Electronic Supplementary Material for Three-Dimensional Multicomponent Vesicles: Dynamics & Influence of Material Properties

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S1. CALCULATION OF INCLINATION ANGLE AND DEFORMATION PARAMETER

The shape and orientation of the three-dimensional vesicles are obtained via eigenvalues of the inertia matrix. Let Ω^- be the region enclosed by the vesicle membrane. The inertia matrix is given by

$$\boldsymbol{I}_0 = \begin{bmatrix} \int_{\Omega^-} \hat{y}^2 + \hat{z}^2 \text{ dV} & -\int_{\Omega^-} \hat{x}\hat{y} \text{ dV} & -\int_{\Omega^-} \hat{x}\hat{z} \text{ dV} \\ -\int_{\Omega^-} \hat{x}\hat{y} \text{ dV} & \int_{\Omega^-} \hat{x}^2 + \hat{z}^2 \text{ dV} & -\int_{\Omega^-} \hat{y}\hat{z} \text{ dV} \\ -\int_{\Omega^-} \hat{x}\hat{z} \text{ dV} & -\int_{\Omega^-} \hat{y}\hat{z} \text{ dV} & \int_{\Omega^-} \hat{x}^2 + \hat{y}^2 \text{ dV} \end{bmatrix}, \tag{S1}$$

where $\hat{x} = x - \bar{x}$, $\hat{y} = y - \bar{y}$, and $\hat{z} = z - \bar{z}$. The center of the vesicle, $(\bar{x}, \bar{y}, \bar{z})$ is defined as

$$\bar{x} = \frac{1}{V}\int_{\Omega^-} x \text{ dV}, \qquad \bar{y} = \frac{1}{V}\int_{\Omega^-} y \text{ dV}, \qquad \bar{z} = \frac{1}{V}\int_{\Omega^-} z \text{ dV}, \tag{S2}$$

while the volume is given by $V = \int_{\Omega^-} \text{dV}$. In all cases the integrals are calculated using a numeric Heaviside function and the level-set describing the interface:

$$\int_{\Omega^-} f \text{ dV} \approx \sum_{ijk} \left(1 - H(\phi_{ijk})\right) f_{ijk} h_x h_y h_z, \tag{S3}$$

where f_{ijk} is the value of the function to be integrated at a grid point, h_x, h_y, and h_z are the grid spacings in the $x-$, $y-$, and $z-$directions, respectively, and $H(\phi)$ is the Heaviside function [1].

The symmetric matrix $\boldsymbol{I}_0$ has three real eigenvalues and orthogonal eigenvectors [2]. The inclination angle is defined as the angle between the eigenvector corresponding to the largest eigenvalue and the $x-$axis. Following the definition of deformation parameter, or asphericity, is defined as [3, 4]

$$D = \frac{\lambda_{max} - \lambda_{min}}{\lambda_{max} + \lambda_{min}}, \tag{S4}$$

where λ_{max} is the largest eigenvalue while λ_{min} is the smallest eigenvalue.

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S2. DESCRIPTION OF MOVIES

There are four movies included in this work. Each corresponds to a result presented in the main text and all have $\bar{c} = 0.4$. The movies includes are:

TABLE S1: Table of movies

File Name	Main Text Figure	α	κ_c^B	Pe
FIG2Movie.mov	2	20	0.6	0.2
FIG3Movie.mov	3	20	0.7	0.5
FIG4Movie.mov	4	20	0.8	1.0
FIG10Movie.mov	10	0.5	0.4	0.3

[1] J. D. Towers, *Journal of Computational Physics*, 2009, **228**, 3478–3489.
[2] A. Laadhari, P. Saramito and C. Misbah, *Journal of Computational Physics*, 2014, **263**, 328–352.
[3] L. Lu, W. J. Doak, J. W. Schertzer and P. R. Chiarot, *Soft Matter*, 2016, **12**, 7521–7528.
[4] S. Messlinger, B. Schmidt, H. Noguchi and G. Gompper, *Phys. Rev. E*, 2009, **80**, 011901.