

Electronic supplementary information

Formation of breathing pyrochlore lattice: structural, thermodynamic and crystal chemical aspects

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Let us consider the theoretical derivation of the $F\bar{4}3m$ ordered pyrochlore structure from the disordered $Fd\bar{3}m$ pyrochlore structure $A^{16d}_2B^{16c}_2X^{48f}_6Y^{8b}$. Choose the origin at Wyckoff position $8a$ of $Fd\bar{3}m$ space group. Then the atomic coordinates of the pyrochlore primitive cell (it contain two formula units) in the structures of the disordered and ordered phases are presented in Tables S1-S4.

Table S1. Atom A coordinates at position $16d$ of the pyrochlore structure

Atom number	1	2	3	4
Coordinates of A atoms in the $Fd\bar{3}m$ structure of pyrochlore	$5/8,$ $5/8,$ $5/8$	$5/8,$ $3/8,$ $3/8$	$3/8,$ $5/8,$ $3/8$	$3/8,$ $3/8,$ $5/8$
Vector basis functions (displacements of atoms A)	111	1-1-1	-11-1	-1-11
Coordinates of A atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $Fd\bar{3}m$ of pyrochlore)	$5/8+\delta_A,$ $5/8+\delta_A,$ $5/8+\delta_A$	$5/8+\delta_A,$ $3/8-\delta_A,$ $3/8-\delta_A$	$3/8-\delta_A,$ $5/8+\delta_A,$ $3/8-\delta_A$	$3/8-\delta_A,$ $3/8-\delta_A,$ $5/8+\delta_A$
Coordinates of A atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $F\bar{4}3m$ pyrochlore)	$x,$ $x,$ x	$x,$ $-x,$ $-x$	$-x,$ $x,$ $-x$	$-x,$ $-x,$ x

Atoms A occupy Wyckoff positions $16e$ (x, x, x), where $x = 5/8+\delta_A=0.625+\delta_A$ in the structure of ordered $F\bar{4}3m$ pyrochlore

Table S2. Atom B coordinates at position 16c of the pyrochlore structure

Atom number	1	2	3	4
Coordinates of B atoms in the $Fd\bar{3}m$ structure of pyrochlore	1/8, 1/8, 1/8	1/8, 3/8, 3/8	3/8, 1/8, 3/8	3/8, 3/8, 1/8
Vector basis functions (displacements of atoms B)	111	1-1-1	-11-1	-1-11
Coordinates of B atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $Fd\bar{3}m$ of pyrochlore)	$1/8+\delta_B$, $1/8+\delta_B$, $1/8+\delta_B$	$1/8+\delta_B$, $3/8-\delta_B$, $3/8-\delta_B$	$3/8-\delta_B$, $1/8+\delta_B$, $3/8-\delta_B$	$3/8-\delta_B$, $3/8-\delta_B$, $1/8+\delta_B$
Coordinates of B atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $F\bar{4}3m$ pyrochlore)	x , x , x	x , $-x$, $-x$	$-x$, x , $-x$	$-x$, $-x$, x

Atoms B occupy Wyckoff positions 16c (x, x, x), where $x = 1/8 + \delta_B = 0.125 + \delta_B$ in the structure of ordered $F\bar{4}3m$ pyrochlore

Table S3. Atom X coordinates at position 48*f* of the pyrochlore structure

Atom number	1	2	3	4	5	6	7	8	9	10	11	12
Coordinates of X atoms in the $Fd\bar{3}m$ structure of pyrochlore	x, 1/2, 1/2	1/2, x, 1/2	1/2, 1/2, x	1/4 +x, 1/4, 1/4	1/4, 1/4 +x, 1/4	1/4, 1/4, 1/4 +x	1-x, 1/2, 1/2	1/2, 1-x, 1/2	1/2, 1/2, 1-x	1/4-x, 1/4, 1/4	1/4, 1/4-x, 1/4	1/4, 1/4, 1/4-x
Scalar basic functions	1	1	1	-1	-1	-1	1	1	1	-1	-1	-1
Vector basis functions (displacements of atoms X)	100	010	001	-100	0-10	00-1	-100	0-10	00-1	100	010	001
Coordinates of X atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $Fd\bar{3}m$ of pyrochlore)	$x+\delta_X$, 1/2, 1/2	1/2, $x+\delta_X$, 1/2	1/2, 1/2, $x+\delta_X$	1/4 +x- δ_X , 1/4, 1/4	1/4, 1/4 +x- δ_X , 1/4	1/4, 1/4, 1/4 +x- δ_X	1-x- δ_X , 1/2, 1/2	1/2, 1-x- δ_X , 1/2	1/2, 1/2, 1-x- δ_X	1/4- $x+\delta_X$, 1/4, 1/4	1/4, 1/4- $x+\delta_X$, 1/4	1/4, 1/4, 1/4- $x+\delta_X$
Coordinates of X atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $F\bar{4}3m$ pyrochlore)	x_1 0, 0	0, x_1 0	0, 0, x_1	x_2 1/4, 1/4	1/4, x_2 , 1/4	1/4, 1/4, x_2	- x_1 0, 0	0, - x_1 0	0, 0, - x_1	x_2 1/4, 1/4	1/4, x_2 , 1/4	1/4, 1/4, x_2
Wyckoff position	24 <i>f</i>	24 <i>f</i>	24 <i>f</i>	24 <i>g</i>	24 <i>g</i>	24 <i>g</i>	24 <i>f</i>	24 <i>f</i>	24 <i>f</i>	24 <i>g</i>	24 <i>g</i>	24 <i>g</i>

Atoms X occupy Wyckoff positions 24*f* ($x_1=x+\delta_X+1/2$) and 24*g* ($x_2=1/4+x-\delta_X$) where x - free parameter of Wyckoff position 48*f* in the initial $Fd\bar{3}m$ -pyrochlore, δ_X -magnitude of displacement

Table S4. Atom Y coordinates at position $8b$ of the pyrochlore structure

Atom number	1	2
Coordinates of Y atoms in the $Fd\bar{3}m$ structure of pyrochlore	1/2, 1/2, 1/2	3/4, 3/4, 3/4
Scalar basic functions	1	-1
Coordinates of Y atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $Fd\bar{3}m$ of pyrochlore)	1/2, 1/2, 1/2	3/4, 3/4, 3/4
Coordinates of Y atoms in the structure of $F\bar{4}3m$ pyrochlore (in the basis of the structure $F\bar{4}3m$ pyrochlore)	1/2, 1/2, 1/2	3/4, 3/4, 3/4
Wyckoff position	$4b$	$4d$

Table S5. The role of order parameters in the $Fd\bar{3}m \rightarrow F\bar{4}3m$ phase transitions in $Ce_2Zr_2O_{7+\sigma}$ ¹ and NH_4NbWO_6 .^{2,3} The amplitudes normalized with respect to the primitive unit cell of the parent phase.

Irrep	Atom Wyckoff position	Amplitudes of distortions (Å)	
		$Ce_2Zr_2O_{7+\sigma}$	NH_4NbWO_6
$\mathbf{k}_{11}\tau_4(\Gamma_1^+)$	48f	0.45373	0.26247
	All	0.45373	0.26247
$\mathbf{k}_{11}\tau_4(\Gamma_2^-)$	16d	0.12223	-*
	16c	0.14445	0.17222
	48f	0.46114	0.26966
	All	0.49845	0.31996
Total	All	0.67404	0.41384*

*- displacement of ion NH_4^+ was not taken into account

References

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