Supplementary Information

On the universality of the Flow Curve for Soft-Particle Glasses

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1. Pair distribution function for suspension at rest for different force laws.
The pair distribution function for suspensions with different pairwise elastic force laws are similar at rest, as noted in Fig. S1.

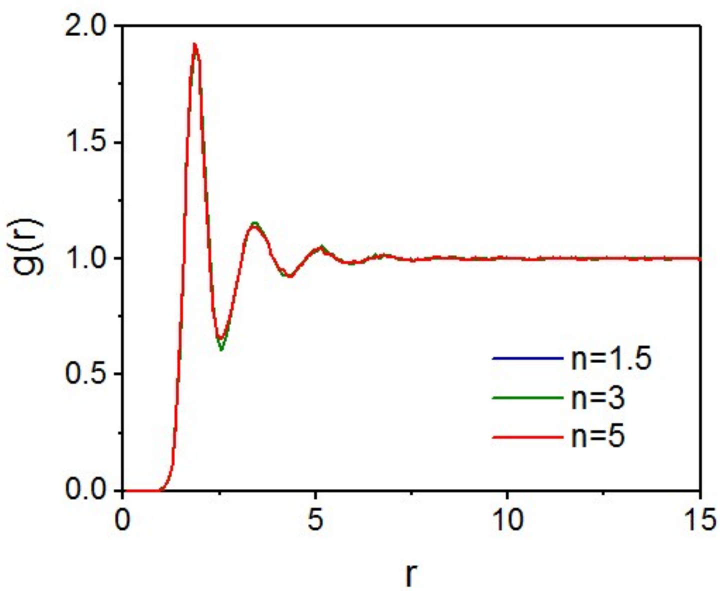

Figure S1 Pairwise particle distribution at rest for volume fraction = 0.8

2. Fitting parameters for curves in Figs. 2-6. Tables S1-S6 list the fitting parameters used for the curves in Figs. 2-6 of the paper. Standard deviations are given inside parentheses following each data point. All the data shown in Table S1 and S5 are based on the following dimensionless equations:

$$\sigma/E^* = \sigma_y/E^* + k_\sigma\left(\dot{\gamma}\eta_s/E^*\right)^m, \tag{S1}$$

$$N_1/E^* = N_{1y}/E^* + k_{N1}\left(\dot{\gamma}\eta_s/E^*\right)^{m_1}, \tag{S2}$$

$$-N_2/E^* = -N_{2y}/E^* + k_{N2}\left(\dot{\gamma}\eta_s/E^*\right)^{m_2}. \tag{S3}$$

Table S1 Fitting parameters for curves in Fig. 2.

Volume Fraction	σ_y	k_σ	m
0.70	1.36×10^{-4} (1.59×10^{-5})	2.93 (2.52)	0.606 (0.065)
0.80	6.29×10^{-4} (4.59×10^{-5})	1.86 (0.79)	0.500 (0.032)
0.90	2.69×10^{-3} (9.61×10^{-5})	3.51 (0.71)	0.488 (0.015)
Volume Fraction	N_{1y}	k_{N1}	m_1
0.70	1.15×10^{-5} (2.00×10^{-6})	0.838 (0.354)	0.693 (0.036)
0.80	6.76×10^{-5} (1.29×10^{-5})	0.301 (0.186)	0.535 (0.053)
0.90	2.49×10^{-4} (2.14×10^{-5})	0.666 (0.117)	0.538 (0.017)
Volume Fraction	N_{2y}	k_{N2}	m_2
0.70	2.49×10^{-5} (1.18×10^{-6})	1.98 (0.33)	0.669 (0.013)
0.80	1.11×10^{-4} (6.59×10^{-6})	2.02 (0.24)	0.615 (0.011)
0.90	3.48×10^{-4} (3.06×10^{-5})	2.77 (0.39)	0.587 (0.013)

(The fitting parameters are similar for simulations with and without near-field draft forces.)

