SUPPLEMENTARY INFORMATION

Appendix A: Derivation of Shape Equations

The purpose of this Appendix is to formulate the capillay shape equation for the splay-bend director and the quartic surface energy.
The nematic capillarity vector is defined by the gradient of the scalar field rγ:

$$\xi(\mathbf{n},\mathbf{k}) = \nabla[r\gamma(\mathbf{k})] \tag{A1}$$

where r is the magnitude of surface position vector **r: r=rk**. Noting that $d(r\gamma) = \nabla(r\gamma).d\mathbf{r}$, the gradient of rγ yield:

$$\xi(\mathbf{n},\mathbf{k}) = \nabla[r\gamma(\mathbf{k})] = \gamma\frac{\partial r}{\partial \mathbf{r}} + r\frac{d\gamma}{d\mathbf{r}} = \gamma\mathbf{k} + I_s\cdot\frac{d\gamma}{d\mathbf{k}} \tag{A2}$$

Thus the normal and tangential components of capilarity vector are:

$$\begin{aligned}\xi_{\perp}(\mathbf{n},\mathbf{k}) &= \gamma\mathbf{k}\\ \xi_{\parallel}(\mathbf{n},\mathbf{k}) &= \mathbf{I}_s\cdot\frac{d\gamma}{d\mathbf{k}} = (\mathbf{I}_s.\mathbf{n})\frac{d\gamma}{d(\mathbf{n.k})} = \gamma'\mathbf{n}_{\parallel}\end{aligned} \tag{A3}$$

Where $\gamma' = \frac{d\gamma}{d(\mathbf{n.k})}$ and $\mathbf{n}_{\parallel} = \mathbf{I}_s.\mathbf{n}$ is the tangential component of the surface director field. Noticing that $\mathbf{I}_s$ is the 2×2 unit surface dyadic: $\mathbf{I}_s$=**I**-**kk** where **I** is the3×3 volumetric unit tensor, we have:

$$\xi_{\parallel}(\mathbf{n},\mathbf{k}) = I_s\cdot\frac{d\gamma}{d\mathbf{k}} = (\mathbf{I}-\mathbf{kk})\cdot\frac{d\gamma}{d\mathbf{k}} = \mathbf{I}.\frac{d\gamma}{d\mathbf{k}} - \mathbf{kk}.\frac{d\gamma}{d\mathbf{k}} \tag{A4}$$

Replacing the quartic surface free energy $\gamma = \gamma_o + \mu_2(\mathbf{n}\cdot\mathbf{k})^2 + \mu_4(\mathbf{n}\cdot\mathbf{k})^4$ to eqn (A3) we get:

$$\xi = \xi_{\perp} + \xi_{\parallel} = \left(\gamma_o + \mu_2(\mathbf{n}\cdot\mathbf{k})^2 + \mu_4(\mathbf{n}\cdot\mathbf{k})^4\right)\mathbf{k} + \left(2\mu_2(\mathbf{n}\cdot\mathbf{k})(\mathbf{n}\cdot\mathbf{t}) + 4\mu_4(\mathbf{n}\cdot\mathbf{k})^3(\mathbf{n}\cdot\mathbf{t})\right)\mathbf{t} \tag{A5}$$

Considering the capillary pressure definition, $\Delta P = (\nabla_s\cdot\xi) \equiv \mathbf{t}\cdot\partial\xi/\partial s$ where ΔP represents the pressure difference between the air and N* substrate, and ∇_s is the surface gradient, the case of zero capillary pressure, $\Delta P = 0$ yields $\nabla_s\cdot\xi = 0 \rightarrow \xi = \mathbf{C}_0 = \text{constant}$, where C_0 is a constant vector:

$$\begin{aligned}(\xi_{\perp})^2 + (\xi_{\parallel})^2 &= \xi\cdot\xi = \mathbf{C}_0^2\\ \xi_{\perp}\mathbf{k} + \xi_{\parallel}\mathbf{t} &= \mathbf{C}_0\end{aligned} \tag{A6}$$

Substituting eqn (A5) to eqn (A6) yields:

$$\left(\gamma_o + \mu_2(\mathbf{n}\cdot\mathbf{k})^2 + \mu_4(\mathbf{n}\cdot\mathbf{k})^4\right)^2 + \left(2\mu_2(\mathbf{n.k})(\mathbf{n.t}) + 4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})\right)^2 = \mathbf{C}_0^2 \tag{A7}$$

