}\right)/2$ and rearranging the right side allows us to rewrite this as

$$2\sum_{l=1}^{N}\omega_{c,l}^2\cos(kld) = \zeta(\zeta-\xi) - \omega^2 + \omega_0^2 + i\omega(\xi - 2\zeta) \tag{S4}$$

Separating the real and imaginary parts results in the dispersion relation given in eqn (2) of the main text:

$$0 = \omega^2 - \zeta(\zeta-\xi) - \omega_0^2 + 2\sum_{l=1}^{N}\omega_{c,l}^2\cos(kld) \quad \text{(Real)} \tag{S5a}$$

$$0 = 2\zeta - \xi \quad \text{(Imaginary)} \tag{S5b}$$

S2: Coupling Strength

For two nanoscale resonators a and b separated by a distance much less than the resonant wavelength, such that the incident illumination exerts the same force $F(t) = F_0\mathrm{e}^{i\omega t}$ on the resonators, we write coupled equations of motion for each resonator:

$$F_0\mathrm{e}^{i\omega t} + Bx_b = m\frac{d^2x_a}{dt^2} + \Gamma\frac{dx_a}{dt} + kx_a \tag{S6a}$$

$$F_0\mathrm{e}^{i\omega t} + Bx_a = m\frac{d^2x_b}{dt^2} + \Gamma\frac{dx_b}{dt} + kx_b \tag{S6b}$$

One method to find solutions to these coupled equations is to take linear combinations of them. Adding the equations and introducing $z = x_a + x_b$ we obtain eqn (3) of the main text

$$\frac{2F_0}{m}\mathrm{e}^{i\omega t} + \omega_c^2 z = \frac{d^2z}{dt^2} + \xi\frac{dz}{dt} + \omega_0^2 z \tag{S7}$$

where we have used our previous definitions of ω_c, ξ, and ω_0. Solutions to this equation take the form $z(t) = C\mathrm{e}^{i\omega t}$. Inserting this into eqn (S7) and solving for C yields

$$C = \frac{2F_0/m}{(\omega_0^2 - \omega_c^2) - \omega^2 + i\omega\xi} \tag{S8}$$

The amplitude of oscillation is therefore given as

$$|C| = \sqrt{C \cdot C^*} = \frac{2F_0/m}{(\omega_0^2 - \omega_c^2 - \omega^2)^2 + \omega^2\xi^2} \tag{S9}$$

and the phase is

$$\tan(\phi) = \frac{\mathrm{Im}(C)}{\mathrm{Re}(C)} = -\frac{\xi\omega}{\omega_0^2 - \omega_c^2 - \omega^2} \tag{S10}$$

which allows us to write the solution to the coupled equation of motion as $z(t) = |C|\mathrm{e}^{i\phi}\mathrm{e}^{i\omega t}$. Only the real part of the force and the solution, however, is physical. We can then express the power delivered to the two oscillators as

$$\dot{W}(t) = \mathrm{Re}[F(t)]\mathrm{Re}\left[\frac{dz}{dt}\right] = -F_0\omega|C|\cos(\omega t)\sin(\omega t + \phi) \tag{S11}$$

When this is averaged over once period of oscillation, we obtain eqn (4) of the main text

$$\langle \dot{W}(t) \rangle = \frac{F_0^2}{\xi m} \cdot \frac{\xi^2\omega^2}{\xi^2\omega^2 + (\omega_0^2 - \omega_c^2 - \omega^2)^2} \tag{S12}$$

The measured or simulated absorption peak corresponds to $d\langle \dot{W}(t) \rangle / d\omega = 0$, which gives us eqn (5) in the main text

