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Elasticity and Yielding of Mesophases of Block Copolymers in Water-Oil Mixtures

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Derivation of elastic constants for micellar cubic system:

$$\Delta F_C = 8\frac{-A_H\, R_C}{12\, {b_1}^2} + 4\frac{-A_H\, R_C}{12\, {c_1}^2} - 12\frac{-A_H\, R_C}{12\, {a_1}^2} \tag{S1}$$

Thus:

$$C_{C11} = \frac{\partial^2 \Delta F_C}{\partial \varepsilon_{C1}^2} = \frac{-A_H R_C}{3}\left(\frac{-3a}{(\varepsilon_{C1}+1)^{5/2}(a(\frac{1}{\sqrt{\varepsilon_{C1}+1}}-1)+a_1)^3} + \frac{3a^2}{(\varepsilon_{C1}+1)^3(a(\frac{1}{\sqrt{\varepsilon_{C1}+1}}-1)+a_1)^4} + \frac{6a^2}{(a\varepsilon_{C1}+a_1)^4}\right) \tag{S2}$$

$$C_{C12} = \frac{\partial^2 \Delta F_C}{\partial \varepsilon_{C1}\partial \varepsilon_{C2}} = \frac{-A_H R_C}{3}\left(\frac{-8a^2}{(\varepsilon_{C2}+1)^3\left(\frac{a}{(\varepsilon_{C2}+1)^2}-2R_C\right)^3} + \frac{3a\sqrt{\varepsilon_{C2}+1}}{(a\varepsilon_{C2}+a_1)^3} - \frac{6a^2(\varepsilon_{C2}+1)^{3/2}}{(a\varepsilon_{C2}+a_1)^4}\right) \tag{S3}$$

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$$C_{C44} = \frac{\partial^2 \Delta F_C}{\partial \gamma_{C12}^2} = -4A_H R_C \left(\left(\frac{\left(\frac{B_1}{K_1} + \frac{\left(O_1 + \frac{H_1}{3{S_1}^{2/3}}\right)}{12a}\right)^2}{\left(3a\left(\sqrt{\frac{1}{12aP_1} + \left(\frac{1}{N_1} - \frac{\left(2\gamma_{C12}+1\right)^2}{2} + \frac{\left(4\gamma_{C12}+1\right)^3}{27} + \frac{1}{2}\right)^2}\right)\right)^3} \right) + \left(\frac{F_1}{12aM_1} - \frac{\left(O_1 + \frac{Q_1}{3{S_1}^{2/3}}\right)^2}{\frac{12a{P_1}^3}{2}} + \right.\right.$$

$$\frac{\left(\frac{416\gamma_{C12}}{9} - \frac{G_1}{9{S_1}^{5/3}} + \frac{\frac{416\gamma_{C12}+92}{9} - \frac{{R_1}^2}{E_1} + \frac{I_1}{D_1}}{3{S_1}^{2/3}} + \frac{92}{9}\right)}{K_1} -$$

$$\left.\left. \frac{\left(\frac{3{A_1}^2}{L_1} + \frac{C_1}{J_1}\right)}{6a\left(\frac{1}{12aP_1} + \sqrt{\frac{1}{\left(a_1 + \frac{\left(32{\gamma_{C12}}^3 + 32{\gamma_{C12}}^2 + 10\gamma_{C12} + 1\right)}{6} + {S_1}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2}}\right)^2} \right)\right) \tag{S4}$$

where,

$$A_1 = 4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} + \frac{4\gamma_{C12} - \frac{7(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8{\gamma_{C12}}^2 + 6\gamma_{C12} + 1)\right) + 2}{\left((3{S_1}^{2/3}) - \left(\frac{4}{3}(8{\gamma_{C12}}^2 + 6\gamma_{C12} + 1)\right) + 2\right)} +$$

$$\frac{\frac{T_1\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8{\gamma_{C12}}^2 + 6\gamma_{C12} + 1)\right) + 2\right)}{2\sqrt{\left(\frac{(32{\gamma_{C12}}^3 + 32{\gamma_{C12}}^2 + 10\gamma_{C12} + 1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^3}}}{\left((3{S_1}^{2/3}) - \left(\frac{4}{3}(8{\gamma_{C12}}^2 + 6\gamma_{C12} + 1)\right) + 2\right)} \tag{S4-a}$$

$$B_1 = \left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8{\gamma_{C12}}^2 + 6\gamma_{C12} + 1)\right) + 2\right) + \frac{Q_1}{3{S_1}^{2/3}} \tag{S4-b}$$

