Supplementary Material for "Nonlinear Bending Deformation of Soft Electrets and Prospects for Engineering Flexoelectricity and Transverse (d_{31}) Piezoelectricity"

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Formulation for the bending of compressible electret

In this supplementary, we relax the incompressible constraint for an electret with surface charge density $\widetilde{\rho}_e = q\delta(X - X_{ch})$ and update relations given in section 4 in the paper. The objective is to assess the effect of the incompressibility assumption on our final results.

In contrast to the deformation of incompressible block given in eqn (36), the deformation of compressible block is expressed as [1, 2]

$$r = r(X), \quad \theta = \theta_0 \frac{Y}{L}, \quad z = Z, \tag{S.1}$$

where θ_0 is an unknown constant. $r(X)$ and θ_0 will to be determined by solving the boundary-value problem. The deformation gradient tensor $\mathbf{F} = d\mathbf{x}/d\mathbf{X}$ is given as

$$\mathbf{F} = \frac{dr}{dX}\mathbf{e}_r \otimes \mathbf{e}_X + \frac{r\theta_0}{L}\mathbf{e}_\theta \otimes \mathbf{e}_Y + \mathbf{e}_z \otimes \mathbf{e}_Z. \tag{S.2}$$

We use the *compressible* neo-Hookean material and the corresponding strain energy function is

$$W^{elast} = \frac{\mu}{2}\left(J^{-2/3}\,\mathrm{tr}(\mathbf{F}^T\mathbf{F}) - 3\right) + \frac{k}{2}(J-1)^2, \tag{S.3}$$

where k is the bulk modulus of the material. Substituting eqn (S.3) and (S.2) into eqn (9) in the paper, the first Piola stress $\widetilde{\mathbf{\Sigma}}$ is obtained as

$$\widetilde{\mathbf{\Sigma}} = \frac{\partial \psi}{\partial \mathbf{F}} = \Sigma_{rX}\mathbf{e}_r \otimes \mathbf{e}_X + \Sigma_{\theta Y}\mathbf{e}_\theta \otimes \mathbf{e}_Y + \Sigma_{zZ}\mathbf{e}_z \otimes \mathbf{e}_Z. \tag{S.4}$$

Furthermore, eqn (S.2) and eqn (8a) and (4) are used to express nominal electric displacement as

$$\widetilde{\mathbf{D}} = D_X\mathbf{e}_X = -\epsilon\frac{r\theta_0}{L}\left(\frac{dr}{dX}\right)^{-1}\frac{d\xi}{dX}\mathbf{e}_X. \tag{S.5}$$

The Piola-Maxwell stress is derived substituting eqn (S.5) into eqn (10) in the paper:

$$\widetilde{\mathbf{\Sigma}}^{MW} = \Sigma^{MW}_{rX}\mathbf{e}_r \otimes \mathbf{e}_X + \Sigma^{MW}_{\theta Y}\mathbf{e}_\theta \otimes \mathbf{e}_Y + \Sigma^{MW}_{zZ}\mathbf{e}_z \otimes \mathbf{e}_Z. \tag{S.6}$$

The equilibrium equations in the reference configuration are expressed as

$$\frac{d}{dX}\left(\Sigma_{rX}+\Sigma_{rX}^{MW}\right)-\frac{\theta_0}{L}\left(\Sigma_{\theta Y}+\Sigma_{\theta Y}^{MW}\right)=0, \tag{S.7a}$$
$$\frac{dD_X}{dX}=q\delta(X-X_{ch}). \tag{S.7b}$$

Boundary conditions are similar to boundary conditions used in the incompressible case:

$$\Sigma_{rX}+\Sigma_{rX}^{MW}=0 \qquad \text{at} \quad X=H \text{ and } X=-H, \tag{S.8a}$$
$$\xi(X=-H)=\xi(X=H)=0, \tag{S.8b}$$
$$\int_{-H}^{H} r\left(\Sigma_{\theta Y}+\Sigma_{\theta Y}^{MW}\right)dX=M. \tag{S.8c}$$

We solve the above system of ordinary differential equations numerically and definition (71) is used to calculate flexoelectric coefficient. Here, we have considered numerical values of $\mu = 1$ MPa, $\epsilon = 2.35\epsilon_0$, $H = 30$ μm, $X_{ch} = 0$ and $q = 10^{-3}$ C/m^2 and present results for different values of Poisson's ratio which was shown in Figs. (I,II and III).