Table S2 Parameters for static properties in Figs. 3 and 4.

n	Parameters	Volume fraction				
		0.70	0.75	0.80	0.85	0.90
1.5	ε	0.079	0.140	0.196	0.251	0.300
	Z	8.04	8.86	9.51	10.04	10.51
	G	9.22×10^{-3} (0.00149)	1.85×10^{-2} (0.00258)	2.79×10^{-2} (0.00224)	3.71×10^{-2} (0.00403)	4.72×10^{-2} (0.00356)
3	ε	0.091	0.159	0.219	0.276	0.329
	Z	7.85	8.57	9.13	9.62	10.04
	G	3.63×10^{-4} (6.09×10^{-5})	1.82×10^{-3} (2.87×10^{-4})	4.95×10^{-3} (4.41×10^{-4})	1.00×10^{-2} (1.13×10^{-3})	1.56×10^{-2} (2.23×10^{-3})
5	ε	0.099	0.169	0.232	0.292	0.346
	Z	7.73	8.41	8.94	9.38	9.75
	G	4.21×10^{-6} (8.98×10^{-7})	7.48×10^{-5} (1.77×10^{-5})	4.28×10^{-4} (8.83×10^{-5})	1.26×10^{-3} (1.74×10^{-4})	3.25×10^{-3} (4.17×10^{-4})

Table S3 Fitting parameters for low frequency modulus in Fig. 3.

n	G_0	ϕ_c	γ (fixed to *n*-0.5)
1.5	0.188 (0.001)	0.651 (0.001)	1.0
3	0.508 (0.021)	0.645 (0.001)	2.5
5	1.460 (0.090)	0.641 (0.002)	4.5

Note: $G = G_0 \left(\phi - \phi_c\right)^\gamma$

Table S4 Fitting parameters in Fig. 4

n	Fig. 4a			Fig. 4b	
	Z_c	Z_0	ζ	ε_0	ϕ_c
1.5	6.0 (0.1)	8.96 (0.03)	0.49 (0.02)	1.24 (0.03)	0.63 (0.01)
3	6.3 (0.1)	7.98 (0.04)	0.54 (0.02)	1.19 (0.03)	0.62 (0.01)
5	6.0 (0.1)	7.32 (0.04)	0.49 (0.02)	1.10 (0.02)	0.62 (0.01)

Note: $Z - Z_c = Z_0 \left(\phi - \phi_c\right)^\zeta$ with ϕ_c fixed to 0.65; $\varepsilon = \varepsilon_0 (\phi - \phi_c)$ with ϕ_c not fixed.

Table S5.1 Fitting parameters for shear stress in Fig. 5.

n	Volume Fraction	σ_y	k_σ	m
1.5	0.70	2.01×10^{-4} (1.6×10^{-6})	0.517 (0.042)	0.438 (0.005)
	0.75	5.69×10^{-4} (6.0×10^{-6})	0.499 (0.055)	0.413 (0.008)
	0.80	1.07×10^{-3} (7.8×10^{-6})	0.637 (0.097)	0.415 (0.011)
	0.85	1.66×10^{-3} (3.5×10^{-5})	0.571 (0.102)	0.398 (0.014)
	0.90	1.83×10^{-3} (4.5×10^{-5})	0.344 (0.086)	0.341 (0.018)
3	0.70	6.71×10^{-6} (3.7×10^{-7})	0.475 (0.073)	0.517 (0.009)
	0.75	4.51×10^{-5} (7.8×10^{-7})	0.349 (0.031)	0.461 (0.006)
	0.80	1.39×10^{-4} (7.0×10^{-7})	0.404 (0.017)	0.441 (0.003)
	0.85	3.00×10^{-4} (1.7×10^{-6})	0.436 (0.016)	0.424 (0.003)
	0.90	5.56×10^{-4} (7.5×10^{-6})	0.504 (0.046)	0.417 (0.006)
5	0.70	1.00×10^{-7} (1.5×10^{-8})	1.015 (0.15)	0.637 (0.007)
	0.75	1.84×10^{-6} (2.0×10^{-7})	0.469 (0.087)	0.551 (0.011)
	0.80	1.06×10^{-5} (2.2×10^{-7})	0.350 (0.054)	0.500 (0.009)
	0.85	3.65×10^{-5} (5.3×10^{-7})	0.394 (0.051)	0.476 (0.008)
	0.90	9.73×10^{-5} (1.1×10^{-6})	0.531 (0.044)	0.470 (0.005)

Table S5.2 Fitting parameters for first normal stress difference N_1 in Fig. 5.