$$C_1 = \left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} + \frac{\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+2\right)+U_1}{\left(\left(3S_1{}^{2/3}\right)-\left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+2\right)}\right)^2 \tag{S4-c}$$

$$D_1 = 2\sqrt{\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^3} \tag{S4-d}$$

$$E_1 = 4\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^{3^{3/2}} \tag{S4-e}$$

$$F_1 = \frac{416\gamma_{C12}}{9} - \frac{\left(\frac{T_1\left(\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+2\right)+2\right)^2}{E_1} + \frac{I_1}{D_1} + 92\right)}{3S_1{}^{2/3}} - \frac{2Q_1{}^2}{9P^{5/3}} + 2\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+2\right)^2 + V_1 \tag{S4-f}$$

$$G_1 = 2\left(\frac{4\gamma_{C12}+T_1\left(\left(\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+4\right)\right)\right)}{2\sqrt{\left(S_1{}^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^3\right)}} - \frac{7(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+2\right)^2 \tag{S4-g}$$

$$H_1 = \left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+2\right) + \frac{R_1}{2\sqrt{\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^3}} \tag{S4-h}$$

$$I_1 = 2\left(\frac{416\gamma_{C12}+92}{9}\right)\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right) - 6\left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)\left(\frac{80\gamma_{C12}+26}{9}\right)^2 + 2\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2 + 6\gamma_{C12} + 1)\right) + 2\right)^2 - \frac{80\left(\frac{(4\gamma_{C12}+1)^2}{9}+\frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^2}{3} \quad \text{(S4-i)}$$

$$J_1 = \left(\frac{1}{\left(a_1+\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6}+S_1{}^{1/3}-\frac{(2\gamma_{C12}+1)^2}{2}+\frac{(4\gamma_{C12}+1)^3}{27}+\frac{1}{2}\right)^2}\right)^{3/2}\left(a_1 + \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} + S_1{}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^6 \quad \text{(S4-j)}$$

$$K_1 = \sqrt{\frac{1}{\left(a_1+\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6}+S_1{}^{1/3}-\frac{(2\gamma_{C12}+1)^2}{2}+\frac{(4\gamma_{C12}+1)^3}{27}+\frac{1}{2}\right)^2}}\left(a_1 + \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} + S_1{}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^3 \quad \text{(S4-k)}$$

$$L_1 = K_1\left(a_1 + \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} + S_1{}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right) \quad \text{(S4-l)}$$

$$M_1 = \left(\left(a_1 + \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} + S_1{}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 + S_1{}^{1/3}\right)^2 \quad \text{(S4-m)}$$

$$N_1 = \left(a_1 + \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} + S_1{}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^{1/3} \quad \text{(S4-n)}$$

$$O_1 = 2\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2 + 6\gamma_{C12} + 1)\right) + 2\right)\left(a_1 + \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} + S_1{}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right) \quad \text{(S4-o)}$$

$$P_1 = \left(a_1 + \left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)\right)^2 + S_1{}^{1/3} \quad \text{(S4-p)}$$

$$Q_1 = 4\gamma_{C12} + \frac{R_1}{2\sqrt{\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^3}} -$$

$$\frac{7(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2 + 6\gamma_{C12} + 1)\right) + 2 \qquad \text{(S4-q)}$$

$$R_1 = 3\left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^2 \left((\frac{80\gamma_{C12}}{9}) + \frac{26}{9}\right) + 2\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} +\right.$$

$$\left.\frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2 + 6\gamma_{C12} + 1)\right) + 2\right) \qquad \text{(S4-r)}$$

$$S_1 = \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1\,)}{6} +$$

$$\sqrt{\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^3} -$$

$$\frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2} \qquad \text{(S4-s)}$$

$$T_1 = 3\left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^2 \left((\frac{80\gamma_{C12}}{9}) + \frac{26}{9}\right) + 2\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} +\right.$$

$$\left.\frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right) \qquad \text{(S4-t)}$$

$$U_1 = U_1 = \frac{T_1\left(4\gamma_{C12} - \frac{(4\gamma_{C12}+1)^2}{9} - \left(\frac{4}{3}(8\gamma_{C12}{}^2+6\gamma_{C12}+1)\right)+2\right)}{2\sqrt{\left(\frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right)^2 - \left(\frac{(4\gamma_{C12}+1)^2}{9} + \frac{(4\gamma_{C12}+1)(2\gamma_{C12}+1)}{3}\right)^3}} \qquad \text{(S4-u)}$$

