Supporting Information: Viscoelastic multistable architected materials with temperature-dependent snapping sequence

Kaikai Che [1], Chao Yuan[1,2], Jerry H. Qi [1], and Julien Meaud *[1]

[1]*G.W.W. School of Mechanical Engineering, Georgia Institute of Technology, Atlanta USA*

[2]*State Key Laboratory for Strength and Vibration of Mechanical Structures,Department of Engineering Mechanics, School of Aerospace Engineering, Xi'an Jiaotong University, China*

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*Corresponding author: julien.meaud@me.gatech.edu

1 Material model parameters

The two materials used in this paper are PEGDA and DM9895. PEGDA is modeled as an elastic material with a constant Young's modulus of 25MPa. DM9895 is modeled as a viscoelastic material, with a 27 term Prony series. The material parameters for DM9895 are the values of E_{eq}, E_i, τ_i^0, C_1, C_2 and T_{ref} (see Eqs. 1-3). These parameters were found in a previous paper by fitting the model to DMA measurements [1]. The DMA data was obtained by first heating the material to 90°C; after waiting 10min to reach equilibrium, the temperature was decreases at a rate of 2°C/min to determine the storage modulus vs temperature curve for a 1Hz frequency. For completeness, the parameters are listed in the Tables S1 and S2:

Parameter	Value
T_M	-3 ^{o}C
C_1	17.44
C_2	42.1 ^{o}C
E_{eq}	3.3 MPa

Table S1: Other parameters in the viscoelastic model

Branch i	E_i (Pa)	τ_i^0 (s)
1	3E+08	0.0001
2	2.75E+08	0.000657
3	2.96E+08	0.003872
4	3.05E+08	0.02
5	3.5E+08	0.1
6	3.78E+08	0.576863
7	2.92E+08	3.401616
8	2.15E+08	20
9	1.47E+08	96.82391
10	2.15E+08	362.9461
11	63127650	1000
12	62092100	2671.527
13	52099306	7912.87
14	42374719	23498.79
15	35205449	71461.38
16	27897552	228551.6
17	20760769	726401
18	15532429	2277776
19	11281878	7091525
20	8305791	21997171
21	5959708	68236585
22	4351312	2.08E+08
23	3329757	6.41E+08
24	2644468	2.07E+09
25	2196711	7.07E+09
26	1578065	2.4E+10
27	107012.2	1E+11

Table S2: Parameters for the viscoelastic branches in the viscoelastic model

2 Analysis of the number of stable configurations of bimaterial architected materials with N layers

The number of stable configurations that can be obtained using a compressive deformation at multiples temperatures was analyzed. While a higher number of stable configurations could be obtained by applying a more complicated loading history (that includes tensile loads) or by deforming individual layers by stimulating them directly, only configurations that are obtained in response to a compressive load are considered here. Let N_{DM} and N_{PEGDA} be the number of DM layers and PEDGA layers, respectively. Consider one of the DM layers. As the temperature is increased, this DM layer will exhibit a snapping sequence switch with each of the PEDGA layers, which results in N_{PEGDA} different critical temperatures for this DM layer. Thus, the total number of critical temperature is:

$$N_{cr} = N_{PEGDA} \times N_{DM} \tag{S1}$$

such that the number of sequences is:

$$N_{seq} = N_{cr} + 1 = N_{PEGDA} \times N_{DM} + 1 \tag{S2}$$

Since $N = N_{PEGDA} + N_{DM}$, the maximum number of snapping sequences that can be obtained with bi-material architected materials with N layers is:

$$N_{seq,max}(N) = \max_{2 \leq N_{DM} \leq N-1} \{(N - N_{DM})N_{DM} + 1\} = \lfloor \frac{N}{2} \rfloor \lfloor \frac{N+1}{2} \rfloor + 1 \tag{S3}$$

where $\lfloor . \rfloor$ denotes the floor function. For the snapping sequence obtained at room temperature, $N + 1$ different configurations are obtained; furthermore, for each of the $N_{seq} - 1$ other snapping sequences, an additional stable configuration is obtained (see Fig. 5B). Hence, the maximum number of stable configurations that can be obtained by loading a bimaterial architected material at different temperatures is:

$$N_{bimat}(N) = (N + 1) + (N_{seq,max}(N) - 1) = \lfloor \frac{N}{2} \rfloor \lfloor \frac{N+1}{2} \rfloor + N + 1 \tag{S4}$$

3 Loading curves for multiple tests of a single sample

A single bi-material sample was loaded two times at room temperature to demonstrate that the deformation of the structure is (visco) elastic. As shown in Fig. S1, the loading force of Trial #1 matches almost exactly the loading force of Trial #2. This suggests that the deformation is fully reversible.

