

## Electronic Supplementary Information (ESI) for publication:

### GHz nanomechanical resonator in an ultraclean suspended graphene p-n junction

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#### S1. Derivation of the Resonance Frequency for a Membrane using a Simplified String Ansatz

We assume that the rectangular graphene membrane with width  $w$  and length  $L$  has uniaxial homogeneous strain along the length direction  $x$  of the membrane. We hereby assume that there is no contraction in the width. In reality, if a membrane is strained in  $x$  direction, it will shrink to some extent in the orthogonal direction  $y$ . Since we neglect this, the model is only fine for sheets with  $w \gg L$ . With this assumption we can treat the two-dimensional (2D) membrane as a one-dimensional (1D) string. This is shown in Fig. S1. To model the electrostatic action, we introduce the gate capacitor  $C_G$ , which we assume to be connected to the middle of the string. The gate voltage  $V_G$  is applied to the capacitor through the left contact. In figure (a) the case for  $V_G = 0$  V is shown. It is assumed that there is a pretension in the string given by the force  $T_0$  in units of Newton. If a voltage  $V_G \neq 0$  is applied, as shown in (b), the string is pulled down by the emergent electrostatic force  $F$ . Consequently the tension increases to  $T = T_0 + T_e$ , where  $T_e$  is the additional tension.

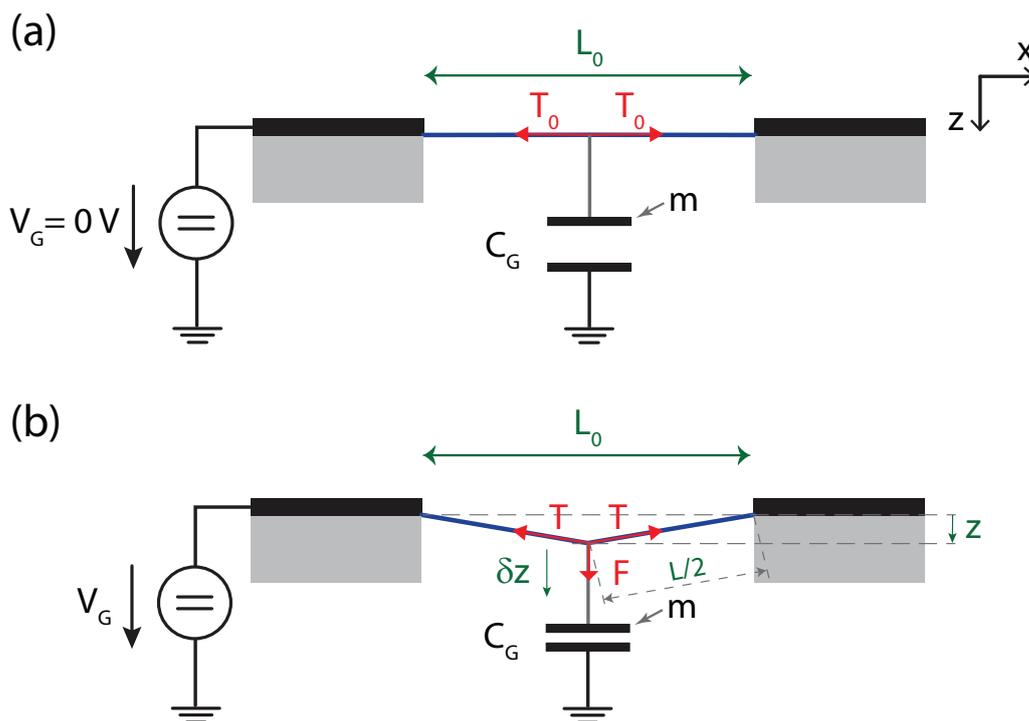


FIG. S 1: (a) String (blue) of length  $L_0$  under pre-tension  $T_0$  connected to a capacitor  $C_G$  to simulate the action of an electrostatic force. In (b) the electrostatic force  $F$  due to  $V_G \neq 0$  is pulling on the string, resulting in an extension  $z$  and an increased tension  $T > T_0$ . The small value  $\delta z$  indicates that the string can also vibrate around its equilibrium position. The resonance frequency is calculated assuming that the string is massless and that all mass  $m$  is in the upper capacitor plate.

