Electronic Supplementary Material (ESI) for Soft Matter.

Electronic Supplementary Information

Cylindrical nematic liquid crystal shell: Effect of saddle-splay elasticity

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Critical K_{24} for the double-twist director configuration in the cylindrical nematic shell

Here we show that the critical value of K_{24} for the spontaneous chiral symmetry breaking in the cylindrical nematic shell is $2K_2$ using eqn (3) and eqn (4) of the main text. In other words, if K_{24} is smaller than $2K_2$, the ground state corresponds to the trivial parallel-axial configuration, $\beta(\rho) = 0$. Note that this criterion is same to the one in the cylinder case.[1] First, we make the substitution $t = \log \rho$ then eqn (3) of the main text becomes

$$2k_2 \frac{d^2\beta}{dt^2} = k_2 \cos 2\beta \, \sin 2\beta + 4 \cos \beta \sin^3 \beta. \tag{S1}$$

Multiplying through by $\frac{\partial \beta}{\partial t}$ and integrating gives

$$k_2 (\frac{d\beta}{dt})^2 = k_2 \cos^2 \beta \, \sin^2 \beta + \sin^4 \beta + C, \tag{S2}$$

where C is the integration constant. Note that eqn (S2) also satifies the boundary condition, eqn (4) of the main text, or equivalently

$$\left(\frac{d\beta}{dt}\right)_{t=t_i} = \frac{(k_{24} - k_2)}{k_2} \sin \beta(t_i) \cos \beta(t_i) = \alpha \sin \beta(t_i) \cos \beta(t_i), \quad i = 1, 2. \tag{S3}$$

Organizing terms and replacing $\sin^2 \beta(t_i)$ into X gives

$$k_2(\alpha^2 - 1)X(1 - X) = X^2 + C. \tag{S4}$$

For $\beta(\rho)$ to be the double-twist director configuration, eqn (S4) should have two real solutions of X with a constant C. Considering $0 \leq X = \sin^2 \beta(t_i) \leq 1$ and utilizing a simple graphical analysis, we find that there cannot be two real solutions of X when $\alpha^2 - 1 \leq 0$, i.e., $K_{24} \leq 2K_2$.

References

[1] Z. S. Davidson, L. Kang, J. Jeong, T. Still, P. J. Collings, T. C. Lubensky and a. G. Yodh, *Physical Review E*, 2015, **91**, 050501.

