Supporting Information

Numbering-up of capillary microreactors for homogeneous processes and its application in free radical polymerization

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Derivation of the dimensionless variables

The first step is usually to list all the variables and their dimensions. Table S1 gives the physical quantities involved in our experiments, their symbols, and their dimensions expressed by three fundamental dimensions of time (t), length (l), and mass (m).

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Quantities	Symbols	Dimensions	
Capillary length	L	l	
Capillary inner diameter	$I.D. (D_i)$	1	
Viscosity of the fluid	μ	$t^{-1}ml^{-1}$	
Velocity of the flow	u	$t^{-1}l$	
The Density of the fluid	ρ	<i>ml</i> ⁻³	

Table S1. Different quantities, their symbols and dimensions

Then the dimensional matrix can be formed from the following table:

Table S2 Formation of the Dimensionless Matrix

	L	D_i	μ	и	ρ
t	0	0	-1	-1	0
l	0	0	1	0	1
т	1	1	-1	1	-3

The numbers in Table S2 represent the exponents of t, l, and m in the dimensional expression for the variables involved. The dimensional matrix can be thus an array of numbers and displayed as follows:

$$M = \begin{pmatrix} 0 & 0 & -1 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & -1 & 1 & -3 \end{pmatrix}$$
 * MERGEFORMAT (S1)

A kernel vector A = [a, b, c, d, e] should be looked for such that the matrix product of M on A yields the zero vector [0, 0, 0], namely $L^a D_i^b \mu^c u^d \rho^e = t^0 l^0 m^0$. This can be satisfied only if a, b, c, d, and e are related as the following relationship:

$$\begin{cases} c+d=0\\ c+e=0\\ a+b-c+d-3e=0 \end{cases}$$
 * MERGEFORMAT (S2)

According to the rank-nullity theorem, for a system of n vectors (the matrix columns) in k linearly independent dimensions, there leaves a nullity p, satisfying Equation 3, where the nullity is the number of extraneous dimensions chosen to be dimensionless.

$$p = n - k$$
 * MERGEFORMAT (S3)

Since there are five quantities and only three fundamental dimensions, two dimensionless variables can be obtained. When we separate the capillary length from the viscosity, i.e., they will not appear in the same dimensionless variable, two vectors are:

$$A_{1} = \begin{bmatrix} a = 0 \\ b \\ c = 1 \\ d \\ e \end{bmatrix}, A_{2} = \begin{bmatrix} a = 1 \\ b \\ c = 0 \\ d \\ e \end{bmatrix} \land * \text{MERGEFORMAT (S4)}$$

Applying them to Equation S2 respectively gives:

$$A_{1} = \begin{bmatrix} 0 & & & 1 \\ -1 & & & \\ 1 & , & A_{2} = \begin{bmatrix} 1 & & \\ -1 & & 0 \\ 0 & & \\ -1 & & & 0 \end{bmatrix}$$
 * MERGEFORMAT (S5)

Thus, the dimensionless variables can be obtained as:

$$\pi_1 = L^0 D_i^{-1} \mu^{-1} u^1 \rho^1 = \frac{D_i u \rho}{\mu} \qquad \land * \text{ MERGEFORMAT (S6)}$$
$$\pi_2 = L^1 D_i^{-1} \mu^0 u^0 \rho^0 = \frac{L}{D_i} \qquad \land * \text{ MERGEFORMAT (S7)}$$

where the former is famous as the Reynolds number (*Re*), and the latter is the lengthdiameter ratio.