n	Volume Fraction	N_{1y}	k_{N1}	m_1
1.5	0.70	1.91×10^{-5} (2.0×10^{-6})	0.650 (0.335)	0.629 (0.041)
	0.75	5.70×10^{-5} (2.0×10^{-6})	0.424 (0.061)	0.554 (0.012)
	0.80	1.07×10^{-4} (3.5×10^{-6})	0.456 (0.059)	0.529 (0.011)
	0.85	1.68×10^{-4} (4.3×10^{-6})	0.487 (0.061)	0.511 (0.010)
	0.90	2.10×10^{-4} (1.4×10^{-5})	0.359 (0.102)	0.464 (0.024)
3	0.70	5.95×10^{-7} (9.9×10^{-8})	0.232 (0.101)	0.645 (0.029)
	0.75	4.42×10^{-6} (3.2×10^{-7})	0.260 (0.091)	0.607 (0.025)
	0.80	1.39×10^{-5} (1.7×10^{-6})	0.402 (0.132)	0.596 (0.026)
	0.85	3.26×10^{-5} (1.6×10^{-6})	0.419 (0.065)	0.572 (0.013)
	0.90	5.70×10^{-5} (1.9×10^{-6})	0.342 (0.048)	0.534 (0.011)
5	0.70	1.45×10^{-8} (2.1×10^{-9})	0.147 (0.072)	0.707 (0.027)
	0.75	1.93×10^{-7} (5.2×10^{-8})	0.304 (0.245)	0.694 (0.051)
	0.80	1.08×10^{-6} (2.5×10^{-7})	0.113 (0.072)	0.589 (0.042)
	0.85	3.60×10^{-6} (3.1×10^{-7})	0.090 (0.029)	0.537 (0.022)
	0.90	1.06×10^{-5} (1.2×10^{-6})	0.408 (0.125)	0.611 (0.024)

Table S5.3 Fitting parameters for second normal stress difference N_2 in Fig. 5.

n	Volume Fraction	$-N_{2y}$	k_{N2}	m_2
1.5	0.70	2.88×10^{-5} (1.1×10^{-6})	0.487 (0.042)	0.533 (0.007)
	0.75	7.40×10^{-5} (9.1×10^{-7})	0.421 (0.014)	0.501 (0.003)
	0.80	1.28×10^{-4} (2.0×10^{-6})	0.466 (0.020)	0.496 (0.004)
	0.85	1.79×10^{-4} (4.9×10^{-6})	0.481 (0.051)	0.487 (0.009)
	0.90	2.19×10^{-4} (8.6×10^{-6})	0.422 (0.054)	0.469 (0.011)
3	0.70	9.35×10^{-7} (1.8×10^{-7})	0.661 (0.121)	0.621 (0.012)
	0.75	6.68×10^{-6} (5.6×10^{-7})	0.577 (0.118)	0.584 (0.014)
	0.80	1.90×10^{-5} (1.1×10^{-6})	0.508 (0.038)	0.551 (0.006)
	0.85	3.96×10^{-5} (1.5×10^{-6})	0.557 (0.048)	0.539 (0.007)
	0.90	6.70×10^{-5} (1.2×10^{-6})	0.527 (0.022)	0.518 (0.004)
5	0.70	1.60×10^{-8} (3.0×10^{-9})	0.990 (0.093)	0.710 (0.005)
	0.75	3.55×10^{-7} (5.6×10^{-8})	0.821 (0.125)	0.664 (0.010)
	0.80	1.77×10^{-6} (2.8×10^{-7})	0.635 (0.139)	0.620 (0.015)
	0.85	5.08×10^{-6} (5.6×10^{-7})	0.511 (0.103)	0.580 (0.014)
	0.90	1.33×10^{-5} (6.4×10^{-7})	0.620 (0.049)	0.572 (0.006)

Table S6. Fitting parameters in Fig. 6**.**

n		$\tilde{k}_\sigma$	$\tilde{m}$
1.5		145.7 (37.2)	0.407 (0.025)
3		224.2 (20.4)	0.428 (0.009)
5		677.0 (73.5)	0.503 (0.012)
n	$\tilde{N}_{1y}$	$\tilde{k}_{N1}$	$\tilde{m}_1$
1.5	0.104 (0.007)	72.5 (15.2)	0.537 (0.025)
3	0.104 (0.005)	112.4 (12.0)	0.588 (0.014)
5	0.110 (0.007)	132.2 (15.5)	0.616 (0.016)
n	$-\tilde{N}_{2y}$	$\tilde{k}_{N2}$	$\tilde{m}_2$
1.5	0.133 (0.023)	112.9 (49.8)	0.532 (0.053)
3	0.136 (0.027)	181.0 (45.6)	0.550 (0.033)
5	0.144 (0.026)	420.2 (75.2)	0.611 (0.023)