From geometry, we have

$$\frac{z}{L/2} = \frac{F/2}{T}. \quad (1)$$

This results into the relation

$$z = L \frac{F}{4T} = \frac{F}{2T} \sqrt{(L_0/2)^2 + z^2}, \quad (2)$$

which for  $z \ll L_0$  can be approximated as

$$z = L_0 \frac{F}{4T}. \quad (3)$$

Hence, the effective force distance relation of the strained string reads

$$F = \frac{4T}{L_0} z. \quad (4)$$

We note here, that the force distance relation is still non-linear, since  $T$  is a function of  $z$ . This we determine next. We first look at the additional tension  $T_e$  arising through the electrostatic force. We assume that the force of the string is linear in elongation and write  $T_e = E\Delta L/2$ , where  $\Delta L = L - L_0$  and  $E$  the 2D Young's modulus of graphene.  $L$  can be written for small extensions  $z \ll L_0$  as

$$L = 2\sqrt{(L_0/2)^2 + z^2} \simeq L_0 + 2L_0 \left(\frac{z}{L_0}\right)^2, \quad (5)$$

resulting in

$$\Delta L = 2L_0 \left(\frac{z}{L_0}\right)^2. \quad (6)$$

Taken all together yields the following expression for the force  $F(z)$  as a function of extension  $z$

$$F(z) = 4z \left( \frac{T_0}{L_0} + E \left(\frac{z}{L_0}\right)^2 \right). \quad (7)$$

Given all constants,  $L_0$ ,  $T_0$  and  $E$ , we see that  $F(z)$  is in general not linear, but has a cubic contribution. This force is balanced by the electrostatic force of the capacitor  $C_G$ , which is given by

$$G = \frac{1}{2} \frac{\partial C_G}{\partial z} V_G^2. \quad (8)$$

Hence, the equilibrium position  $z = z_0$  needs to be calculated from the equation

$$4z_0 \left( \frac{T_0}{L_0} + E \left(\frac{z_0}{L_0}\right)^2 \right) = \frac{1}{2} \frac{\partial C_G}{\partial z} \Big|_{z_0} V_G^2. \quad (9)$$

The symbol  $|_{z_0}$  denotes that the value of the function to the left has to be taken at  $z = z_0$ .

Around the equilibrium position  $z_0$  the string vibrates measured by the excursion  $\delta z(t)$ , which is a function of time  $t$ . The dynamics is given by Newton's law:

$$m\ddot{\delta z} = - \frac{\partial F}{\partial z} \Big|_{z_0} \delta z + \frac{1}{2} \frac{\partial^2 C_G}{\partial z^2} \Big|_{z_0} V_G^2 \delta z. \quad (10)$$

Here, we have assumed that the excursion  $\delta z$  is sufficiently small, which results in a simple harmonic oscillator model with a fixed spring constant. The first part of the spring constant, the derivative of the force  $F(z)$ , can be calculated in a straightforward manner:

$$\frac{\partial F}{\partial z} = 4 \frac{T_0}{L_0} + 12E \left(\frac{z}{L_0}\right)^2. \quad (11)$$

The second part corresponds to a negative spring constant. It is known as Coulomb softening. Inserting back into Newton's equation equ. 10 yields then

$$m\ddot{\delta z} = - \left[ 4 \frac{T_0}{L_0} + 12E \left(\frac{z_0}{L_0}\right)^2 - \frac{1}{2} \frac{\partial^2 C_G}{\partial z^2} \Big|_{z_0} V_G^2 \right] \delta z. \quad (12)$$

We then obtain for resonance frequency  $\omega_o$ :

$$m\omega_0^2 = 4\frac{T_0}{L_0} + 12E\left(\frac{z_0}{L_0}\right)^2 - \frac{1}{2}\frac{\partial^2 C_G}{\partial z^2}\Bigg|_{z_0} V_G^2. \quad (13)$$

There are now two equations that need to be solved together: equation 9 and 12. The first one determines the equilibrium position  $z_0$  and the second the vibration eigenfrequency  $\omega_0$ . We can distinguish two cases. In the first case (a)  $T_0 \rightarrow 0$ ,  $z_0 \propto V_G^{2/3}$ , according to equation 9. According to equation 13 the last term will then dominate, resulting into a *negative* dispersion relation  $\omega(V_G) \propto -V_G^2$ . That is to say that  $\omega(V_G)$  changes to smaller values with increasing magnitude of  $V_G$  until sufficient tension is built up. Case (b) is easier to treat. Here we assume that  $T_0$  is initially already large. This is the case in the experiments we have performed. More precisely, we assume that the first term on the left in equation 9 dominates over the second. We then obtain for  $z_0$ :

$$z_0 = \frac{1}{8}\frac{L_0}{T_0}\frac{\partial C_G}{\partial z}\Bigg|_{z_0} V_G^2. \quad (14)$$