$$V_1 = 2\left(\frac{416\gamma_{C12}+92}{9}\right)\left(a_1 + \frac{(32\gamma_{C12}{}^3+32\gamma_{C12}{}^2+10\gamma_{C12}+1)}{6} + S_1{}^{1/3} - \frac{(2\gamma_{C12}+1)^2}{2} + \frac{(4\gamma_{C12}+1)^3}{27} + \frac{1}{2}\right) \qquad \text{(S4-v)}$$

$$\gamma_{C12} = \frac{\varepsilon_{C1} - \varepsilon_{C2}}{2} \qquad \text{(S4-w)}$$

Derivation of elastic constants for lamellar system:

$$\Delta F_L = \frac{-A_H}{6\,\pi\,{D_1}^3} - \frac{-A_H}{6\,\pi\,D^3} \tag{S5}$$

Intermicellar distances (D_1 and D_2) in the lamellar case is defined as $D_1 = \delta - V_{ppo}\delta$ and $D_2 = (d - \delta)(1 - V_{peo})$, where V_{ppo} and V_{peo} are the volume fraction of polypropylene oxide block polyethlene oxide block, respectively; and δ is the apolar domain size, which in turn is a function of lattice parameter, d, and apolar volume fraction, f. Therefore, $D_1 = (1 - V_{ppo})fd = \beta_1 d$ and $D_2 = (1 - V_{ppeo})(1 - f)d = \beta_2 d$. Similarly, after deformation, the intermicellar distances change to $D_3 = \beta_1 d_1$ and $D_4 = \beta_2 d_1$.

Therefore:

$$C_{L11} = \frac{\partial^2 \Delta F_L}{\partial \varepsilon_{L1}^2} = \frac{-3A_H({\beta_1}^3 + {\beta_2}^3)}{6\,\pi{\beta_1}^3{\beta_2}^3 d^3}\left(\frac{4}{(\varepsilon_{L1}+1)^5}\right) \tag{S6}$$

$$C_{L12} = \frac{\partial^2 \Delta F_L}{\partial \varepsilon_{L1}\partial \varepsilon_{L2}} = \frac{-3A_H({\beta_1}^3 + {\beta_2}^3)}{6\,\pi{\beta_1}^3{\beta_2}^3 d^3}\left(-4\left(\frac{1}{\varepsilon_{L1}+1}\right)^3\right) \tag{S7}$$

$$C_{L44} = \frac{\partial^2 \Delta F_L}{\partial \gamma_{L12}^2} = \frac{-3A_H({\beta_1}^3 + {\beta_2}^3)}{6\,\pi{\beta_1}^3{\beta_2}^3 d^3}\left(\frac{\left(3\sqrt{{\gamma_{L12}}^2+1}+\gamma_{L12}\right)}{({\gamma_{L12}}^2+1)^{\frac{3}{2}}\left(\sqrt{{\gamma_{L12}}^2+1}+\gamma_{L12}\right)^3}\right) \tag{S8}$$

where $\gamma_{L12} = \frac{\varepsilon_{L1} - \frac{1}{\varepsilon_{L1}+1} + 1}{2}$.

Derivation of elastic constants for hexagonal system:

$$\Delta F_H = 2\frac{-A_H\sqrt{R_H/2}}{8\sqrt{2}\,x_1{}^{13/5}} + 8\left(\frac{-A_H\sqrt{R_H/2}}{16\sqrt{2}\,l_1{}^{13/5}} + \frac{-A_H\sqrt{3}\sqrt{R_H/2}}{16\sqrt{2}\,l_1{}^{13/5}}\right) + 2\frac{-A_H\sqrt{R_H/2}}{8\sqrt{2}\,(\frac{x+l}{2}-2R)^{13/5}} \tag{S9a}$$

$$\Delta F_H = \frac{-A_H\sqrt{R_H/2}}{8\sqrt{2}}\left(\frac{2}{\left(h\left(\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)}-1\right)+h_1\right)^{13/5}} + \frac{4+4\sqrt{3}}{(h\varepsilon_1+h_1)^{13/5}} + \frac{2}{\left(\frac{h\left(\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)}-1\right)+2h+h\varepsilon_1}{2}-2R\right)^{13/5}}\right)2\frac{}{x_1{}^{13/5}} + 8\left(\frac{-A_H\sqrt{R_H/2}}{16\sqrt{2}\,l_1{}^{13/5}} + \frac{-A_H\sqrt{3}\sqrt{R_H/2}}{16\sqrt{2}\,l_1{}^{13/5}}\right) + 2\frac{-A_H\sqrt{R_H/2}}{8\sqrt{2}\,(\frac{x+l}{2}-2R)^{13/5}} \tag{S9b}$$

