## **Supplemental Material**



**Fig. S1**. DSC thermogram of aqueous glycerol solution with a mass fraction of water of  $X_w = 0.4$ . The cooling rate and heating rate are 3.0 and 0.7 K/min., respectively.  $T_g$ is defined as the onset temperature of glass-liquid transition, and  $T_c$  that of cold-crystallization of water.

**Table S1**. Ratio of the cold-crystallization temperature,  $T_c$ , to the glass transition temperature,  $T_g$ , for some metallic glasses, polymers, and aqueous solutions of electrolytes and small organic molecules.

	$T_g$	T <sub>c</sub>	$T_{\rm c}/T_g$
Glycerol·5H <sub>2</sub> O <sup>1</sup>	155.68	185.63	1.19
Glycerol·4H <sub>2</sub> O <sup>1</sup>	156.92	179.36	1.14
$DMSO \cdot 5H_2O^1$	139.88	189.22	1.35
$EG \cdot 4H_2O^1$	141.54	169.3	1.19
Butanetriol $\cdot 5.3 H_2 O^1$	160.57	194.98	1.21
$PEG300{\cdot}14H_2O^1$	171.32	210.94	1.23
$Ca(NO_3)_2 \cdot 10H_2O^1$	172.4	206.21	1.19
$CrCl_3 \cdot 22H_2O^1$	150.66	206.26	1.36
$CrCl_3 \cdot 24H_2O^1$	149.07	174.23	1.16
$CrCl_3 \cdot 26H_2O^1$	149.53	168.08	1.12
$CaCl_2 \cdot 13H_2O^1$	154.44	204.92	1.32
$CaCl_2 \cdot 14H_2O^1$	152.83	202.95	1.32
$MnCl_2{\cdot}13H_2O^1$	154.01	203.49	1.32
$MnCl_2{\cdot}15H_2O^1$	150.13	190.81	1.27
$ZnCl_2 \cdot 12H_2O^{[1]}$	153.99	193.57	1.25
$MgCl_2{\cdot}18H_2O^1$	151.98	194.5	1.27
$FeCl_3 \cdot 22H_2O^1$	154.68	192.97	1.24
Sucrose.0.19H <sub>2</sub> O <sup>2</sup>	333.15	398.15	1.19
Sucrose.0.28H <sub>2</sub> O <sup>2</sup>	331.15	388.15	1.17
Sucrose.0.38H <sub>2</sub> O <sup>2</sup>	323.15	373.15	1.15
Sucrose. $0.61H_2O^2$	305.15	365.15	1.19
Sucrose <sup>2</sup>	347.15	403.15	1.16
Lactose <sup>2</sup>	381.15	458.15	1.20
Poly(L-lactic acid) <sup>3</sup>	328.35	377.35	1.14
$PEEK^4$	419.0	446	1.06