Replacing the unit tangent, **t** unit normal, **k** in terms of the tangent angle ϕ: $\mathbf{t}(x) = (\sin\phi(x),\ \cos\phi(x),\ 0)$ $\mathbf{n}(x) = (\cos\theta,\ \sin\theta,\ 0)$, and the director field, **n** in terms of the director angle, θ: $\theta = qx;\ q = \frac{2\pi}{P_0}$: $\mathbf{n}(x) = (\cos\theta,\ \sin\theta,\ 0)$ and following the parametrization: at s=0, $d\kappa(s=0)/ds = 0;\ \kappa(s=0) = \kappa_o;\ \ \phi(s=0) = \pi/2;$ $(\mathbf{n}.\mathbf{k})=0$, we obtain the constant C_0:

$$(\gamma_o)\boldsymbol{\delta}_y = \mathbf{C}_0 \tag{A8}$$

The dot product of the capillary vector, eqn (A6) with the unit vectors δ_y and δ_x respectively, gives:

$$\left(\gamma_o+\mu_2(\mathbf{n}\cdot\mathbf{k})^2+\mu_4(\mathbf{n}\cdot\mathbf{k})^4\right)\sin\varphi-\left(2\mu_2(\mathbf{n.k})(\mathbf{n.t})+4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})\right)\cos\varphi=\gamma_o \tag{A9-a}$$

$$\left(\gamma_o+\mu_2(\mathbf{n}\cdot\mathbf{k})^2+\mu_4(\mathbf{n}\cdot\mathbf{k})^4\right)\cos\varphi+\left(2\mu_2(\mathbf{n.k})(\mathbf{n.t})+4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})\right)\sin\varphi=0 \tag{A9-b}$$

Multiplying eqn (A9-a) and eqn (A9-b) by $\cos\varphi$ and $\sin\varphi$ respectively, we get:

$$\left(\gamma_o+\mu_2(\mathbf{n}\cdot\mathbf{k})^2+\mu_4(\mathbf{n}\cdot\mathbf{k})^4\right)\sin\varphi\cos\varphi-\left(2\mu_2(\mathbf{n.k})(\mathbf{n.t})+4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})\right)\cos^2\varphi=\gamma_o\cos\varphi \tag{A10-a}$$

$$\left(\gamma_o+\mu_2(\mathbf{n}\cdot\mathbf{k})^2+\mu_4(\mathbf{n}\cdot\mathbf{k})^4\right)\cos\varphi\sin\varphi+\left(2\mu_2(\mathbf{n.k})(\mathbf{n.t})+4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})\right)\sin^2\varphi=0 \tag{A10-b}$$

Subtracting eqn (A10-a) from eqn (A10-a) gives:

$$\begin{gathered}\left(2\mu_2(\mathbf{n.k})(\mathbf{n.t})+4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})\right)\cos^2\varphi+\left(2\mu_2(\mathbf{n.k})(\mathbf{n.t})+4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})\right)\sin^2\varphi=-\gamma_o\cos\varphi\\ 2\mu_2(\mathbf{n.k})(\mathbf{n.t})+4\mu_4(\mathbf{n.k})^3(\mathbf{n.t})+\gamma_o\cos\varphi=0\end{gathered} \tag{A11}$$

Replacing **t**(x), **k**(x), and **n**(x) in terms of the normal angle φ and the director angle θ yields the algebraic shape equation for the normal angle $\varphi(x, \mu_2/\gamma_0, \mu_4/\gamma_0, P_0)$:

$$\frac{2\mu_2}{\gamma_0}\cos(\varphi-\theta)\sin(\varphi-\theta)+\frac{4\mu_4}{\gamma_0}\cos^3(\varphi-\theta)\sin(\varphi-\theta)+\cos\varphi=0 \tag{A12}$$

Appendix B: Linear Model

The purpose of this Appendix is to formulate the linear model corresponding to the capillay shape equation for the splay-bend director and the quartic surface energy.

The dot product of the capillary vector, eqn (A5) with the unit tangent, **t**, gives:

$$\left(2\mu_2(\mathbf{n.k})+4\mu_4(\mathbf{n.k})^3\right)(\mathbf{n}\cdot\mathbf{t})=\gamma_0\delta_y\cdot\mathbf{t} \tag{B1}$$

Expressing the unit tangent, **t** and unit normal, **k** in terms of the surface amplitude, h_x gives:

$$\mathbf{t}=\frac{1}{\sqrt{1+h_x^2}}\begin{pmatrix}1\\-h_x\end{pmatrix};\ \mathbf{k}=\frac{1}{\sqrt{1+h_x^2}}\begin{pmatrix}h_x\\1\end{pmatrix} \tag{B2}$$

As we focus on the surface nano-wrinkling, h_x<<1, the linear approximation of **t** and **k** is valid: $\mathbf{t}=\boldsymbol{\delta}_x$; $\mathbf{k}=\boldsymbol{\delta}_y$. Replacing the unit tangent, **t** and unit normal, **k** into eqn (B1) gives:

$$2\mu_2 n_y n_x+4\mu_4 n_y{}^3 n_x=-\gamma_o h_x \tag{B3}$$

Rearranging eqn (B3) yields:

$$\left(2\mu_2(\mathbf{n}.\delta_y)+4\mu_4(\mathbf{n}.\delta_y)^3\right)(\mathbf{n}\cdot\delta_x)=-\gamma_o h_x \tag{B4}$$