Thus:

$$C_{H11} = \frac{\partial^2\Delta F_H}{\partial\varepsilon_{H1}^2} = \frac{-A_H\sqrt{R_H/2}}{40\sqrt{2}}\left(\frac{468h^2\left(-\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)^2}-1\right)^2}{5\left(h\left(\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)}-1\right)+h_1\right)^{23/5}} + \frac{117\left(h\left(\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)^2}-1\right)+h\right)^2}{5\left(\frac{h\left(\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)}-1\right)+h\varepsilon_1+2h)}{2}-2R_H\right)^{23/5}} + \frac{234\,(4+4\sqrt{3})h^2}{5(h\varepsilon_1+h_1)^{23/5}} - \frac{26h\left(\frac{3-(\varepsilon_1+1)^2}{(\varepsilon_1+1)^3}+\frac{1}{\varepsilon_1+1}\right)}{\left(h\left(\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)}-1\right)+h_1\right)^{\frac{18}{5}}} - \frac{13h\left(\frac{3-(\varepsilon_1+1)^2}{(\varepsilon_1+1)^3}+\frac{1}{\varepsilon_1+1}\right)}{\left(\frac{h\left(\frac{3-(\varepsilon_1+1)^2}{2(\varepsilon_1+1)}-1\right)+h\varepsilon_1+2h)}{2}-2R_H\right)^{18/5}}\right) \tag{S10}$$

$$C_{H12} = \frac{\partial^2 \Delta F_H}{\partial \varepsilon_{H1} \partial \varepsilon_{H2}} = \frac{-A_H\sqrt{R_H/2}}{40\sqrt{2}} \left(\frac{3-(\varepsilon_{H1}+1)^2}{3+(\varepsilon_{1H}+1)^2}\right) \left(\frac{234\left(4+4\sqrt{3}\right) h^2}{5\,(h\varepsilon_{H1}+h_1)^{23/5}} + \frac{468 h^2 {\varepsilon_{H2}}^2}{5\,(h\varepsilon H_2+h_1)^{23/5}} - \right.$$

$$\left. \frac{26h\left(\frac{1}{\varepsilon_{H1}+1} - \frac{(\varepsilon_{H1}+1)^2-3}{(\varepsilon_{1H}+1)^3}\right)}{(h\varepsilon_{H2}+h_1)^{\frac{18}{5}}} + \frac{13h\left(\frac{1}{\varepsilon_{H1}+1} - \frac{(\varepsilon_{H1}+1)^2-3}{(\varepsilon_{H1}+1)^3}\right)}{\left(\frac{h\varepsilon_{H2}+h\varepsilon_{H1}+2h}{2} - 2R_H\right)^{\frac{18}{5}}} - \frac{117h^2(-\varepsilon_{H2}-1)^2}{5\left(\frac{h\varepsilon_{H2}+h\varepsilon_{H1}+2h}{2} - 2R_H\right)^{23/5}} \right) \tag{S11}$$

where $\varepsilon_{H2} = \frac{3-(\varepsilon_{H1}+1)^2}{2(\varepsilon_{H1}+1)} - 1$. Elastic constants associated with direction 3 are zero, as there is no strain in that direction.

$$C_{H13} = \frac{\partial^2 \Delta F_H}{\partial \varepsilon_{H1} \partial \varepsilon_{H3}} = 0 \tag{S12}$$

$$C_{H33} = \frac{\partial^2 \Delta F_H}{\partial \varepsilon_{H3}^2} = 0 \tag{S13}$$

