# Supplementary Materials

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## 1 Understanding cell migration with a Fokker-Planck equation

The distribution of cell orientation $\theta$ (Figure 9(a)-(c)) shows two peaks in the probability density function. This indicates that the cell prefers to move either way along the stretching direction, $\theta = \pm\pi/2$. We can therefore propose a Langevin equation for $\theta$:

$$\frac{d\theta(t)}{dt} = -N_{fiber}k_c \sin(2\theta(t) - 2\theta_0) + \eta(t) \tag{1}$$

where $\theta(t)$ is the orientation of the cell at time $t$, $\theta_0 = \pi/2$ which is the fiber alignment direction, $\eta$ a white noise, and $k_c$ is a coupling strength between the cell moving direction (orientation) and fiber alignment. The strength of the white noise is defined by $<\eta(t)\eta(t')>= 2B\delta(t-t')$. The corresponding Fokker-Planck equation is:

$$\frac{\partial\rho(\theta,t)}{\partial t} = \frac{\partial}{\partial\theta}\left[N_{fiber}k_c \sin(2\theta(t) - 2\theta_0)\rho(\theta,t)\right] \tag{2}$$

$$+B\frac{\partial^2\rho(\theta,t)}{\partial\theta^2} \tag{3}$$

The steady state solution is:

$$\rho(\theta) = \exp(N_{fiber}k'_c \cos(2\theta - 2\theta_0))/const \tag{4}$$

Here $k'_c = k_c/B$. We can try to fit the distribution of the cell orientation with Eq. (4) by tuning $k'$.

As we can see in Figure 9 a-c, Eq. (4) is consistent with the distribution obtained from simulations. We can further calculate the nematic order of the cell orientation $N_{cell} =< \cos(2\theta - 2\theta_0) >$:

$$N_{cell} = \int_{-\pi}^{\pi} \cos(2\theta - 2\theta_0)\rho(\theta)d\theta \tag{5}$$

$$= \int_{-\pi}^{\pi} \cos(2\theta)\exp(N_{fiber}k'_c \cos(2\theta))d\theta/C \tag{6}$$

where $C$ is a normalization constant.For small strain $\epsilon$, the fiber alignment is weak, hence:

$$C \approx \int_{-\pi}^{\pi} (1 + N_{fiber}k'_c \cos(2\theta))\, d\theta = 2\pi \tag{7}$$

Similarly,

$$N_{cell} \approx \int_{-\pi}^{\pi} \cos(2\theta)[1 + N_{fiber} k'_c \cos(2\theta)] d\theta \quad (8)$$

$$= \int_{-\pi}^{\pi} N_{fiber} k'_c \cos^2(2\theta)] d\theta = N_{fiber} k'_c \pi / const \quad (9)$$

$$= N_{fiber} k'_c / 2 \quad (10)$$

This shows that the initial slope of the curve $N_{cell}$ vs. $N_{fiber}$ should be $k'_c/2$. Indeed, the same slope obtained from simulation is quite close to the predicted $k'_c/2 = 5.5$ (Figure 9d).

As we have seen above, the Langevin equation with a first order coupling between cell orientation and fiber alignment can match the simulation results very well. Although the Langevin equation and Fokker-Planck equation completely leave out the mechanical details, they present a useful picture of the role of contact guidance in the cell migration. .

# 2 Addiitonal example of creating stiffness gradient by spatially varying $p$

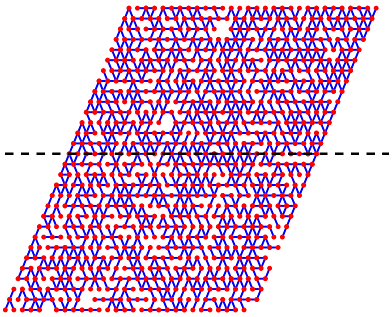

Figure 1: Fibrous network case. $p_{upper} = 0.60$ and $p_{lower} = 0.55$ The dashed line shows the location of the interface. In this network setting, it is very hard to distinguish the $p$ difference directly from the network's configuration, yet the cell is able to sense the small difference in the stiffness and move toward the stiffer direction. (Figure 7 (b) and (e) in the main text)

# 3 Parameter set

| parameter | meaning | value |
|---|---|---|
| $\lambda$ | contraction | 0.4 |
| $k_s$ | FA spring constant | 0.1 |
| $k_{-,b}$ | detachment rate on the back | 0.02 |
| $k_{-,f}$ | detachment rate on the front | 0.005 |
| $\alpha$ | off-rate parameter | 25 |
| $R$ | cell radius | $4l_0$(lattice spacing) |
| $k_+$ | on rate | 0.01 |
| $k$ | network bond spring constant | 1 |
| $\kappa$ | network bending stiffness | see the main text |
| $p$ | network bond occupation probability | see the main text |
| $k_m$ | maturation parameter | 1 |
| $f_{threshold}$ | maturation force threshold | 0.0005 |