Since $n_x dx=\frac{1}{q}dn_y$, the eqn (B4) becomes:

$$-\frac{1}{q}\left(\mu_2(\mathbf{n}.\delta_y)^2+\mu_4(\mathbf{n}.\delta_y)^4\right)=\gamma_o h \tag{B5}$$

Rearranging eqn (B5) gives the surface profile, h(x) as the ratio of anisotropic anchoring energy to isotropic surface tension (divided by wave vector, q):

$$h(x) = -\frac{\gamma_{anc}}{\gamma_o q} = -\frac{1}{\gamma_o q}\left(\mu_2(\mathbf{n}\cdot\boldsymbol{\delta}_y)^2 + \mu_4(\mathbf{n}\cdot\boldsymbol{\delta}_y)^4\right) = -\frac{1}{\gamma_o q}\left(\mu_2(\sin qx)^2 + \mu_4(\sin qx)^4\right) \tag{B6}$$

The surface curvature h_{xx} then can be obtained from:

$$h_{xx} = -2\frac{\mu_2}{\gamma_o}q\left(\cos qx^2 - \sin qx^2\right) - 4\frac{\mu_4}{\gamma_o}q\left(3\sin qx^2\cos qx^2 - (\sin qx)^4\right) \tag{B7}$$

Appendix C: Multiple Scales

The purpose of this Appendix is to formulate the surface amplitudes for the planar, $h_{\|}$, homeotropic, $h_{\perp}$ and oblique, h_o orientations as a function of $r = \mu_2 / 2\mu_4; -1 \le r \le 0$.

Considering eqn (B6), we can find the planar, $h_{\|}$ homeotropic, $h_{\perp}$ and oblique, h_o amplitudes as follows:

(a) Planar

$$n_x = 1,\ \sin qx = 0 \ \rightarrow h_{\|} = 0 \tag{C1}$$

(b) Homeotropic

$$n_y = 1,\ \cos qx = 0(\mathbf{n}\text{ along } y) \rightarrow h_{\perp} = -\frac{1}{\gamma_o q}(\mu_2 + \mu_4) = -\frac{\mu_4}{\gamma_o q}\left(2\frac{\mu_2}{2\mu_4} + 1\right) = -\frac{\mu_4}{\gamma_o q}(2r + 1) \tag{C2}$$

(c) Oblique

for $-1 < \mu_2 / 2\mu_4 < 0,\ \mu_2 < 0$

$$\sin qx^2 = -\frac{\mu_2}{2\mu_4},\ \cos qx^2 = 1 + \frac{\mu_2}{2\mu_4} \ \rightarrow h_o = -\frac{1}{\gamma_o q}\left(-\mu_2\left(\frac{\mu_2}{2\mu_4}\right) + \mu_4\left(\frac{\mu_2}{2\mu_4}\right)^2\right) = \frac{\mu_4}{\gamma_o q}\left(\left(\frac{\mu_2}{2\mu_4}\right)^2\right) = \frac{\mu_4}{\gamma_o q}\left(r^2\right) \tag{C3}$$

Appendix D: Scaling Laws

The purpose of this Appendix is to formulate the two-scale ratio SR=h_o/h_2 as a function of $r = \mu_2 / 2\mu_4$. Using straight algebra, the distance h_2 (shown in Fig 8) can be expressed in terms of amplitudes for the homeotropic, $h_{\perp}$ and oblique, h_o orientations:

$$h_2 = h_o - \mathrm{sign}(h_{\perp}) \times |h_{\perp}| = \frac{\mu_4}{\gamma_i q}\left(\left(\frac{\mu_2}{2\mu_4}\right)^2 + 2\frac{\mu_2}{2\mu_4} + 1\right) = \frac{\mu_4}{\gamma_o q}\left(r^2 + 2r + 1\right) > 0 \tag{D1}$$

Using the oblique amplitude, h_o, eqn (C3) gives the following linear scaling law:

$$\frac{h_o}{h_2} = \frac{+\frac{\mu_4}{\gamma_o q}\left(\left(\frac{\mu_2}{2\mu_4}\right)^2\right)}{\frac{\mu_4}{\gamma_o q}\left(\left(\frac{\mu_2}{2\mu_4}\right)^2 + 2\frac{\mu_2}{2\mu_4} + 1\right)} = \frac{r^2}{r^2 + 2r + 1} = \frac{r^2}{(1+r)^2} \tag{D2}$$

We note that the same scaling law (D2) are obtained for the H^+ / P_{12} modes.