**Supplementary Information** 

## Evaluation of energy loss at Sn anodes based on phase transition behaviors and formation of electrically resistive phases of the Na-Sn battery

Yong-Seok Choi<sup>1</sup>, Young-Woon Byeon<sup>1</sup>, Jae-Pyoung Ahn<sup>2</sup>, and Jae-Chul Lee<sup>1,\*</sup>

<sup>1</sup>Department of Materials Science and Engineering, Korea University, Seoul 136-701, South Korea

<sup>2</sup>Advanced Analysis Center, Korea Institute of Science and Technology, Seoul 136-791, South Korea

\*To whom correspond should be addressed, E-mail: jclee001@korea.ac.kr

Calculation of the voltage values via Gibbs free energy curves for Stage I – IV transitions.



**Figure S1.** Changes in the Gibbs free energy calculated for the intermediate reaction states occurring during (a) Stage I ( $^{\Delta G_{i,\eta}^{I}}$ ), (b) Stage II ( $^{\Delta G_{i,\eta}^{II}}$ ), (c) Stage III ( $^{\Delta G_{i,\eta}^{III}}$ ), and (d) Stage IV ( $^{\Delta G_{i,\eta}^{IV}}$ ) transitions under the external voltage  $\eta$ . In these graphs, two representative voltage value corresponding to discharge ( $\eta_{dis}$ ) and charge ( $\eta_{chg}$ ) are chosen. The greek alphabets indicated in the *x*-axis denote the intermediate states comprising metastable phases;  $\alpha = 0.3$ Na + Na<sub>0.2</sub>Sn,  $\beta = 0.17$ Na + Na<sub>0.33</sub>Sn,  $\gamma =$ Na<sub>0.75</sub>Sn + 0.25Na,  $\delta =$ Na<sub>2</sub>Sn + 0.25Na,  $\varepsilon =$  Na<sub>3</sub>Sn + 0.75Na.

By applying the same thermodynamic model as explained in Fig. 2 in the manuscript, the discharge ( $\eta_{dis}$ ) and charge voltages ( $\eta_{chg}$ ) corresponding to Stage I – IV can be determined. In this section, we evaluate the  $\eta_{dis}$  and  $\eta_{chg}$  values by calculating the changes in the Gibbs free energy in Stage I – IV during charging and discharging. For this purpose, we first evaluated the Gibbs free energy change ( $\Delta G_i^I$ ) associated with the *i*<sup>th</sup> intermediate reaction in Stage I– IV. This was achieved by add up the Gibbs free energies of all phases (Na, Sn, and Na<sub>x</sub>Sn phases) comprising the *i*<sup>th</sup> intermediate reaction state according to;

$$\Delta G_{i,\eta}^{\ I} = 0.5G_{Na} + G_{Sn,\ i=1.} \tag{S1.1}$$

$$\Delta G_i^I = (0.5 - x)G_{Na} + xG_{Na} S_n, i = 2, 3, 4.$$
(S1.2)

(Stage II transition)

(Stage I transition)

$$\Delta G_i^{II} = (1.0 - x)G_{Na} + G_{Na} x^{Sn}, i = 1, 2, 3.$$
(S2)

(Stage III transition)

$$\Delta G_{i}^{III} = (2.25 - x)G_{Na} + G_{Na}{}_{x}Sn, i = 1, 2, 3.$$
(S3)

(Stage IV transition)

$$\Delta G_{i}^{IV} = (3.75 - x)G_{Na} + G_{Na_{x}}S_{n}, i = 1, 2, 3.$$
(S4)

When an external voltage/potential ( $\eta$ ) is applied to the Na-Sn system, the Gibbs free energy of each intermediate reaction in Eqs. (S1-4) decreases linearly in proportion to the *x* value of Na in Na<sub>*x*</sub>Sn. Therefore, under the application of the external voltage  $\eta$ , the Gibbs free energy of each intermediate state in Stage I – IV transitions can be evaluated from

...

(Stage I transition)

$$\Delta G_{i,\eta}^{\ l} = 0.5G_{Na} + G_{Sn} - (0.5 - x)\eta, \quad i = 1.$$
(S5.1)

$$\Delta G_{i,\eta}^{\ I} = (0.5 - x)G_{Na} + xG_{Na}{}_{x}Sn - (0.5 - x)\eta, \quad i = 2, 3, 4.$$
 (S5.2)

$$\Delta G_i^{II} = (1.0 - x)G_{Na} + G_{Na_x Sn} - (1.0 - x)\eta, \quad i = 1, 2, 3.$$
(S6)

(Stage II transition)

$$\Delta G_{i}^{III} = (2.25 - x)G_{Na} + G_{Na_x}S_n - (2.25 - x)\eta, \quad i = 1, 2, 3.$$
 (S7)

(Stage III transition)

(Stage IV transition)

$$\Delta G_{i}^{IV} = (3.75 - x)G_{Na} + G_{Na_{x}Sn} - (3.75 - x)\eta, \quad i = 1, 2, 3.$$
 (S8)

As shown in Eqs (S5–8), the external voltage ( $\eta$ ) changes the Gibbs free energies of the *i*<sup>th</sup> intermediate reaction states and thus, alters to overall shape of the Gibbs free energy curve during charging and discharging. According to the definitions made in the manuscript, the  $\eta_{dis}$  value can be determined as the highest voltage that makes the all intermediate states for discharge process to be the free-energy downhill, while the  $\eta_{chg}$  value as the lowest voltage that causes the all intermediate states for charge process to be the free-energy downhill. By substituting the various  $\eta$  values, we obtained the  $\eta_{dis}$  and  $\eta_{chg}$  values for Stage I – IV transitions that satisfied the above-mentioned criteria (Fig. S1). Therefore, the  $\eta_{dis}$  and  $\eta_{chg}$  corresponding to Stage I – IV transitions are determined to be  $\eta_{dis} = 0.43$  (Stage I), 0.31 (Stage II), 0.14 (Stage III), 0.11 V (Stage IV) and  $\eta_{chg} = 0.58$  (Stage I), 0.51 (Stage II), 0.42 (Stage III), 0.13 V (Stage IV), which is also summarized in Table 2 in the manuscript.