## Table SI. Continued.

	Tg	Тс	Tc/Tg
Isotactic polypropylene <sup>5</sup>	278.15	298.15	1.07
$Pd_{40}Ni_{10}Cu_{30}P_{20}{}^{6}$	586	671	1.14
$La_{68}Ga_{10}Cu_{22}^{7}$	375	424	1.13
$La_{68}Ga_{12}Cu_{20}^{7}$	380	432	1.14
$La_{68}Ga_{14}Cu_{18}^{7}$	391	435	1.11
$La_{70}Ga_{10}Cu_{20}{}^7$	379	409	1.08
$La_{70}Ga_{12}Cu_{18}^{7}$	382	438	1.15
$La_{70}Ga_{14}Cu_{16}^{7}$	394	433	1.10
$La_{70}Ga_{16}Cu_{14}^{7}$	401	452	1.13
$Y_{50}Ni_{20}Al_{30}{}^8$	655	701	1.07
Y <sub>55</sub> Ni <sub>20</sub> Al <sub>25</sub> <sup>8</sup>	639	669	1.05
$Y_{60}Ni_{20}Al_{20}{}^8$	588	634	1.08
$La_{60}Ni_{20}Al_{20}^{8}$	446	495	1.11
$Y_{60}Ni_{15}Al_{25}{}^8$	588	664	1.13
Y <sub>60</sub> Co <sub>15</sub> Al <sub>25</sub> <sup>8</sup>	648	678	1.05
$Y_{60}Fe_{15}Al_{25}{}^8$	667	697	1.04
La <sub>60</sub> Ni <sub>15</sub> Al <sub>25</sub> <sup>9</sup>	455	507	1.11
La59Ni15Al25Cu19	454	508	1.12
La <sub>58</sub> Ni <sub>15</sub> Al <sub>25</sub> Cu <sub>2</sub> <sup>9</sup>	454	509	1.12
La55Ni15Al25Cu59	457	538	1.18
La52Ni15Al25Cu89	459	535	1.17
La <sub>50</sub> Ni <sub>15</sub> Al <sub>25</sub> Cu <sub>10</sub> <sup>9</sup>	464	537	1.16
La <sub>60</sub> Ni <sub>14</sub> Al <sub>25</sub> Cu <sub>1</sub> <sup>9</sup>	454	510	1.12
La <sub>60</sub> Ni <sub>13</sub> Al <sub>25</sub> Cu <sub>2</sub> <sup>9</sup>	449	499	1.11
La <sub>60</sub> Ni <sub>10</sub> Al <sub>25</sub> Cu <sub>5</sub> <sup>9</sup>	442	493	1.12
La <sub>60</sub> Ni <sub>15</sub> Al <sub>24</sub> Cu <sub>1</sub> <sup>9</sup>	449	519	1.16
La <sub>60</sub> Ni <sub>15</sub> Al <sub>23</sub> Cu <sub>2</sub> <sup>9</sup>	443	509	1.15

## Table SI. Continued.

	Tg	Tc	Tc/Tg
La <sub>60</sub> Ni <sub>15</sub> Al <sub>20</sub> Cu <sub>5</sub> <sup>9</sup>	426	514	1.21
La <sub>60</sub> Ni <sub>15</sub> Al <sub>17</sub> Cu <sub>8</sub> <sup>9</sup>	416	485	1.17
La <sub>60</sub> Ni <sub>15</sub> Al <sub>15</sub> Cu <sub>10</sub> <sup>9</sup>	412	464	1.13
La <sub>60</sub> Ni <sub>15</sub> Al <sub>10</sub> Cu <sub>15</sub> <sup>9</sup>	401	434	1.08
$Tm_{39}Ce_{16}Co_{20}Al_{25}{}^{10}$	608	663	1.09
$Tm_{39}Pr_{16}Co_{20}Al_{25}{}^{10}$	611	666	1.09
$Tm_{39}Nd_{16}Co_{20}Al_{25}{}^{10}$	621	701	1.13
$Tm_{39}Sm_{16}Co_{20}Al_{25}{}^{10}$	631	693	1.10
$Tm_{39}Gd_{16}Co_{20}Al_{25}{}^{10}$	646	717	1.11
$Tm_{39}Tb_{16}Co_{20}Al_{25}{}^{10}$	650	711	1.09
$Tm_{39}Dy_{16}Co_{20}Al_{25}{}^{10}$	657	701	1.07
$Tm_{39}Ho_{16}Co_{20}Al_{25}{}^{[10]}$	666	719	1.08
$Tm_{39}Er_{16}Co_{20}Al_{25}{}^{10}$	668	723	1.08
$Gd_{53}Al_{24}Co_{20}Zr_{3}^{10}$	599	653	1.09
$Gd_{51}Al_{24}Co_{20}Zr_{4}N{b_{1}}^{10}$	598	653	1.09
$Dy_{36}Ho_{2}0Al_{24}Co_{20}{}^{10}$	633	687	1.09
$Ho_{36}Dy_{20}Al_{24}Co_{20}{}^{10}$	553	629	1.14
$Er_{50}Al_{24}Co_{20}Y_{6}^{10}$	651	702	1.08
$Pd_{40}Ni_{22.5}Fe_{17.5}P_{20}{}^{10}$	600	666	1.11
$Sr_{60}Mg_{18}Zn_{22}^{11}$	331	374	1.13
$Sr_{60}Li_{11}Mg_9Zn_{20}^{-11}$	299	323	1.08
$Sr_{60}Li_5Mg_{15}Zn_{20}{}^{11}$	315	336	1.07
$Sr_{60}Mg_{20}Zn_{15}Cu_{5}{}^{11}$	335	374	1.12
$Sr_{40}Yb_{20}Mg_{20}Zn_{15}Cu_{5}{}^{11}$	336	378	1.13
$Sr_{20}Yb_{20}Ca_{20}Mg_{20}Zn_{20}{}^{11}$	353	389	1.10
$Sr_{20}Yb_{20}Ca_{20}Mg_{20}Zn_{10}C{u_{10}}^{11}$	351	391	1.11
$Ca_{60}Mg_{25}Cu_{15}{}^{12}$	390	416	1.07