Note: $\sigma / \sigma_y = 1 + \tilde{k}_\sigma \left(\dot{\gamma}\eta_s / G_0\right)^{\tilde{m}}$

$$N_1 / \sigma_y = \tilde{N}_{1y} + \tilde{k}_{N1}\left(\dot{\gamma}\eta_s / G_0\right)^{\tilde{m}_1}; -N_2 / \sigma_y = -\tilde{N}_{2y} + \tilde{k}_{N2}\left(\dot{\gamma}\eta_s / G_0\right)^{\tilde{m}_2}$$

Table S7. Fitting parameters in Fig 7.

	$\bar{k}_\sigma$	$\bar{m}$
	42.6 (7.9)	0.405 (0.025)
$\bar{N}_{1y}$	$\bar{k}_{N1}$	$\bar{m}_1$
0.104 (0.005)	14.7 (1.4)	0.540 (0.018)
$-\bar{N}_{2y}$	$\bar{k}_{N2}$	$\bar{m}_2$
0.130 (0.026)	20.1 (6.3)	0.517 (0.058)

(Note: $\sigma / \sigma_y = 1 + \bar{k}_\sigma \left(\dot{\gamma}\eta_s / \gamma_y^2 E^*\right)^{\bar{m}}$

$$N_1 / \sigma_y = \bar{N}_{1y} + \bar{k}_{N1}\left(\dot{\gamma}\eta_s / \gamma_y^2 E^*\right)^{\bar{m}_1}; -N_2 / \sigma_y = -\bar{N}_{2y} + \bar{k}_{N2}\left(\dot{\gamma}\eta_s / \gamma_y^2 E^*\right)^{\bar{m}_2}$$)

3. Scaling of yield stress with distance to jamming.

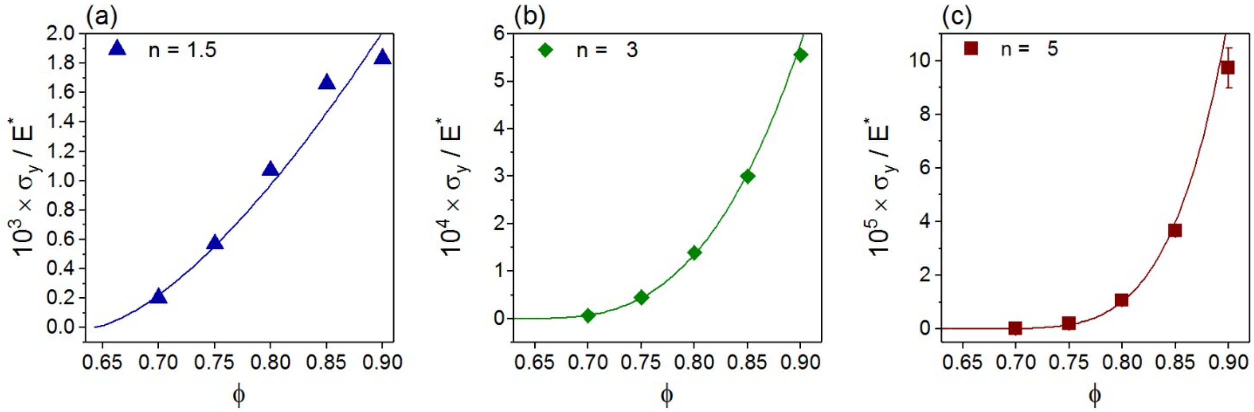

Figure S2 Scaling of yield stress with volume fraction. ϕ_c is assumed to be 0.64. The fitted curves are: (a) $\sigma_y / E^* = 0.0152(\phi - \phi_c)^{1.5}$; (b) $\sigma_y / E^* = 0.0331(\phi - \phi_c)^3$; (c) $\sigma_y / E^* = 0.0980(\phi - \phi_c)^5$.