

Supplementary Information

Correction: Characterizing surface wetting and interfacial properties using enhanced sampling (SWIPES)

Sean M. Marks, Hao Jiang, Zachariah Vicars, Suruchi Fialoke,
Amish J. Patel*
(amish.patel@seas.upenn.edu)

*Department of Chemical and Biomolecular Engineering, University of Pennsylvania
Philadelphia, PA 19104, USA*

1 Comparing h_{COM} and h

In this appendix, we present the derivation which leads to Eq. 1. Consider the model setup illustrated in Fig. S1. The simulation box is a fixed-volume orthorhombic box with side lengths L_x , L_y , and L_z maintained at constant temperature, T . It contains N_{tot} water molecules in contact with a solid surface a of length $l_{S,x}$ that runs from $x_{S,0}$ to $x_{S,1}$ along the x -axis. Its surface is normal to the z -axis and the separation between opposing faces across periodic boundary conditions is l_z . We will refer to the space between the two faces of the solid surface as the *inner* region, and the space outside as the *outer* region. The vapor-liquid interface in the inner region is located at $x = H$, the position of its midpoint.

For a single-component system of point masses (or rigid molecules, whose positions are identified by their centers of mass), $\langle x_{\text{COM}} \rangle_{\kappa, N^*}$ for the entire system is related to the total number density profile at equilibrium, $\rho(x, y, z)$, by

$$\langle x_{\text{COM}} \rangle_{\kappa, N^*} \equiv \frac{1}{N_{\text{tot}}} \int_V dV x \rho(x, y, z) \quad (1)$$

where the integral is taken over the entire simulation box (volume V) and the dependence of ρ on the biasing parameters κ and N^* is suppressed for clarity. Assuming that ρ is constant along y due to periodic boundary conditions,

$$\langle x_{\text{COM}} \rangle_{\kappa, N^*} = \frac{L_y}{N_{\text{tot}}} \int_0^{L_x} dx \int_0^{L_z} dz x \rho(x, z) \quad (2)$$

In evaluating Eq. 2, we distinguish between several different regions of the simulation box to simplify the calculation. First, we assume that $l_{S,x}$ is large enough there are regions to the left and right of the inner box vapor-liquid interface in which

*To whom correspondence should be addressed

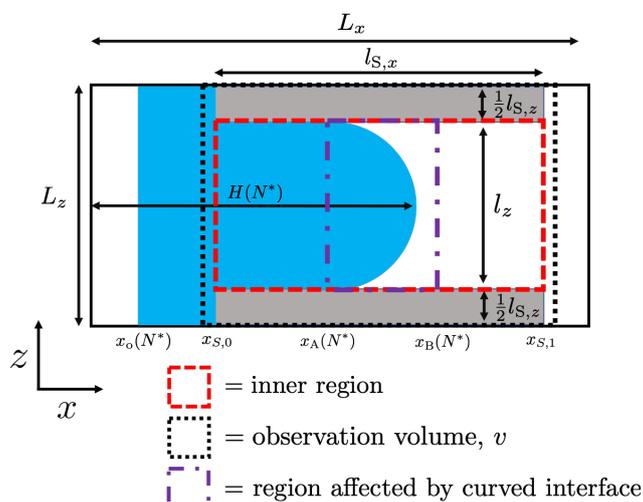


Figure S1: Illustration of the model system employed to estimate the accuracy of using x_{COM} to track the location of the vapor-liquid interface.

the density is unaffected by the inner vapor-liquid interface or edges of the surface, and is therefore constant along x . Denote the density of the liquid and vapor in these "bulk" inner regions by $\rho_{L,S}(z)$ and $\rho_{V,S}(z)$, respectively. We assume that deviations of the density field away from these profiles due to the presence of the inner vapor-liquid interface are confined to the region from $x_A(N^*)$ to $x_B(N^*)$. Provided that the inner vapor-liquid interface maintains its shape as N^* increases, $\Delta x_{\text{AB}} = x_B - x_A$ is constant and $x_A(N^*) \propto H(N^*)$. To account for edge effects, we place the Gibbs dividing surface between the inner and outer liquid phases at $x_{S,0}$ and assume that the edge effects are confined to the region $x_{S,0} \pm \Delta x_{S,0}$ with width $2\Delta x_{S,0}$. We use the same approach at the right edge of the slab and confine the inhomogeneity to the region $x_{S,1} \pm \Delta x_{S,1}$. The corresponding surface excesses are Γ_L and Γ_V , respectively. The probe volume, v , is chosen such that it encompasses the region $x = x_{S,0} - \Delta x_{S,0}$ to $x = x_{S,1} + \Delta x_{S,1}$.

