Electronic Supplementary Information

Synchronization of self-propelled soft pendulums

Satoshi Nakata^{*}a, Katsuhiko Kayahara^a, Masakazu Kuze^a, Elliott Ginder^b, Masaharu Nagayama^c,

Hiraku Nishimori^a

^a Graduate School of Science, Hiroshima University, 1-3-1 Kagamiyama, Higashi-Hiroshima 739-8526, Japan

^b Dept. of Mathematical Sciences Based on Modeling and Analysis, Meiji University, 4-21-1 Nakano, Tokyo 164-8525, Japan

^c Research Institute for Electronic Science, Hokkaido University, N20W10, Kita-Ward, Sapporo 001-0020, Japan

*To whom correspondence should be addressed. Tel.: +81-82-424-7409, E-mail: nakatas@hiroshima-u.ac.jp

1. Movies for experimental results in Figures 2, 3, and 4

Movie S1. Movie of pendulum motion in Fig. 2 (real time)

Movie S2. Movie of in-phase synchronization in Fig. 3 (real time)

Movie S3. Movie of out-phase synchronization in Fig. 4 (real time)

2. Phase differences

To confirm the softness of two strings when they exhibit synchronization, the phase of the free edge (θ_b) and that close to the fixed edge (θ_a) were analyzed for the strings 1 and 2. Figure S1 shows the time-course of the phase difference ($\Delta \theta_a = \theta_{a2} - \theta_{a1}$, $\Delta \theta_b = \theta_{b2} - \theta_{b1}$) for two strings which exhibit in-phase synchronization (see Fig.3). Here, the pendulum behavior for one string is the same as that for another one if $\Delta \theta_a = \Delta \theta_b = 0$. If both $\Delta \theta_a$ and $\Delta \theta_b$ oscillate, synchronization occurs while changing the phase difference between the two strings. If $\Delta \theta_a = \Delta \theta_b$, the behavior of free edge is the same as that near the fixed edge, i.e., there is no softness in the two strings. Both $\Delta \theta_a$ and $\Delta \theta_b$ were not constant but oscillated. $\Delta \theta_a$ was not equal to $\Delta \theta_b$.



Figure S1. Time-course of (a) the phase difference between two strings ($\Delta \theta_a$: dotted line, $\Delta \theta_b$: solid line) and (b) $\Delta \theta$ for (1) string 1 and (2) string 2 when they exhibited in-phase synchronization at l = 50 mm. The data of Fig.S1 corresponds to that of Fig.3.

3. Simultaneous measurement of surface tension and motion of a string on water

The surface tension of water was measured simultaneously when a string exhibited pendulum motion, in order to clarify the relationship between pendulum motion and the surface tension. Figure S2a shows the experimental system. The Wilhelmy plate method was used for measuring the surface tension. A Pt wire (diameter: 0.5 mm) was used to measure the surface tension as the blade was attached to the water surface, and the distance between the tip of the string and the Pt wire was at a minimum (~ 10 mm) when the angle θ was π rad. Figure S2b shows the surface tension was maximized at $\theta \sim 0$ or π rad, but was the minimized at $\theta \sim \pi/2$ rad.



Figure S2. (a) Experimental setup for simultaneous measurement of the surface tension and pendulum motion of a string, and (b) the surface tension when $\theta \sim \pi/6$, $\pi/3$, $\pi/2$, $2\pi/3$, or $5\pi/6$ under the pendulum motion.

4. Numerical simulation distant filament motion.

In addition, we simulated the motion of two interfaces which are separated by a relatively large distance. In particular, we take their initial configurations as follows:

$$x_1(s,0) = (-0.5 - 1 + 0.36s)^T, x_2(s,0) = (0.5, -1 + 0.36s)^T,$$

where the filaments are again held fixed at the boundary using Dirichlet boundary conditions and the initial velocities along the curves are prescribed to have differing magnitudes:

$$\dot{x}_1(s,0) = (y_1,0)^T, \dot{x}_2(s,0) = (-2y_2,0)^T.$$

The computational domain is enlarged to the region $[-1,1]\times[-1,-0.4]$ and parameters values are $m = 1, \mu = 0.1, \alpha = 6, K_b = 4 \times 10^5, k = 100$ and $\Delta t = 6.4 \times 10^{-5}$. The filament motions are shown in Figure S3a, and time-evolution of the angles θ_{b1} , and θ_{b2} are shown in Figure S3b. We remark that our observations resemble the case of a single filament. In particular, noting the differences in the magnitude of the initial velocity fields, we observe relatively constant slopes of θ_{b1} , and θ_{b2} . This indicates a higher level of velocity independence between the filaments and can be attributed to the increase in their distance *l*. In this way, the coupling strength between the filaments' motions can be seen to decrease with an increase in distance.



Figure S3. No synchronization. (a) Evolution of two filaments, located relatively far apart. (b) The corresponding time series of θ_{b1} and θ_{b2} .