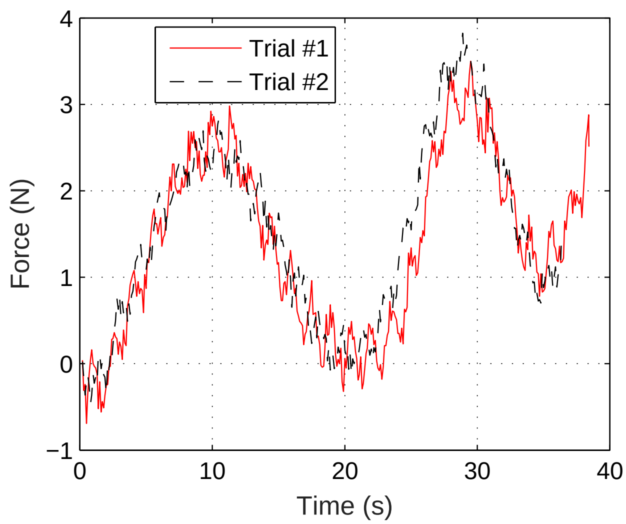


Figure S1: Loading force vs time for a bi-layer bi-material sample (made of PEDGA and DM) loaded at room temperature at a loading velocity of 10mm/min. The geometric parameters are $Q_1 = Q_2 = 5$, $P_1 = 9.33$ and $P_2 = 14$. The sample was tested twice on the same day.

4 Determination of the design with maximum stiffness tunability

To determine the set of design parameters with maximum stiffness tunability for a 4-layer bi-material architected material, we evaluated the stiffness of the stable configurations for a wide range of values for the design parameters. The considered values for the non-dimensional parameters P_j and Q_j of each layer are shown Fig. S2: $2.8 \leq Q_j \leq 5$ and $8.53 \leq P_j \leq 27$, where each parameter is allowed to take 12 different value. Only the values of P_j and Q_j that resulted in a multistable design are represented in Fig. S2. The stiffness values of each individual layer was first calculated using FEA simulations with a Static, Linear perturbation in ABAQUS. As shown in Fig. S3, the boundary conditions for the bottom edge of the model are the following: for the top 3 layers, the bottom edge has a fixed vertical displacement and the bottom middle node is fixed in both directions; for the bottom layer, the bottom edge is fixed (since the bottom edge of the 4-layer model is also fixed). For all simulations, a uniformed vertical displacement was applied to the top edge of the model.

The value of the stiffness of individual layers was normalized by $E \times b$ (where E is the Young's modulus and b is the out-of-plane thickness) and plotted as function of $1/P$ and Q in Fig. S4. The use of Eq. 7 made it possible to evaluate the effective stiffness of 4-layer architected materials at room temperature for a very large number of different designs.

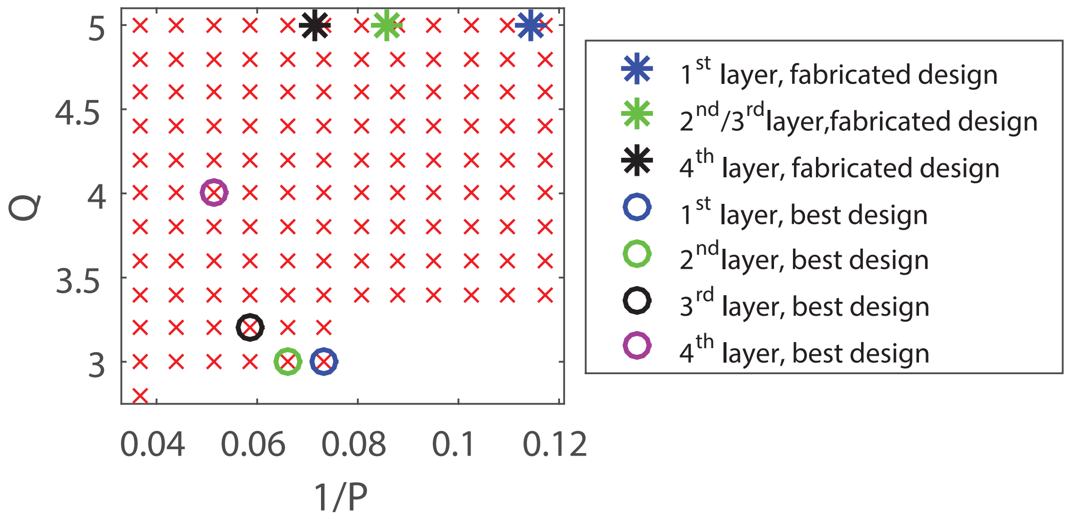


Figure S2: Design space. Each red cross correspond to a value of P_j and Q_j that was considered for each layer. The parameters of the fabricated design (Fig. 5A) and of the best design are shown with stars and circles, respectively.

