Supporting information for "Facilitated polymer capture by charge inverted electroosmotic flow in voltage-driven polymer translocation"

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I. COMPUTATION OF THE 1L-LEVEL ELECTROSTATIC POTENTIAL $\phi(r)$

We explain here the computation of the correlationcorrected average electrostatic potential $\phi(r)$ required for the calculation of the drift velocity in Eq. (10) of the main text. The underlying 1l formalism being valid for dielectrically continuous media, the potential $\phi(r)$ will be computed by neglecting the dielectric jumps in the system. According to the 1l-theory of charge correlations [1, 2], the average potential in the pore is given by

$$\phi(r) = \phi_0(r) + \phi_c(r). \tag{1}$$

In Eq. (1), the MF component $\phi_0(r)$ solves the radial PB equation

$$\frac{1}{4\pi\ell_B r}\partial_r \left[r\partial_r\phi_0(r)\right] + \sum_{i=1}^p q_i n_i(r)$$
$$= -\sigma_m \delta(r-d) - \sigma_p \delta(r-a), \tag{2}$$

with the ionic number density function

$$n_i(r) = \rho_{bi} e^{-q_i \phi_0(r)} \theta(d-r) \theta(r-a).$$
(3)

Together with the Gauss' laws $\phi'_0(a^+) = -4\pi\ell_B\sigma_p$ and $\phi'_0(d^-) = 4\pi\ell_B\sigma_m$, Eq. (2) can be easily solved by numerical discretization. Then, the potential component $\phi_c(r)$ associated with charge correlations reads

$$\phi_c(r) = \int_a^d \mathrm{d}r' r' \tilde{G}_0(r, r'; k = 0) \delta\sigma(r'). \tag{4}$$

Eq. (4) includes the Fourier-transform of the electrostatic Green's function $G(\mathbf{r}, \mathbf{r'})$ defined by

$$G(\mathbf{r}, \mathbf{r}') = \sum_{n=-\infty}^{\infty} e^{in(\theta-\theta')} \int_{-\infty}^{\infty} \frac{\mathrm{d}k}{4\pi^2} \tilde{G}_n(r, r; k) e^{ik(z-z')},$$
(5)

with θ the polar angle in the x - y plane. The Green's function is the solution of the kernel equation

$$\left[\nabla^2 - \chi^2(r)\right] G(\mathbf{r}, \mathbf{r}') = -4\pi \ell_B \delta(\mathbf{r} - \mathbf{r}'), \qquad (6)$$

with the screening function $\chi^2(r) = 4\pi \ell_B \sum_i q_i^2 n_i(r)$. Eq. (4) also contains the charge density excess

$$\delta\sigma(r) = -\frac{1}{2}\sum_{i=1}^{p} q_i^3 n_i(r)\delta G(r),\tag{7}$$

with the ionic self-energy corresponding to the renormalized equal-point correlation function

$$\delta G(r) = \sum_{n=-\infty}^{\infty} \int_0^\infty \frac{\mathrm{d}k}{2\pi^2} \left[\tilde{G}_n(r,r;k) - \tilde{G}_{bn}(r,r;k) \right],\tag{8}$$

where the Fourier-transformed bulk Green's function is

$$\tilde{G}_{bn}(r,r';k) = 4\pi\ell_B \mathbf{I}_n(p_b r_{<}) \mathbf{K}_n(p_b r_{>}).$$
(9)

In Eq. (9), we used the radial variables

$$r_{<} = \min(r, r'); \quad r_{>} = \max(r, r').$$
 (10)

According to Eqs. (4) and (7), the computation of the correction term $\phi_c(\mathbf{r})$ necessitates the knowledge of the Green's function $G(\mathbf{r}, \mathbf{r}')$ solving Eq. (6). In order to solve this kernel equation, we exploit the cylindrical symmetry and insert the Fourier expansion (5) into Eq. (6). Then, we use the definition of the Green's function $\int d\mathbf{r}'' G^{-1}(\mathbf{r}, \mathbf{r}'') G(\mathbf{r}'', \mathbf{r}') = \delta(\mathbf{r} - \mathbf{r}')$ to recast Eq. (6) as an integral relation,

$$\tilde{G}_{n}(r,r';k) = \tilde{G}_{n}^{(0)}(r,r';k)$$

$$+ \int_{a}^{d} \mathrm{d}u u \tilde{G}_{n}^{(0)}(r,u;k) \delta n(u) \tilde{G}_{n}(u,r';k).$$
(11)

Eq. (11) involves the excess ion density function

$$\delta n(r) = \sum_{i=1}^{p} \rho_{bi} q_i^2 \left[1 - e^{-q_i \phi_0(r)} \right]$$
(12)

and the reference potential $\tilde{G}_n^{(0)}(r, r'; k)$. The latter corresponds to the Fourier-transform of the DH Green's function [1, 2]

$$\tilde{G}_{n}^{(0)}(r,r';k) = \tilde{G}_{bn}(r,r';k) + \delta \tilde{G}_{n}^{(0)}(r,r';k), \qquad (13)$$

where the inhomogeneous part reads

$$\delta \tilde{G}_{n}^{(0)}(r, r'; k) = \frac{4\pi \ell_{B}}{g_{1}g_{2} - 1} \{ g_{1}I_{n}(p_{b}r_{<})I_{n}(p_{b}r_{>}) \quad (14) \\ + g_{2}K_{n}(p_{b}r_{<})K_{n}(p_{b}r_{>}) \\ + I_{n}(p_{b}r_{<})K_{n}(p_{b}r_{>}) \\ + K_{n}(p_{b}r_{<})I_{n}(p_{b}r_{>}) \} .$$

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$$g_1 = \frac{I_n(ka)K'_n(p_ba) - I'_n(ka)K_n(p_ba)}{I'_n(ka)I_n(p_ba) - I_n(ka)I'_n(p_ba)},$$
 (15)

$$g_{2} = \frac{K_{n}(kd)I'_{n}(p_{b}d) - K'_{n}(kd)I_{n}(p_{b}d)}{K'_{n}(kd)K_{n}(p_{b}d) - K_{n}(kd)K'_{n}(p_{b}d)}$$
(16)

accounting for the presence of the concentric nanopore and the DNA molecule.

After obtaining the MF potential $\phi_0(r)$ in Eq. (12) from the solution of Eq. (2), the integral Eq. (11) can be numerically solved by iteration. The details of this iterative scheme can be found in Ref. [2]. The resulting Green's function $\tilde{G}_n(r, r'; k)$ is to be used next in Eqs. (4) and (7)-(8) in order to obtain the potential correction

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 $\phi_c(r)$. The substitution of the 1l potential (1) into Eq. (10) of the main text provides us with the correlationcorrected drift velocity. We finally note that in Eq. (29) of the main text, the liquid charge density is defined as

$$\rho_c(r) = \sum_{i=1}^p q_i \rho_i(r), \qquad (17)$$

with the correlation-corrected ionic number density

$$\rho_i(r) = \rho_{ib} e^{-q_i \phi_0(r)} \left[1 - q_i \phi_c(r) - \frac{q_i^2}{2} \delta G(r) \right].$$
(18)

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