We now enter this value into resonance equation 13 and obtain:

$$m\omega_0^2 = 4\frac{T_0}{L_0} + \frac{3}{16}E\left(\frac{\partial C_G}{\partial z}\Bigg|_{z_0}\frac{V_G^2}{T_0}\right)^2 - \frac{1}{2}\frac{\partial^2 C_G}{\partial z^2}\Bigg|_{z_0} V_G^2. \quad (15)$$

This is the equation that appears in the publication. Let us now define the two limits using this equation. In the limit that  $T_0$  is ‘small’ the first term is small and for the two gate-dependent terms the second term dominates over the third. This results in a *positive* dispersion with  $\omega \propto V_G^4$ . The equation reads:

$$m\omega_0^2 \approx 4\frac{T_0}{L_0} + \frac{3}{16}E\left(\frac{\partial C_G}{\partial z}\Bigg|_{z_0}\frac{V_G^2}{T_0}\right)^2. \quad (16)$$

In contrast, if  $T_0$  is large, the second term can be neglected. The eigenfrequency is now large and the dispersion is initially negative. The equation reads:

$$m\omega_0^2 \approx 4\frac{T_0}{L_0} - \frac{1}{2}\frac{\partial^2 C_G}{\partial z^2}\Bigg|_{z_0} V_G^2. \quad (17)$$

In this later case, by fitting the theoretical model with the experimental curve, one obtains with quite good accuracy as fitting parameters directly the mass  $m$ , the built-in tension  $T_0$  and the second derivative of  $C_G$  at the equilibrium position. Almost all measured dispersion curves are of this kind, i.e. show negative dispersion, indicating that there is substantial built-in strain  $T_0$ .

## S2. Fitting Procedure

We briefly describe how the measured response curves  $f_0(V_g)$  are fitted. In our experiment, we performed several current annealing steps to clean graphene. At a few intermediate steps, we measured the dispersion curves. We repeated this process until we observe Fabry-Pérot (FP) oscillations in the conductance, which is evidence for ballistic transport. Then, we assumed that the sample is clean from adsorbates, so that the final mass density should be very close to the monolayer graphene mass density ( $\rho \approx \rho_0$ ). Hence, we fixed  $\rho$  at  $\rho_0$ . By fixing this, only two parameters are free to be fitted: the built-in tension  $T_0$  and  $C_G$ . While ideally one could think to fit all three parameters, this is not possible in most of the measured dispersion curves. Having fixed the mass density  $\rho$  for ballistic devices showing FP oscillations to  $\rho_0$ , we obtain then values for  $C_G$  and  $T_0$  from the data. Next, we used the extracted value,  $C_G$  in measurements, which we obtain before the “final” step when the sample did not yet show FP oscillations, to extract  $T_0$  and  $\rho$ . To summarize,  $C_G$  is fixed in the “final” step, assuming ( $\rho = \rho_0$ ), and  $\rho$  and  $T_0$  are obtained using this single  $C_G$  for all data obtained before.

In the following table we list the calculated and the fitted capacitance values for each device, using a simple parallel-plate capacitor model to calculate capacitances. It is seen that there is a significant difference. The fitted capacitance turns out to be bigger by as much as a factor of 50. This is probably due to two reasons. First, there is an additional stray capacitance. The biggest difference, however, comes from the assumption of a parallel plate capacitor. In this model the two plates move parallel to each other with coordinate  $z$ . In reality, the graphene membrane is clamped on both sides and there is a homogeneous force acting on the surface. The effective bending of the membrane is therefore much less. One cannot assume that  $C_G \propto 1/z$ . In contrast,  $C_G \propto 1/z^p$  with a power  $p \ll 1$ . In the fitting procedure, we obtain  $C_G$  through the negative curvature of the dispersion relation. This curvature is given by the second derivative of  $C_G$ , and, hence, proportional to  $1/p$ . As  $p$  could easily be  $1/10$ , the extracted capacitance comes out wrong by more than an order of magnitude. Hence, the difference can be anticipated to be large. It is due to the simplified assumptions. We stress, however, that this has no effect on the extracted tension, since this value is determined by the frequency at zero gate voltage.