Breaking the simulation box into regions as described above,

$$\begin{aligned} \langle x_{\text{COM}} \rangle_{\kappa, N^*} = & \frac{L_y}{N_{\text{tot}}} \left[\underbrace{\int_0^{x_{S,0}-\Delta x_{S,0}} dx \int_0^{L_z} dz x \rho(x, z)}_{\text{to the left of the surface}} + \underbrace{\int_{x_{S,0}-\Delta x_{S,0}}^{x_{S,0}-\Delta x_{S,0}} dx \int_0^{L_z} dz x \rho(x, z)}_{\text{left edge of surface}} \right. \\ & + \underbrace{\int_{x_{S,0}+\Delta x_{S,0}}^{x_A(N^*)} dx \int_0^{L_z} dz x \rho_{L,S}(z)}_{\text{"bulk" liquid between surface}} + \underbrace{\int_{x_A(N^*)}^{x_B(N^*)} dx \int_0^{L_z} dz x \rho(x, z)}_{\text{inner vapor-liquid interface}} + \underbrace{\int_{x_B(N^*)}^{x_{S,1}-\Delta x_{S,1}} dx \int_0^{L_z} dz x \rho_{V,S}(z)}_{\text{"bulk" vapor between surface}} \\ & \left. + \underbrace{\int_{x_{S,1}-\Delta x_{S,1}}^{x_{S,1}+\Delta x_{S,1}} dx \int_0^{L_z} dz x \rho(x, z)}_{\text{right edge of surface}} + \underbrace{\int_{x_{S,1}+\Delta x_{S,1}}^{L_x} dx \int_0^{L_z} dz x \rho(x, z)}_{\text{to the right of the surface}} \right] \quad (3) \end{aligned}$$

After taking the derivative with respect to N^* , it can be shown that

$$\begin{aligned} \frac{h_{\text{COM}}}{h} \equiv \frac{d\langle x_{\text{COM}} \rangle_{\kappa, N^*} / dN^*}{dH/dN^*} = & 1 - \frac{(\Gamma_L + \Gamma_V) A_o}{N_{\text{tot}}} - (1 - \zeta r) \left[1 - \frac{\langle N_v \rangle_{\kappa, N^*}}{N_{\text{tot}}} - \frac{\Delta x_{S,0} \rho_{L,b}}{L_x \rho_{\text{tot}}} - \frac{\Delta x_{S,1} \rho_{V,b}}{L_x \rho_{\text{tot}}} \right] \\ & - \zeta \frac{\rho_{V,b}}{\rho_{\text{tot}}} \left[\frac{\bar{\rho}_{V,S} l_{S,x}}{\rho_{V,b} L_x} + r \left(1 - \frac{l_{S,x}}{L_x} \right) \right] \quad (4) \end{aligned}$$

where $A_o = L_y L_z$ is the outer cross-sectional area, $\rho_{\text{tot}} \equiv N_{\text{tot}}/V$ is the average density, Γ_L and Γ_V are the surface excesses at either end of the surface,

$$\begin{aligned} \Gamma_L & \equiv \int_{x_{S,0}-\Delta x_{S,0}}^{x_{S,0}} dx [\bar{\rho}_o(x) - \rho_{L,b}] + \zeta \int_{x_{S,0}}^{x_{S,0}+\Delta x_{S,0}} dx [\bar{\rho}_i(x) - \bar{\rho}_{L,S}], \\ \Gamma_V & \equiv \zeta \int_{x_{S,1}-\Delta x_{S,1}}^{x_{S,1}} dx [\bar{\rho}_i(x) - \bar{\rho}_{V,S}] + \int_{x_{S,1}}^{x_{S,1}+\Delta x_{S,1}} dx [\bar{\rho}_o(x) - \rho_{V,b}], \end{aligned}$$

$\zeta = l_z/L_z$ is the fraction of the box along z that is between the surfaces, $r \equiv \Delta \bar{\rho}_S / \Delta \rho_b = (\bar{\rho}_{L,S} - \bar{\rho}_{V,S}) / (\rho_{L,b} - \rho_{V,b})$ is the ratio of differences in average density between the inner and outer regions, $\bar{\rho}_{L,S} = \frac{1}{l_z} \int_{l_z} dz \rho_{L,S}(z)$ and $\bar{\rho}_{V,S} = \frac{1}{l_z} \int_{l_z} dz \rho_{V,S}(z)$ are the average densities in the bulk inner regions, and $\bar{\rho}_i(x) = \frac{1}{l_z} \int_{l_z} dz \rho(x, z)$ and $\bar{\rho}_o(x) = \frac{1}{L_z} \int_0^{L_z} dz \rho(x, z)$ are the densities averaged over the inner and outer cross-sectional areas (respectively).

If we assume that the vapor and liquid densities are uniform and equal to their bulk values ($\rho_{V,b}$ and $\rho_{L,b}$, respectively), then $\bar{\rho}_{V,S} = \rho_{V,b}$ and $\bar{\rho}_{L,S} = \rho_{L,b}$; since edge effects are therefore also neglected, $\Delta x_{S,0} = \Delta x_{S,1} = 0$ and $\Gamma_L = \Gamma_V = 0$. Eq. 4 then greatly simplifies to

$$\frac{h_{\text{COM}}}{h} = 1 - \alpha \lambda - (1 - \alpha) \frac{\rho_{V,b}}{\rho_{\text{tot}}} \quad (5)$$

where $\alpha \equiv l_{S,z}/L_z = 1 - \zeta$ is the fraction of the box occupied by the solid surface along z , and $\lambda \equiv 1 - \langle \tilde{N}_v \rangle_{\kappa, N^*} / N_{\text{tot}}$ is the fraction of molecules outside the observation volume. Lastly, for fluids far from their triple point (such as water at ambient conditions), $\rho_{V,b} \ll \rho_{\text{tot}}$ when the liquid occupies a substantial part of the simulation box. and therefore the last term in Eq. 5 is negligible. This leads to Eq. 1 of the main text: $h_{\text{COM}}/h \approx 1 - \alpha \lambda$.