Supplementary material to

Effect of microchannel structure and fluid properties on non-inertial particle migration

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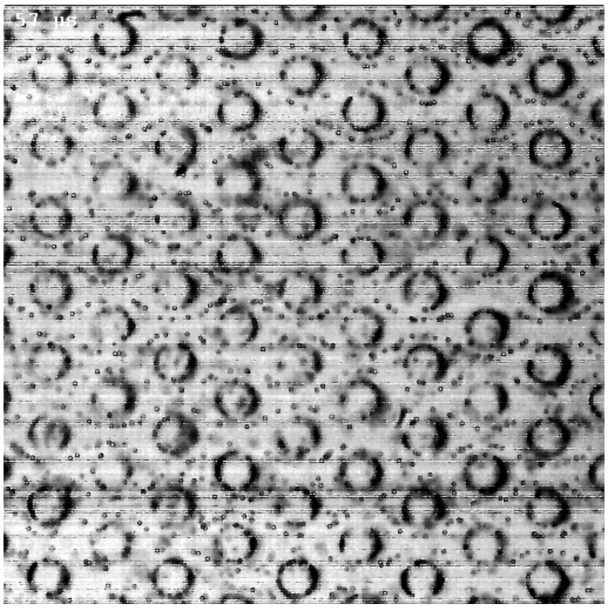
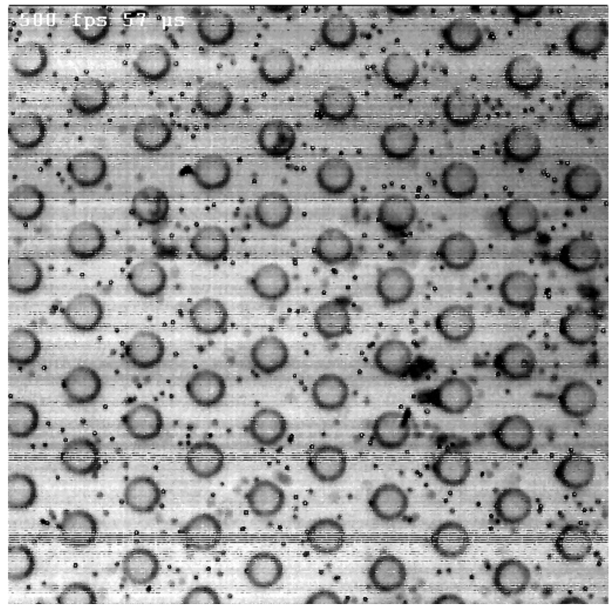
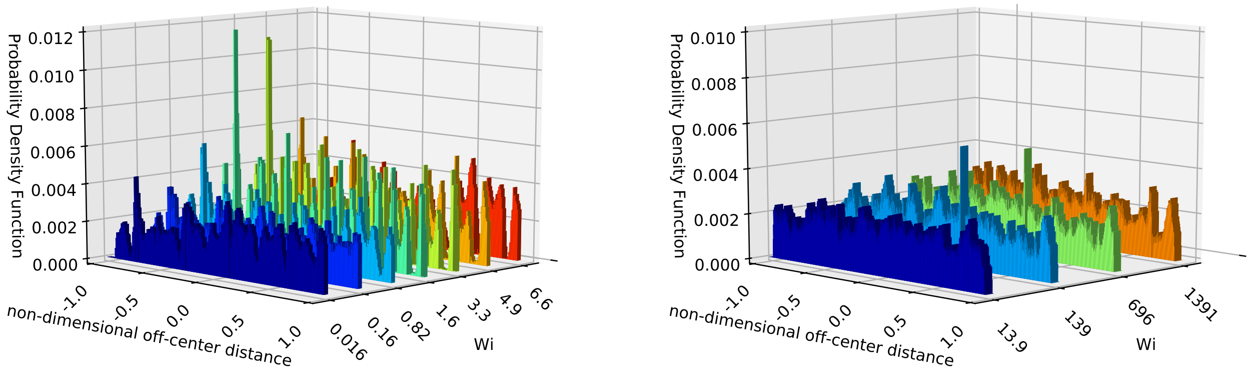


(a)



(b)

Figure 1: Instantaneous image of experiments in an orderly arranged pillared microchannel of porosity, $\epsilon = 0.76$ (a) Boger fluid (F2) at Wi = 3.3 (b) HPAM3630 (F4) at Wi = 1391.



(a) (b)

Figure 2: Experimental results for particle fraction in an orderly arranged pillared microchannel of porosity, $\epsilon = 0.76$ (a) Boger fluid (F2) (b) HPAM3630 (F4).

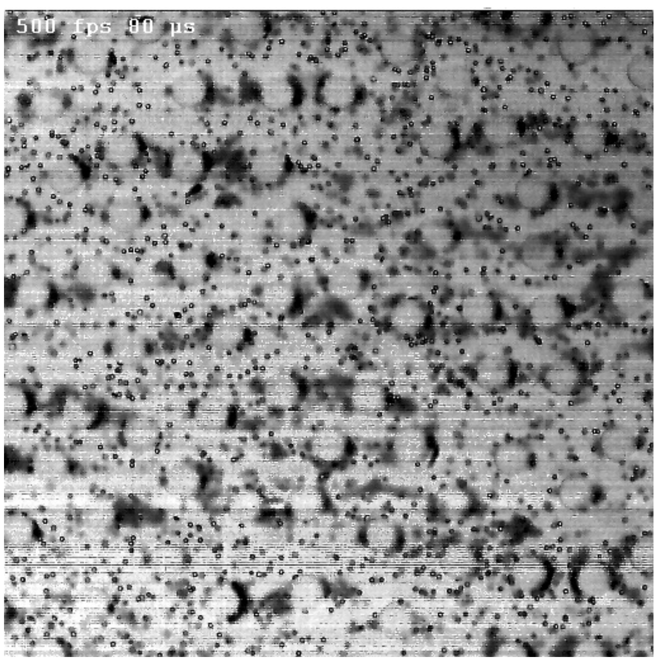


Figure 3: Instantaneous image of experiments in a randomly arranged pillared microchannel of porosity, $\epsilon = 0.81$ in Boger fluid (F2) at Wi $= 2.7$

