

## 1. DLP MODEL

The complex permittivity  $\varepsilon^*(\omega)$  can be derived into the contribution of counterions fluctuation along chains axes  $\varepsilon_l^*(\omega)$  and that perpendicular to chains axes  $\varepsilon_h^*(\omega)$ , respectively:

$$\varepsilon^* = \varepsilon_l^*(\omega) + \varepsilon_h^*(\omega) + A\omega^{-m} - \frac{B\omega^{1-m}}{i\omega\varepsilon_0} \quad (S1)$$

Where  $\omega (=2\pi f, f$  is the measured frequency) is the angular frequency,  $A\omega^{-m}$  is the electrode polarization (EP) term of the permittivity data and the corresponding  $B\omega^{1-m}/i\omega\varepsilon_0$  is the electrode polarization (EP) term of the dielectric loss data. A, B and m are the empirical parameters, m is related to the electrical phase angle  $\delta$  ( $=\pi(1-m)/2$ ) to characterize electrode polarization.<sup>1-3</sup>

According to the DLP theory of flexible polyelectrolyte,<sup>4</sup> the contribution of low-frequency(LF) relaxation can be expressed as:

$$\varepsilon_l^*(\omega) = \frac{\Delta\varepsilon_l}{1 + \left( \frac{-i\omega\xi^2}{D_i} \right)^\alpha} \quad (S2)$$

where  $\xi$  is the correlation length,  $D_i (=k_B T M_i)$  is the diffusivity of the counterions,  $M_i$  is the mobility of the counterions,  $k_B T$  is the thermal energy, and  $\alpha$  is the the distribution coefficient of low-frequency(LF) relaxation. On the basis of the perturbation calculation of PE solution, the dielectric increment can be defined as:

$$\Delta\varepsilon_l = \frac{16\pi}{9} \frac{\varepsilon_s \varphi \xi}{\nu \kappa^2} \left( \frac{G_l}{G_0} \right)^2 \quad (S3)$$

Where  $\varphi$  is the volume fraction of PE chains,  $G_0 (=M_i q^2/l_B)$  is the linear conductance of the counterions in the bulk,  $l_B$  is the distance at which the electrostatic interaction between two elementary charges in the medium is equal to the thermal energy  $k_B T$ , The contribution of counterion fluctuation along chains axis to linear conductance can be given as:

$$G_l = M_i q \rho_l \quad (S4)$$

Where the linear charged density of counterions that cause LF relaxation can be expressed as:

$$\rho_l = -2\pi\epsilon_0\epsilon_s \frac{\kappa a K_1(\kappa a)}{K_0(\kappa a)} \zeta \quad (S5)$$

Where  $\epsilon_0$  is vacuum dielectric constant and  $K_\alpha(\alpha=0,1)$  is the modified Bessel function, which was used to describe the distribution of electric potential around the charged cylinder.

The contribution of high-frequency (HF) relaxation can be expressed as:

$$\epsilon_h^*(\omega) = \frac{\Delta\epsilon_h}{1 + \left(\frac{-i\omega}{\omega_h}\right)^\beta} \quad (S6)$$

Where  $\omega_h$  is the relaxation angular frequency and  $\beta$  is the distribution coefficient of high frequency relaxation. The dielectric increment can be defined as:

$$\Delta\epsilon_h = \frac{2}{9\pi^2} \frac{\varphi G_h^2 \kappa}{\omega_h^2 \epsilon_0^2 \epsilon_s \nu} \quad (S7)$$

The contribution of counterion fluctuation perpendicular to chain axes to linear conductance can be given as:

$$G_h = 2qM_i\rho_h \left(1 - \frac{1}{2\pi\eta l_B M_i}\right) \quad (S8)$$

Where  $\eta$  is the viscosity of the solvent, and the linear charged density of counterions that cause HF relaxation can be given as:

$$\rho_h = \frac{\pi\epsilon_0\epsilon_s\zeta^2}{4k_B T} \left( \frac{\kappa a K_1(\kappa a)}{K_0(\kappa a)} \right)^2 - \frac{\pi\epsilon_0\epsilon_s\zeta^2(\kappa a)^2}{4k_B T} \quad (S9)$$

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