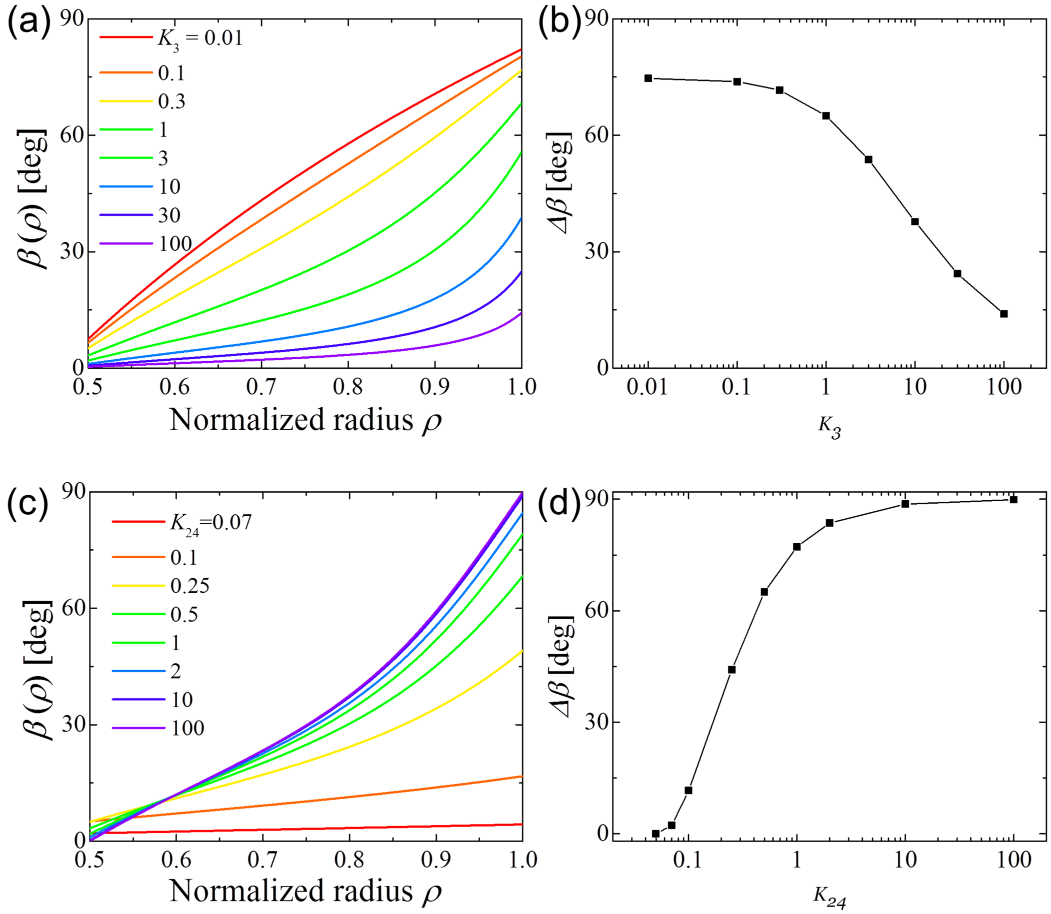

Figure S1: Effects of elastic constants on twist angle profiles. (a) and (b) A numerically calculated twist angle profile and the amount of twist $\Delta\beta = |\beta_2 - \beta_1|$ in the double-twist director configuration with $K_2 = \frac{1}{30}$, $K_{24} = \frac{1}{2}$, and varying K_3. Note that dimensionless elastic constants were used in the calculation. The normalized inner shell radius ρ_1 is fixed to be 0.5. With a large K_3, the twist angle increases slowly when ρ is small. It is because of a large bending penalty with a small radius of curvature. Consequently, the larger the K_3 is, the smaller $\Delta\beta$ is. (c) and (d) A numerically calculated twist angle profile and the amount of twist $\Delta\beta = |\beta_2 - \beta_1|$ in the double-twist director configuration with $K_3 = 1$, $K_2 = \frac{1}{30}$, and varying K_{24}. Note again that dimensionless elastic constants were used in the calculation. With a large K_{24}, β_1 and β_2 are close to 0 and 90 degree, respectively. It is because K_{24} works like a curvature-induced azimuthal anchoring and the large K_{24} correspond to strong azimuthal anchoring along the direction of the largest principal curvature when the surface normal vectors at the interfaces are pointing toward the nematic LC. In contrast, the small K_{24} should compete with the twist elastic constant K_2 so that large slip angles, i.e., deviations from 90 degree, occurs at the outer shell radius ρ_2; the slip angles at the inner shell radius ρ_1 are relatively small because the larger bending penalty and the enhanced K_{24} effect at the smaller radius of curvature.

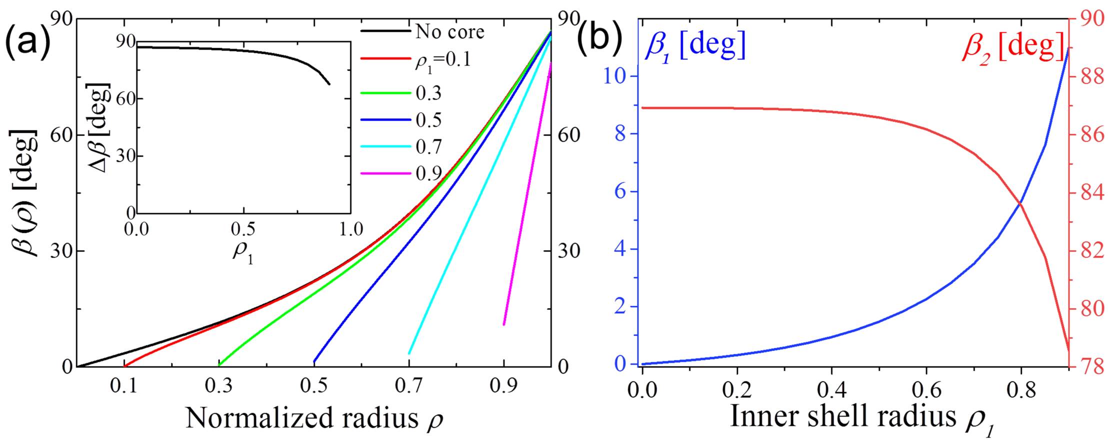

Figure S2: A numerically calculated twist angle profile in the double-twist director configuration with $k_2 = \frac{1}{20}$ and $k_{24} = 6$ which are the approximate elastic constants of 30.0% (wt/wt) nematic SSY at 25.0 °C. (a) The twist angle β is plotted as a function of the normalized radius $\rho = r/R_2$ according to different normalized inner shell radius $\rho_1 = R_1/R_2 = 0.1, ..., 0.9$. (b) The twist angles at the inner and outer wall are plotted respectively as a function of inner shell radius ρ_1.