Dimensionless number influencing particle migration

Using Oldroyd-B constitutive equation for evaluating first normal stress difference, N_1 ([1]):

$$N_1 \sim \eta \lambda \dot{\gamma}^2 \tag{1}$$

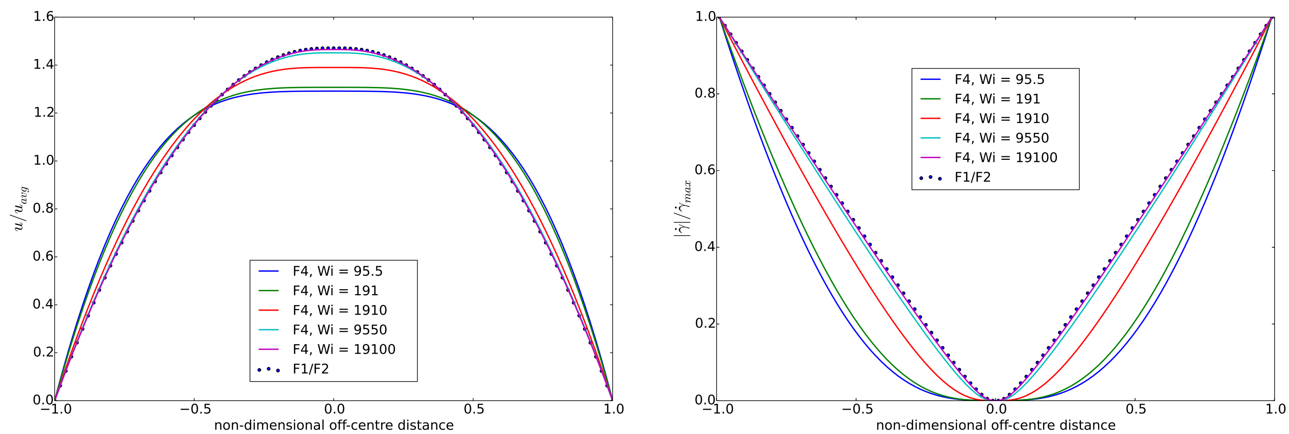


Figure 4: Distribution of (a) velocity and (b) shear rate along the width for Newtonian fluid(F1), Boger fluid(F2) and HPAM3630 (F4) at various Wi.

Lateral force on the particle can be estimated as:

$$F_y \sim \frac{\pi d_p^3}{6} \frac{\partial N_1}{\partial y} \tag{2}$$

Lateral velocity of a particle based on Stokes flow assumption:

$$u_y = \frac{F_y}{3\pi \eta d_p} \sim d_p^2 \lambda \dot{\gamma} \frac{\partial \dot{\gamma}}{\partial y} \tag{3}$$

Assuming parabolic velocity profile,

$$u_x = u\left(1 - \left(\frac{2y}{w}\right)^2\right) \tag{4}$$

$$\dot{\gamma} = \frac{\partial u_x}{\partial y} = \frac{4uy}{w^2} \tag{5}$$

$$\frac{\partial \dot{\gamma}}{dy} = \frac{4u}{w^2} \tag{6}$$

$$u_y \sim \frac{\lambda u^2 d_p^2 y}{w^4} \tag{7}$$

$$\frac{dy}{dx} = \frac{u_y}{u_x} \sim \frac{\lambda u d_p^2 y}{w^4 \left(1 - \left(\frac{2y}{w}\right)^2\right)} \tag{8}$$

Non-dimensionalizing the y distance with half-width ($w/2$) and x distance with the length of a channel (l): $\tilde{y} = 2y/w$, $\tilde{x} = x/l$

$$\frac{\tilde{y}}{\tilde{x}} \sim \frac{l}{w} \frac{\lambda u d_p^2 \tilde{y}}{w^3 \left(1 - \tilde{y}^2\right)} \tag{9}$$

Relevant dimensionless group for particle migration is:

$$\frac{\lambda u d_p^2 l}{w^4} = \frac{\lambda u}{w}\frac{l}{w}\left(\frac{d_p}{w}\right)^2 = Wi(l/w)\beta^2 \tag{10}$$

Similar dimensionless group is also proposed by [2, 3] and is valid in the limit of small Weissenberg numbers ($Wi \ll 1$).

References

[1] D. F. James, "Boger fluids," *Annual Review of Fluid Mechanics*, vol. 41, pp. 129–142, 2009.

[2] P. Brunn, "The motion of rigid particles in viscoelastic fluids," *Journal of Non-Newtonian Fluid Mechanics*, vol. 7, no. 4, pp. 271–288, 1980.

[3] G. Romeo, G. D'Avino, F. Greco, P. A. Netti, and P. L. Maffettone, "Viscoelastic flow-focusing in microchannels: Scaling properties of the particle radial distributions," *Lab on a Chip*, vol. 13, no. 14, pp. 2802–2807, 2013.