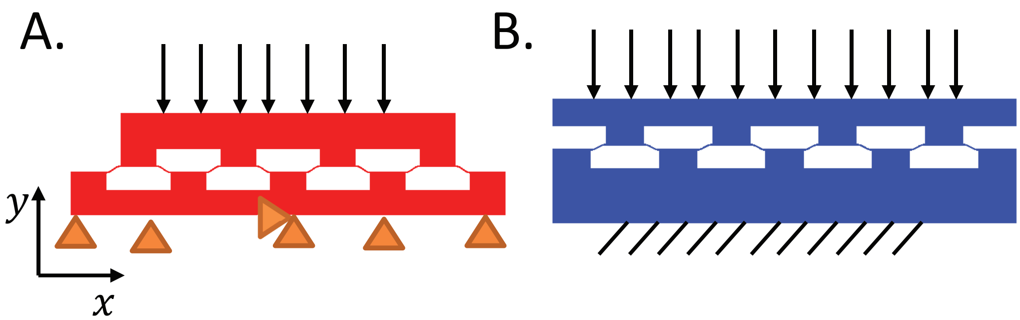


Figure S3: FEA model of single layer. A. Boundary conditions for the top 3 layers. B. Boundary condition for the bottom layer.

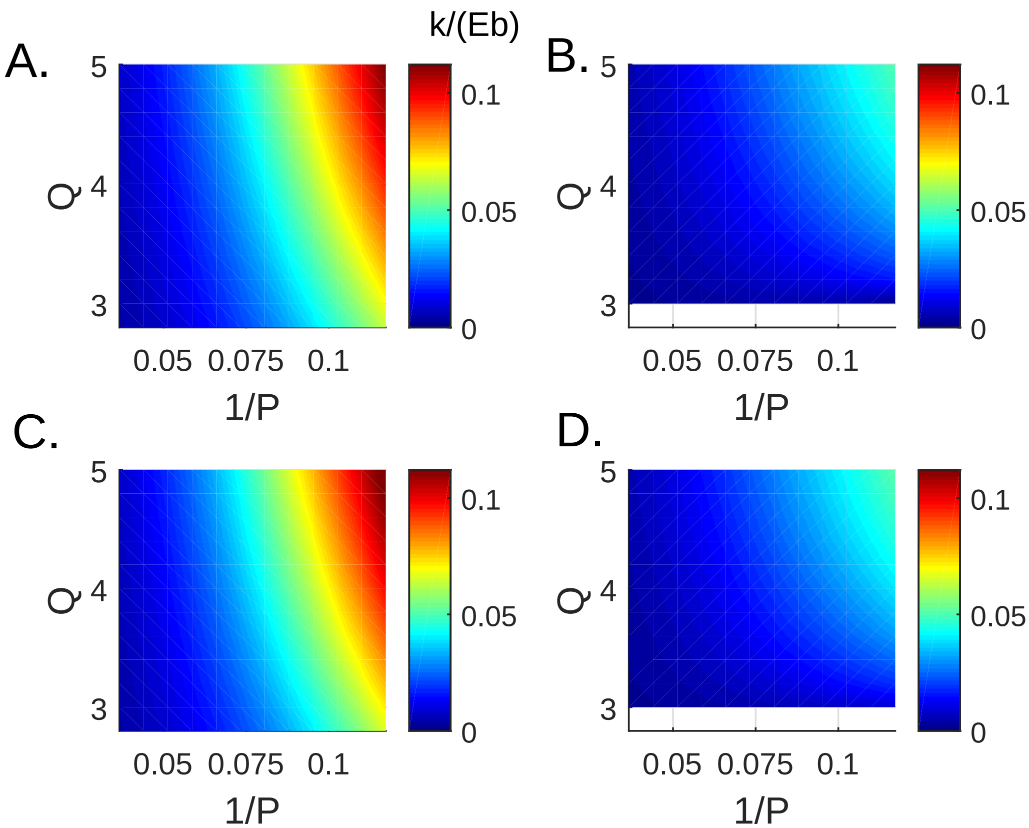


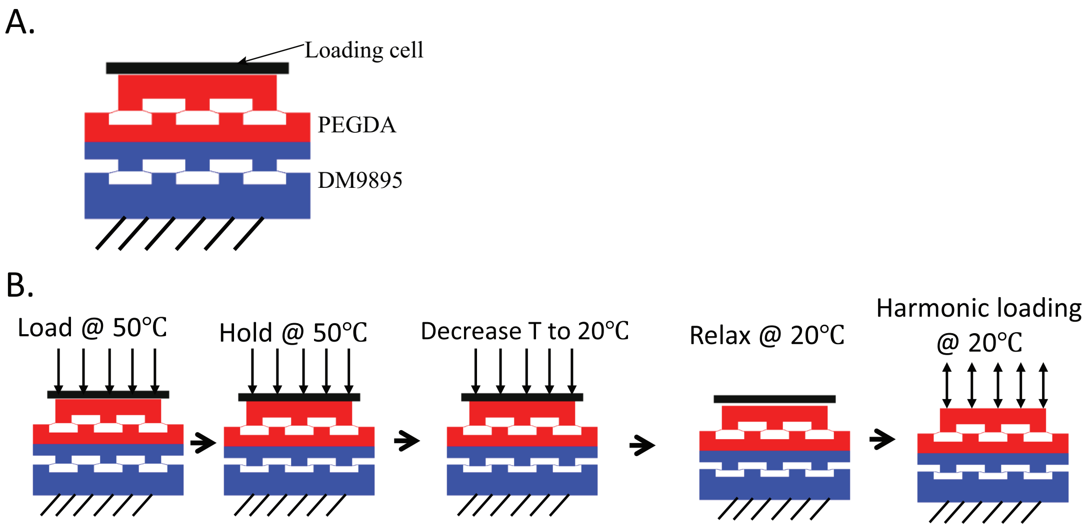
Figure S4: Value of the normalized stiffness ($k/(Eb)$) in the undeformed configuration (k_0 in A and C) and deformed configurations (k_1 in B and D) for a single layer as function of $1/P$ and Q. The results of A and B were obtained using the boundary conditions of Fig. S3A; the results of C and D were obtained using the boundary conditions of Fig. S3B.

5 Direct calculations of the stiffness of bimaterial architected materials with multiple layers

The effective stiffness of the stable configurations for the best design was also directly calculated using FEA simulations. All the stiffness values were calculated at room temperature (20^oC). To illustrate the simulation process, the following discussion takes a bi-layer model as an example.

For a bi-layer bimaterial architected material, there are 4 different stable configurations (00, 01, 10 and 11). Three steps are used in the simulations: compression loading, relaxation, and periodic loading. In the compression loading step, the loading cell is initially utilized to compress the structure with a loading velocity of 10mm/min; the loading cell is then held for 1000 seconds, before unloading the loading cell. The deformed structure is allowed to relax freely in the relaxation step for 6000 seconds. In the last periodic loading step, a harmonic velocity of frequency 1Hz is directly applied at the top edge of the model for 5 cycles. The effective stiffness is computed using the last cycle of the simulation.