Also, the shear elastic constant will be:

$$C_{H44} = \frac{\partial^2 \Delta F_H}{\partial \gamma_{H12}^2} = \frac{-A_H\sqrt{R_H/2}}{8\sqrt{2}} \left(- \frac{468h^2\left(-\frac{\left(\frac{4\gamma_{H12}}{3\sqrt{4{\gamma_{H12}}^2+9}}+\frac{2}{3}\right)\left(3-(\varepsilon_{H1}+1)^2\right)}{2(\varepsilon_{H1}+1)^2} - \frac{4\gamma_{H12}}{3\sqrt{4{\gamma_{H12}}^2+9}} - \frac{2}{3} \right)^2}{25\left(h\left(\frac{3-(\varepsilon_{H1}+1)^2}{2(\varepsilon_{H1}+1)}-1\right)+h_1\right)^{\frac{23}{5}}} + \right.$$

$$\frac{117\left(h\left(\frac{\left(\frac{4\gamma_{H12}}{3\sqrt{4{\gamma_{H12}}^2+9}}+\frac{2}{3}\right)\left(3-(\varepsilon_{H1}+1)^2\right)}{2(\varepsilon_{H1}+1)^2} \right)\right)}{25\left(\frac{h\left(\frac{3-(\varepsilon_{H1}+1)^2}{2(\varepsilon_{H1}+1)}-1\right)+h\varepsilon_{H1}+2h}{2} - 2R_H\right)^{\frac{23}{5}}} -$$

$$\frac{26h\left(-\frac{\left(\frac{4}{3\sqrt{4\gamma_{H12}{}^{2}+9}}-\frac{16\gamma_{H12}{}^{2}}{3\left(4\gamma_{H12}{}^{2}+9\right)^{\frac{3}{2}}}\right)\left(3-\left(\varepsilon_{H1}+1\right)^{2}\right)}{2\left(\varepsilon_{H1}+1\right)^{2}}+\frac{\left(3-\left(\varepsilon_{H1}+1\right)^{2}\right)}{\left(\varepsilon_{H1}+1\right)^{3}}+\frac{\left(\frac{4\gamma_{H12}}{3\sqrt{4\gamma_{H12}{}^{2}+9}}+\frac{2}{3}\right)^{2}}{\left(\varepsilon_{H1}+1\right)}-\frac{4}{3\sqrt{4\gamma_{H12}{}^{2}+9}}+\frac{16\gamma_{H12}{}^{2}}{3\left(4\gamma_{H12}{}^{2}+9\right)^{\frac{3}{2}}}\right)}{5\left(h\left(\frac{3-\left(\varepsilon_{H1}+1\right)^{2}}{2\left(\varepsilon_{H1}+1\right)}-1\right)+h_1\right)^{\frac{18}{5}}}-$$

$$\frac{13\left(h\left(-\frac{\left(\frac{4}{3\sqrt{4\gamma_{H12}{}^{2}+9}}-\frac{16\gamma_{H12}{}^{2}}{3\left(4\gamma_{H12}{}^{2}+9\right)^{\frac{3}{2}}}\right)\left(3-\left(\varepsilon_{H1}+1\right)^{2}\right)}{2\left(\varepsilon_{H1}+1\right)^{2}}+\frac{\left(\frac{4\gamma_{H12}}{3\sqrt{4\gamma_{H12}{}^{2}+9}}+\frac{2}{3}\right)^{2}\left(3-\left(\varepsilon_{H1}+1\right)^{2}\right)}{\left(\varepsilon_{H1}+1\right)^{3}}+\frac{\left(\frac{4\gamma_{H12}}{3\sqrt{4\gamma_{H12}{}^{2}+9}}+\frac{2}{3}\right)^{2}}{\left(\varepsilon_{H1}+1\right)}\right)\right)}{5\left(\frac{h\left(\frac{3-\left(\varepsilon_{H1}+1\right)^{2}}{2\left(\varepsilon_{H1}+1\right)}-1\right)+h\varepsilon_{H1}+2h}{2}-2R_H\right)^{\frac{18}{5}}}-$$

$$\left.\frac{13h(4+4\sqrt{3})\left(\frac{4}{3\sqrt{4\gamma_{H12}{}^{2}+9}}-\frac{16\gamma_{H12}{}^{2}}{3\left(4\gamma_{H12}{}^{2}+9\right)^{\frac{3}{2}}}\right)}{5(h\varepsilon_{H1}+h_1)^{18/5}}+\frac{234h^{2}(4+4\sqrt{3})\left(\frac{4\gamma_{H12}}{3\sqrt{4\gamma_{H12}{}^{2}+9}}+\frac{2}{3}\right)^{2}}{25(h\varepsilon_{H1}+h_1)^{23/5}}\right) \tag{S14}$$

where $\gamma_{H12}=\frac{\varepsilon_{H1}-\frac{\frac{4R_H-2h+2h\varepsilon H_1+4R_H\varepsilon_{H1}+h\varepsilon_{H1}{}^{2}}{(2\varepsilon_{H1}+2)}}{h}-h_1}{2}$.