

1. DLP MODEL

The complex permittivity $\varepsilon^*(\omega)$ can be derived into the contribution of counterions fluctuation along chains axes $\varepsilon_l^*(\omega)$ and that perpendicular to chains axes $\varepsilon_h^*(\omega)$, respectively:

$$\varepsilon^* = \varepsilon_l^*(\omega) + \varepsilon_h^*(\omega) + A\omega^{-m} - \frac{B\omega^{1-m}}{i\omega\varepsilon_0} \quad (\text{S1})$$

Where $\omega(=2\pi f, f$ is the measured frequency) is the angular frequency, $A\omega^{-m}$ is the electrode polarization (EP) term of the permittivity data and the corresponding $B\omega^{1-m}/i\omega\varepsilon_0$ is the electrode polarization (EP) term of the dielectric loss data. A, B and m are the empirical parameters, m is related to the electrical phase angle δ ($=\pi(1-m)/2$) to characterize electrode polarization.¹⁻³

According to the DLP theory of flexible polyelectrolyte,⁴ the contribution of low-frequency(LF) relaxation can be expressed as:

$$\varepsilon_l^*(\omega) = \frac{\Delta\varepsilon_l}{1 + \left(\frac{-i\omega\xi^2}{D_i} \right)^\alpha} \quad (\text{S2})$$

where ξ is the correlation length, D_i ($=k_B T M_i$) is the diffusivity of the counterions, M_i is the mobility of the counterions, $k_B T$ is the thermal energy, and α is the distribution coefficient of low-frequency(LF) relaxation. On the basis of the perturbation calculation of PE solution, the dielectric increment can be defined as:

$$\Delta\varepsilon_l = \frac{16\pi}{9} \frac{\varepsilon_s \varphi \xi}{\nu \kappa^2} \left(\frac{G_l}{G_0} \right)^2 \quad (\text{S3})$$

Where φ is the volume fraction of PE chains, $G_0(=M_i q^2/l_B)$ is the linear conductance of the counterions in the bulk, l_B is the distance at which the electrostatic interaction between two elementary charges in the medium is equal to the thermal energy $k_B T$, The contribution of counterion fluctuation along chains axis to linear conductance can be given as:

$$G_l = M_i q \rho_l \quad (\text{S4})$$

Where the linear charged density of counterions that cause LF relaxation can be expressed as:

$$\rho_l = -2\pi\epsilon_0\epsilon_s \frac{\kappa a K_1(\kappa a)}{K_0(\kappa a)} \zeta \quad (S5)$$

Where ϵ_0 is vacuum dielectric constant and $K_\alpha(\alpha=0,1)$ is the modified Bessel function, which was used to describe the distribution of electric potential around the charged cylinder.

The contribution of high-frequency (HF) relaxation can be expressed as:

$$\epsilon_h^*(\omega) = \frac{\Delta\epsilon_h}{1 + \left(\frac{-i\omega}{\omega_h} \right)^\beta} \quad (S6)$$

Where ω_h is the relaxation angular frequency and β is the distribution coefficient of high frequency relaxation. The dielectric increment can be defined as:

$$\Delta\epsilon_h = \frac{2}{9\pi^2} \frac{\phi G_h^2 \kappa}{\omega_h^2 \epsilon_0^2 \epsilon_s \nu} \quad (S7)$$

The contribution of counterion fluctuation perpendicular to chain axes to linear conductance can be given as:

$$G_h = 2qM_i\rho_h \left(1 - \frac{1}{2\pi\eta l_B M_i}\right) \quad (S8)$$

Where η is the viscosity of the solvent, and the linear charged density of counterions that cause HF relaxation can be given as:

$$\rho_h = \frac{\pi\epsilon_0\epsilon_s\zeta^2}{4k_B T} \left(\frac{\kappa a K_1(\kappa a)}{K_0(\kappa a)} \right)^2 - \frac{\pi\epsilon_0\epsilon_s\zeta^2(\kappa a)^2}{4k_B T} \quad (S9)$$

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