## Electronic Supplementary Information

## Harmonic Analysis of Surface Instability Patterns on Colloidal Particles

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Table S1 Numerical values ( $\pm 95 \%$ confidence interval) of particle parameters: root-mean-square surface roughness $\sigma$, initial aerosol droplet radius $R_{\mathrm{I}}$, buckling transition radius $R_{\mathrm{B}}$, final mean spherical radius $\left\langle R_{\mathrm{F}}\right\rangle$, spherical harmonic mean sphere radius $R_{\mathrm{S}}$, final surface area $A_{\mathrm{F}}$ and final volume $V_{\mathrm{F}}$.

| Parameter | C 1 | C 2 | W 3 |
| :--- | ---: | ---: | ---: |
| $\sigma(\mathrm{~nm})$ | 26.1 | 36.0 | 29.7 |
| $R_{\mathrm{I}}(\mathrm{nm})$ | 498 | 668 | 1181 |
| $R_{\mathrm{B}}(\mathrm{nm})$ | 140 | 196 | 283 |
| $\left\langle R_{\mathrm{F}}\right\rangle(\mathrm{nm})$ | 103.7 | 138.8 | 235.9 |
| $R_{\mathrm{S}}(\mathrm{nm})$ | 92.3 | 126.2 | 233.8 |
| $A_{\mathrm{F}}\left(\mathrm{nm}^{2}\right)$ | $2.456 \times 10^{5}$ | $4.842 \times 10^{5}$ | $1.007 \times 10^{6}$ |
| $V_{\mathrm{F}}\left(\mathrm{nm}^{3}\right)$ | $3.979 \times 10^{6}$ | $9.624 \times 10^{6}$ | $5.303 \times 10^{7}$ |



Fig. S1 (a) A schematic illustration of the particle at time $t=t_{\mathrm{F}}$ (left) and at its pre-buckled state accompanied with different variables used in this study: radius $R$, mean spherical radius $\left\langle R_{\mathrm{F}}\right\rangle$, crust thickness $h$, volume $V$ and surface area $A$. (b) Estimation of the hidden surface area fraction due to creasing (self-contact) for C1. In the left vertical axis, crust thickness as a function of sphere radius needed to reproduce the final particle volume $V_{\mathrm{F}}$. The two annuli represent schematically the shape transition as $R$ increases, whereby a hollow cavity of larger volume is needed to keep the solid volume constant. Following the blue dashed line, an estimate for $\delta A=0.07_{-0.09}^{+0.12}$ can be calculated from the measured crust thickness $h_{\mathrm{F}}=17 \mathrm{~nm}$ (for $\mathrm{C} 2, \delta A=0.01_{-0.07}^{+0.09}$ ). The blue dotted lines represent the $95 \%$ confidence bounds for the crust thickness measurement ( $\pm 2 \mathrm{~nm}$ ). In the right vertical axis, sphere surface area normalized with respect to the final surface area $A_{\mathrm{F}}$ determined from the tomogram. The red dash-dot line shows the buckling transition radius $R_{\mathrm{B}}$ as well as the calculated crust thickness $\tilde{h}_{\mathrm{F}}=18.6 \mathrm{~nm}$ when the volume of the crust equals $V_{\mathrm{F}}$.


Fig. S2 Surface area $A$ as a function of volume $V$. The dashed line represents $A(V)$ of a sphere whereas the dotted lines represent the $A(V)$ evolution of the spherical harmonic models as the maximum harmonic degree $\ell_{\max }$ used in the reconstructions is increased from zero to its final value 64 (marked by $\times$ ). Hollow circles show the final surface area $A_{\mathrm{F}}$ as a function of the final volume $V_{\mathrm{F}}$ determined from the tomograms, with the solid line showing their power law fit.


Fig. S3 Value of multipole $r_{\ell}(\theta, \phi)$ planarity measure $L_{\ell}^{2}\left(\hat{n}_{\ell}\right)$ when the multipole is aligned with its preferred axis $\hat{n}_{\ell}$.