	Tg	Тс	Tc/Tg
$Ca_{55}Mg_{25}Cu_{20}^{12}$	398	428	1.08
$Ca_{50}Mg_{25}Cu_{25}^{12}$	400	439	1.10
$Ca_{45}Mg_{25}Cu_{30}{}^{12}$	400	438	1.10
$Ca_{40}Mg_{25}Cu_{35}^{12}$	399	436	1.09
$Ca_{35}Mg_{25}Cu_{40}{}^{12}$	386	429	1.11
$Ca_{30}Mg_{25}Cu_{45}{}^{12}$	387	430	1.11
$Ta_{42}Ni_{42}Co_{16}^{13}$	998	1038	1.04
$Ta_{42}Ni_{40}C{o_{18}}^{13}$	993	1032	1.03
$Ta_{42}Ni_{38}Co_{20}{}^{13}$	1003	1038	1.03
$Ta_{42}Ni_{36}Co_{22}{}^{13}$	1005	1035	1.03
$Ta_{46}Ni_{36}Co_{18}{}^{13}$	991	1027	1.04
$Ta_{50}Ni_{32}Co_{18}{}^{13}$	980	1015	1.04
$Yb_{70}Zn_{20}Mg_{10}{}^{14}$	347	386	1.11
$Yb_{62.5}Zn_{20}Mg_{17.5}{}^{14}$	367	398	1.08
$Yb_{64}Zn_{20}Mg_{15}Cu_{1}{}^{14}$	357	402	1.13
$Yb_{65}Zn_{20}Mg_{10}C{u_5}^{14}$	384	419	1.09
$Yb_{62.5}Zn_{15}Mg_{17.5}Cu_5^{-14}$	381	401	1.05

## Table SI. Continued.

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**Fig. S2.** Frequency dependence of  $\varepsilon'$  (square),  $\varepsilon''$  (circle) and the derivative quantity  $\varepsilon_{der}^{"} = -\pi/2 \cdot d\varepsilon'(\omega)/d\ln(\omega)$  (diamond) for pure glycerol at (a) 243 K and (b) 293 K, respectively. Different from previous report, the low-frequency relaxation process in  $\varepsilon_{der}^{"}$  around the frequency at which  $\varepsilon' = \varepsilon''$  is not observable.



Fig. S3. Frequency dependence of  $\varepsilon'$  (square),  $\varepsilon''$  (circle) and the derivative quantity  $\varepsilon_{der}^{"} = -\pi/2 \cdot d\varepsilon'(\omega)/d\ln(\omega)$  (diamond) for aqueous glycerol solution with  $X_w = 0.4$  measured during heating the sample from 167 K up to room temperature at a rate of 0.7 K/min. Sample was previously cooled from room temperature down to 167 K at a rate of ~10 K/min. For reference, the peak frequency of  $\varepsilon_{der}^{"}$  for pure glycerol at the corresponding temperature is indicated by a vertical dashed line. It can be concluded that dehydrated glycerol also appears during cold-crystallization finishes at about 208.6 K (f). Peak II in the derivative quantity  $\varepsilon_{der}^{"}$  marked by arrow may come from dc conductivity, gas and/or impurities in solution.



