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Spin waves collimation using flat metasurface

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We consider the system of two semi-infinite ferromagnetic media FM-1 and FM-2, separated with an ultra thin interface in the yz-plane in the uniform static external magnetic field $\mathbf{H}_0$, which is parallel to the y-axis. To calculate the phase shifts and intensities of the SW refracted at the interface we use the Landau-Lifshitz (LL) equation that describes magnetization vector dynamics in the effective magnetic field $\mathbf{H}^j_{\text{eff}}$:

$$\frac{\partial \mathbf{M}_j}{\partial t} = |\gamma|(\mathbf{M}_j \times \mathbf{H}^j_{\text{eff}}), \tag{S1}$$

where γ is a gyromagnetic ratio, $\mathbf{M}_j$ indicates the magnetization vector, a damping term is neglected in our calculations. $\mathbf{H}^j_{\text{eff}}$ denotes the effective magnetic field of the j-th ferromagnet and is determined as the functional derivative of the system total magnetic energy with respect to the magnetization vector: $\mathbf{H}^j_{\text{eff}} = -\delta w/\delta \mathbf{M}_j$.

The total magnetic energy of the system is written as follows:

$$W = \int_V \left[A_{12}\delta(x)\left(\mathbf{M}_1 \cdot \mathbf{M}_2\right) + \sum_{j=1}^{2} \theta((-1)^j x) w_j\right] dV, \tag{S2}$$

where the first term denotes the surface energy density at the interface with the interlayer exchange constant A_{12}, $\delta(x)$ corresponds to the Dirac δ-function and $\theta(x)$ to the Heaviside step function, w corresponds to the bulk energy density of the j-th ferromagnet:

$$w_j = \frac{\alpha_j}{2}\left(\frac{\partial \mathbf{M}_j}{\partial x}\right)^2 - \frac{1}{2}\beta_j\left(\mathbf{M}_j \cdot \mathbf{n}_j\right) - (\mathbf{H}_0 \cdot \mathbf{M}_j). \tag{S3}$$

Eq. (S3) includes the non-uniform exchange energy density term for j-th ferromagnet with the $\alpha_j = A_{\text{ex}_j}/M_{0j}^2$, where A_{ex_j} being the j-th ferromagnet exchange stiffness constant and M_{0j} is the saturation magnetization of the j-th ferromagnet. Anisotropy energy density term contains $\beta_j = K_j/M_{0j}^2$, where K_j is the uniaxial magnetic anisotropy constant of j-th ferromagnet and the unit vector of the easy axis is $\mathbf{n}_j$. The last term in Eq. (S3) represents the j-th ferromagnet Zeeman energy density. In this Letter we relate the parameter A_{12} to the RKKY interactions between the two ferromagnetic materials through the interface δ [S1–S3].

We treat the magnetization dynamics as the small deviations of the magnetization vector $\mathbf{M}_j$ from the ground state $\mathbf{M}_{j,y}$ in the form: $\mathbf{M}_j = \mathbf{M}_{j,y} + \mathbf{m}_j$, where $\mathbf{m}_j$ denotes the magnetization vector dynamical component in the j-th ferromagnet. As the the magnetization vector is preserved, then the following equality holds $m_{j,x}^2 + M_{j,y}^2 + m_{j,z}^2 = M_{0j}^2$. Thus $M_{j,y} = \sqrt{M_{0j}^2 - m_{j,x}^2 - m_{j,z}^2} \approx M_{0j} - \left(m_{j,x}^2 + m_{j,z}^2\right)/2M_{0j}$, which in the linear approximation with respect of $|\mathbf{m}|$ results in the magnetization vector y-component coincidence with the saturation magnetization value $M_{j,y} \approx M_{0j}$.

Solutions of the LL-equations in the homogeneous media could be found in the form of plane waves, therefore for FM-1 and FM-2 correspondingly:

$$\begin{aligned} m_{1x} + i m_{1z} &= I_1 \exp(i(\mathbf{k}_1 x - \omega t)) + \\ &\quad + I_{1R}\exp(i(\mathbf{k}_1 x - \omega t + \varphi_R)), \\ m_{2x} + i m_{2z} &= I_T \exp(i(\mathbf{k}_2 x - \omega t + \varphi_T)), \end{aligned} \tag{S4}$$

where I_1, I_{1R}, I_T are the amplitudes of the incident, reflected and refracted waves and φ_R, φ_T are the phase shifts of the reflected and refracted waves, respectively.

