Supporting information for 'The Downside of Dispersity: Why the Standard Deviation is a More Intuitive Measure of Dispersion in Precision Polymerization'

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Derivation of the expression for dispersity in terms of standard deviation and mean.

In a polymer molecular weight distribution, in which *x* is the degree of polymerization, and *f*(*x*) is the frequency of chains of length *x*, normalized such that $\sum f(x) = 1$, the number and weight average degrees of polymerization DP_n and DP_w are defined as follows:

$$DP_n = \mu = \frac{\sum xf(x)}{\sum f(x)}$$

$$DP_w = \frac{\sum x^2 f(x)}{\sum xf(x)}$$

Their ratio gives the dispersity, Đ:

$$Đ = \frac{DP_w}{DP_n} = \frac{\sum x^2 f(x). \sum f(x)}{\left(\sum xf(x)\right)^2}$$

The standard deviation, *σ*, is defined as the root mean square difference from the mean, giving:

$$\sigma^2 = \sum (x - \mu)^2 f(x)$$

Expanding the sum gives:

$$\sigma^2 = \sum x^2 f(x) - 2\mu \sum xf(x) + \mu^2 \sum f(x) = \sum x^2 f(x) - \mu^2$$

This is the well-known expression for the variance as the mean of the squares minus the square of the mean.

Substituting this formula into the expression for Đ gives:

$$Đ = \frac{DP_w}{DP_n} = \frac{\mu^2 + \sum x^2 f(x) - \mu^2}{\mu^2} = 1 + \frac{\sigma^2}{\mu^2}$$