Table 1: The comparison of calculated capacitance value and fitted capacitance value by fitting

Device	$W(\mu m)$	$L(\mu m)$	Calculated capacitance [F]	Fitted capacitance [F]
A	4.1	1.1	$6.6 \times 10^{-17}$	$3.5 \times 10^{-15}$
B	3	1.2	$5.3 \times 10^{-17}$	$3.9 \times 10^{-15}$
C	2	1.5	$4.4 \times 10^{-17}$	$1.2 \times 10^{-16}$

### S3. Comparison between the mixing current and calculated transconductance

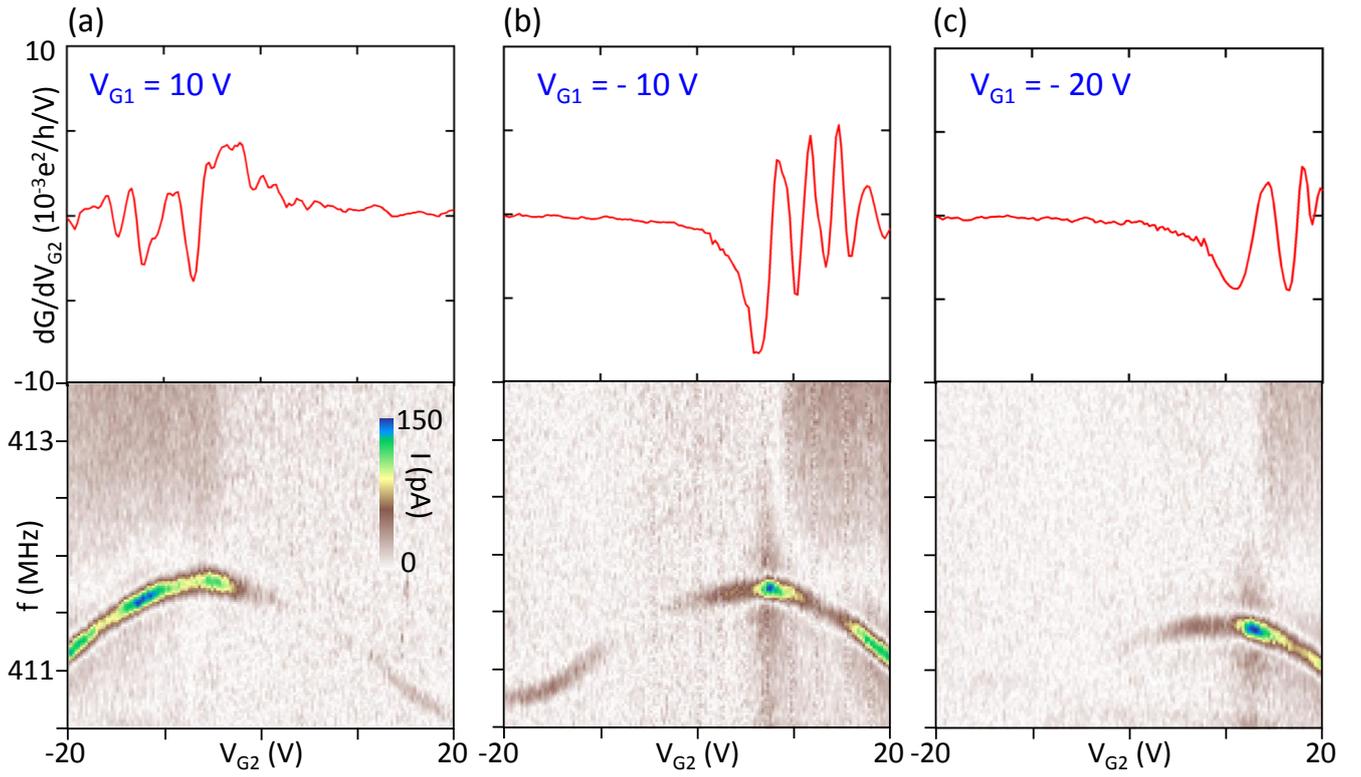


FIG. S 2: The comparison between the mixing current of the resonator and calculated transconductance at (a)  $V_{G1} = 10$  V, (b)  $V_{G1} = -10$  V, and (c)  $V_{G1} = -20$  V. The upper panels are the calculated transconductance, while the lower panels are the mixing current measured as a function of frequency  $f$  and  $V_{G2}$ .