For the undeformed stable configuration (Configuration 00), only the periodic loading step is needed. All these three steps are necessary for the other three stable configurations. Since the PEGDA layer snaps before the DM layer at room temperature, all three steps are at room temperature to obtain the stiffness values for deformed configurations 01 and 11. However, in order to derive the stiffness in the last stable configuration (Configuration 10), the compression loading step has to be applied at a temperature, $T_{loading}$, above the critical temperature, T_{cr}, as shown in Fig. S5B. Once the material has snapped to configuration 10, the temperature is decreased to room temperature before the relaxation step, such that the effective stiffness is always calculated at room temperature. The loading temperatures that were used for the nine stable configurations of the 4-layer architected material shown in the manuscript are shown in Fig. S5C.



C.

Configuration	$T_{loading}$(°C)	$T_{harmonic}$(°C)
0000	–	20
0001	20	20
0011	20	20
0111	20	20
1111	20	20
0100	50	20
1100	50	20
1101	50	20
0101	26	20

Figure S5: A. FEA model of one bi-layer bimaterial architected material. B. Stiffness calculation process for configuration 01 of a bi-layer bimaterial architected material. C. Loading temperature for 4-layer architected material (Fig. 6C of the manuscript).

References

[1] Jiangtao Wu, Chao Yuan, Zhen Ding, Michael Isakov, Yiqi Mao, Tiejun Wang, Martin L Dunn, and H Jerry Qi. Multi-shape active composites by 3d printing of digital shape memory polymers. *Scientific Reports*, 6:24224, 2016.