For the plane waves (S4) with in-plane $\mathbf{k}_j$ the dispersion relation in the j-th ferromagnetic film of the thickness L_j could be represented by the well-known dispersion relation [S4]:

$$\begin{aligned} \omega_j^2(k_j) &= \left(\omega_{0j} + 4\pi|\gamma| M_{0j}(1 - \psi_j(k_j L_j))\right) \times \\ &\times \left(\omega_{0j} + 4\pi|\gamma| M_{0j}\psi_j(k_j L_j)\sin^2(\theta)\right), \end{aligned} \tag{S5}$$

where $\omega_{0j} = |\gamma|\left(H_0 + M_{0j}\left(\beta_j + \alpha_j k_j^2\right)\right)$, $\psi(k_j L_j) = 1 - \left(1 - \mathrm{e}^{-k_j L_j}\right)/k_j L_j$, θ is an angle between SW direction of propagation ($\mathbf{k}_j$) and magnetization orientation ($\mathbf{M}_j$) and for the monochromatic SWs (S4) $\omega_1 = \omega_2 = \omega$. Regarding the limit of the exchange SWs, i.e. $k_j L_j >> 1$ and taking into account the Damon-Eshbach geometry ($\theta = \pi/2$) we derive wave vector value in the j-th ferromagnet:

$$k_j = \left(\frac{1}{\alpha_j}\left(\sqrt{\frac{\omega^2}{\gamma^2 M_{0j}^2} + 4\pi^2} - \frac{H_0}{M_{0j}} - (2\pi + \beta_j)\right)\right)^{\frac{1}{2}}. \tag{S6}$$

Implementation of the total energy (S2) to the LL-equations (S1) in the linear approximation and integration in the infinitely small neighborhood of a point $x = 0$ gives the boundary conditions for the magnetization dynamical components [S5, S6]:

$$\begin{cases} \left(A_{12}(m_{2n} - \xi m_{1n}) + \alpha_1 \dfrac{\partial m_{1n}}{\partial x}\right)\Bigg|_{x=0} = 0, \\ \left(A_{12}(m_{1n} - \dfrac{m_{2n}}{\xi}) - \alpha_2 \dfrac{\partial m_{2n}}{\partial x}\right)\Bigg|_{x=0} = 0, \end{cases} \tag{S7}$$

where $\xi = M_{02}/M_{01}$ and $n = x, z$.

To find the transmitted SW phase shift φ_T and intensity $T = (I_T/I_0)^2$ using the dynamic magnetization components (S4) and wave vector modulus (S6) we solve the system of equations (S7) for the infinitely thin interface and derive:

$$\varphi_T = \arctan\left(A_{12}\frac{\xi\alpha_2 k_2 + \alpha_1 k_1/\xi}{\alpha_1\alpha_2 k_1 k_2}\right) - \begin{cases}\pi, A_{12} < 0 \\ 0, A_{12} > 0\end{cases},$$
$$T = \frac{4\alpha_1^2 k_1^2}{(\alpha_1\alpha_2 k_1 k_2/A_{12})^2 + (\xi\alpha_2 k_2 + \alpha_1 k_1/\xi)^2}. \tag{S8}$$

Let us introduce the interlayer exchange constant A_{12} as a normalized parameter $A_{ex_{12}}$, which is suitable for further comparison of analytical results with ones from numerical experiment. For that we made an evaluation of the energies from both models in Gaussian and SI units, i.e. $\int_V \left[A_{12}\delta(x)\big(\mathbf{M}_1 \cdot \mathbf{M}_2\big)\right] dV$ and $\int_V \left[A_{\mathrm{ex}_j}\left(\partial \mathbf{m}_j/\partial x_i\right)^2\right] dV$ where $\mathbf{m}_j = \mathbf{M}_j/M_{0j}$ is reduced magnetization. To get dimensionless exchange parameter $A_{\mathrm{ex}_{12}}$ it is convenient to make normalization with its limit values $A_{\mathrm{ex}} = A_{12_{Co}}$ (i.e. for exchange constant for Co): $A_{ex_{12}} = A_{12}/A_{\mathrm{ex}}$ With the few steps of easy calculations one can receive equality: $A_{\mathrm{ex}_{12}} = A_{12}\left(\dfrac{\Delta \cdot M_0^2}{2A_{\mathrm{ex}}}\right)$, where M_0 is the Co magnetization saturation and Δ is the unit cell size along the x direction in micromagnetic simulations.

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