$$\omega = \omega_2 = \sqrt{\omega_0^2 - \omega_c^2} \tag{S13}$$

S3: Supplemental Figures

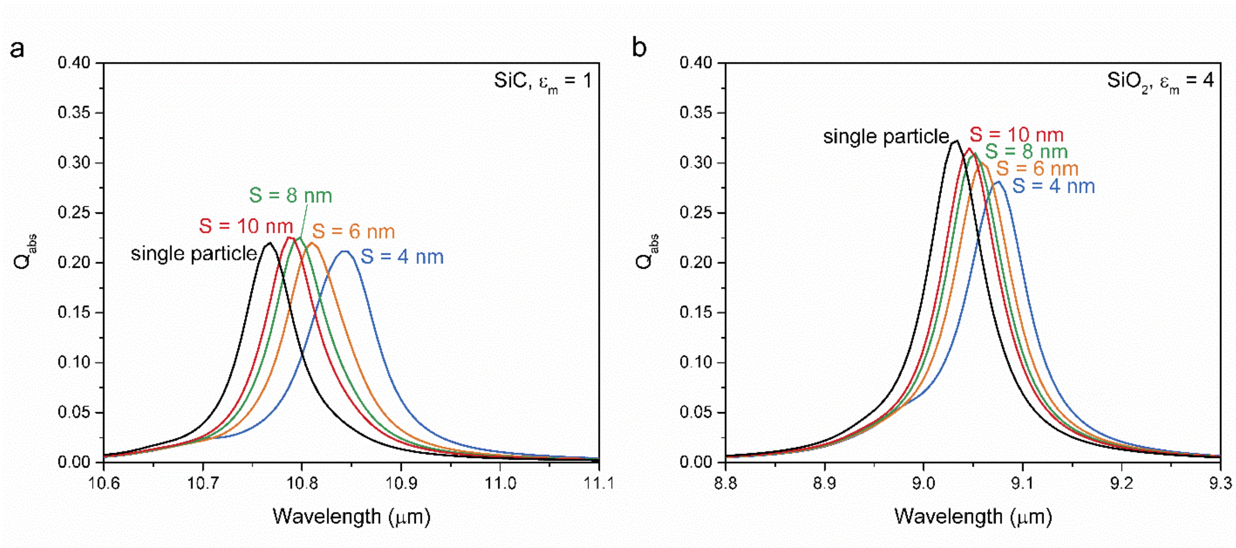

Fig. S1 Absorption efficiency spectra calculated from the discrete dipole approximation for (a) SiC nanoparticles embedded in a background material of permittivity $\varepsilon_m = 1$ and (b) SiO_2 nanoparticles embedded in a background material of permittivity of $\varepsilon_m = 4$. The particles are 12 nm in diameter and separated by distances S of 4, 6, 8, and 10 nm. The spectrum for a single particle is also shown in black, which is used for comparison to the results from Mie theory shown in Fig. S2. As S decreases, the coupling strength and shift in absorption peak from the single particle case increase.

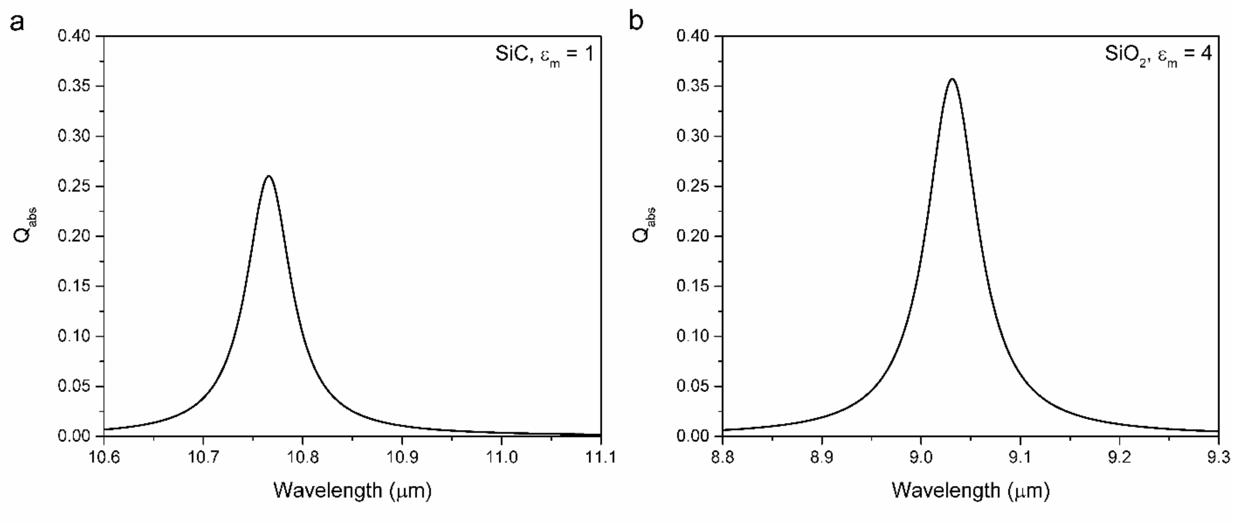

Fig. S2 Absorption efficiency spectra calculated from Mie theory[3, 4] for a 12 nm diameter (a) SiC nanoparticle embedded in a background material of permittivity $\varepsilon_m = 1$ and (b) SiO_2 nanoparticle embedded in a background material of permittivity of $\varepsilon_m = 4$. The absorption peaks agree well with the results from the discrete dipole approximation shown in Fig. S1.

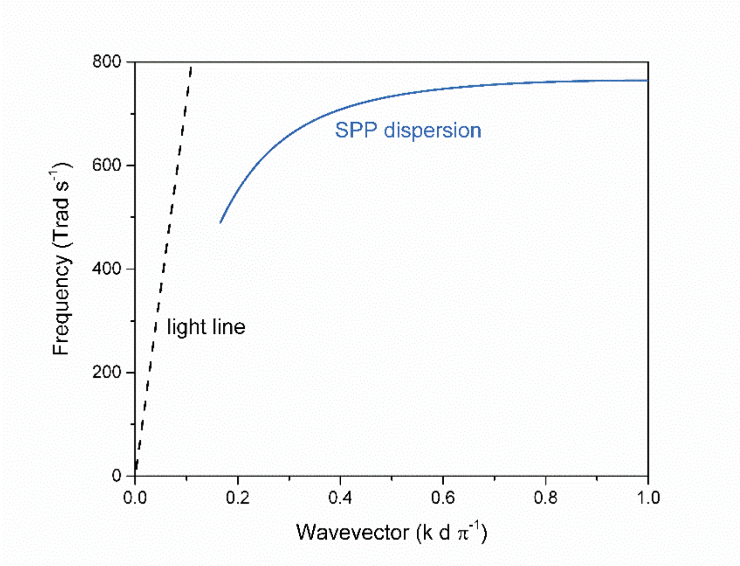

Fig. S3 Dispersion relation (solid blue line) for doped Si resonator chain in an intrinsic Si nanowire with N_e = 1×10^{21} cm^{-3}, AR = 0.8, S = 10 nm, and a diameter of 150 nm. The light line in intrinsic Si is also shown by the dashed black line. For resonators that exhibit very strong coupling, such as these, the dispersion becomes very steep at low frequencies and wavevectors, such that the group velocities exceed the speed of light in the medium. These portions of the dispersion relation are therefore removed from the solution.

References

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