Figure I shows the effect of compressibility on the apparent flexoelectric coefficient. Apparent flexoelectric coefficient versus average dimensionless curvature has been plotted in this figure. For all levels of compressibility, there is no qualitative change in the results compare to incompressible case (except at very high curvatures). Our results remain of the same order e.g. approximate reduction of 30% is observed in the value of flexoelectric coefficient when Poisson's ratio decreases from 0.5 to 0.25 in Fig. I. We remark that a Poisson ratio of 0.25 represents an unusually high compressibility for a soft material.

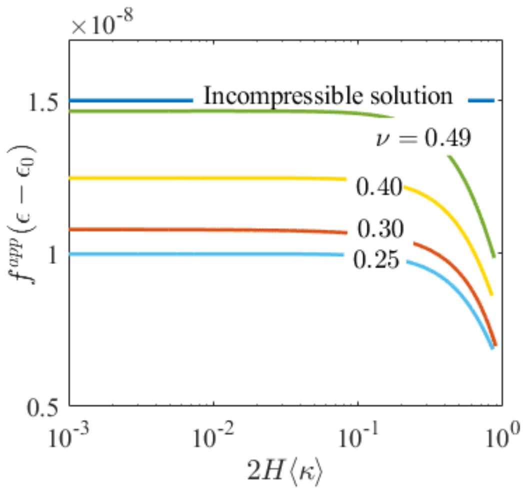

Figure I: Effect of compressibility on the flexoelectric coefficient of electret.

Qualitative agreements of the compressible and incompressible cases can also be found in Figs. II and III. Bending moment versus dimensionless curvature has been plotted in Figure II for materials with different values of Poisson's ratio. It is clear that all materials show the same qualitative behavior.

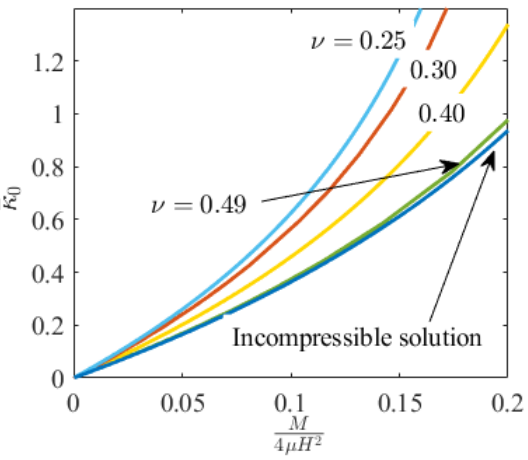

Figure II: Bending moment versus curvature for blocks for different Poisson's ratio.

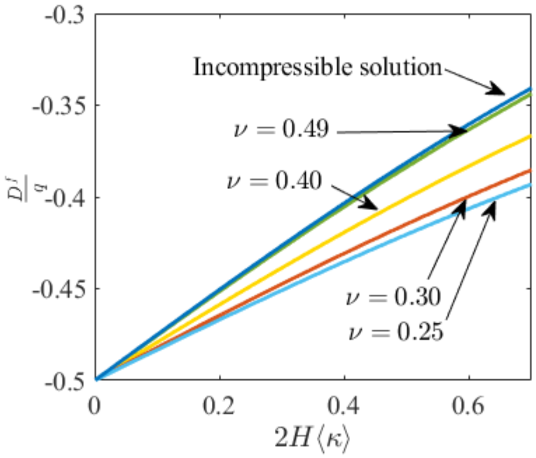

Figure III: Change of the electric displacement with respect to the change of the curvature.

References

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