Fig. S4. Frequency dependence of  $\varepsilon'(\text{square})$ ,  $\varepsilon''(\text{circle})$  and the derivative quantity  $\varepsilon_{der}^{"} = -\pi/2 \cdot d\varepsilon'(\omega)/d\ln(\omega)$  (diamond) for aqueous glycerol solution with  $X_w = 0.4$  measured during heating the sample from 183 K up to room temperature. Sample was previously cooled from room temperature down to 183 K. For reference, the peak frequency of  $\varepsilon_{der}^{"}$  for pure glycerol at the corresponding temperature is indicated by a vertical dashed line. Peak I of  $\varepsilon_{der}^{"}$  becomes observable when dehydrated glycerol emerges in the process.



**Fig. S5**. Frequency dependence of  $\varepsilon'(\text{square})$ ,  $\varepsilon''(\text{circle})$  and the derivative quantity  $\varepsilon_{der}^{"} = -\pi/2 \cdot d\varepsilon'(\omega)/d\ln(\omega)$  (diamond) for aqueous glycerol solution with  $X_w = 0.4$  measured during heating the sample from **188 K** up to room temperature. Sample was previously cooled from room temperature down to **188 K**. For reference, the peak frequency of  $\varepsilon_{der}^{"}$  for pure glycerol at the corresponding temperature is indicated by a vertical dashed line. In this case the derivative quantity  $\varepsilon_{der}^{"}$  manifests a peak (arrowed), but it keeps shifting with varied temperature. This low-frequency relaxation process does not come from pure glycerol, but from the dc conductivity instead.



**Fig. S6**. Inversed temperature dependence of (**a**) relaxation time  $\tau$  and (**b**) dielectric strength  $\Delta \varepsilon$  for  $\alpha$ -,  $\beta$ -relaxations (circle), and for those of ice (hexagon), freeze-concentrated solution (FCS, diamond), retrieved solution (Sol., triangle), and a low-frequency relaxation attributable to pure glycerol (Gly., pentagon) for the aqueous glycerol solution with  $X_w = 0.4$ , measured during heating from  $T_a = 163$  K up to room temperature. Two annealing times, 3 min. (solid symbols) and 300 min. (open symbols), at 163 K were chosen to reveal the effect of holding time on cold-crystallization.



**Fig. S7.** Cold-crystallization of glycerol solution with  $X_w = 0.44$ . Frequency dependence of relaxation time  $\tau$  (A) and dielectric strength  $\Delta \varepsilon$  (B) for  $\alpha$ -,  $\beta$ -relaxations (circles), and for those of ice (hexagons), freeze-concentrated solution (FCS, diamonds), and retrieved solution (Sol., triangle) measured during heating process from 125 K up to room temperature. The sample was precooled from room temperature to 125 K at ~10K/min. For this aqueous solution,  $T_g = 155$  K, and  $T^* = 183.5$  K.

It is not easy to directly determine the amount of ice at temperatures lower than  $T^*$ , because of the closeness of  $\tau_{ice}$  to  $\tau_{\alpha}$  and even  $\tau_{\beta}$  for intermediate-concentrated glycerol solutions. Fortunately, the relaxation time of freeze-concentrated phase,  $\tau_{con.}$ , is obviously longer than  $\tau_{\alpha}$  in the whole measured temperature range. The corresponding  $\Delta \varepsilon_{con.}$  can help us to estimate  $\Delta \varepsilon_{ice}$  at temperatures lower than  $T^*$  based on the ratio of  $\Delta \varepsilon_{ice}$  to  $\Delta \varepsilon_{con.}$  shown at temperatures above  $T^*$ . The fraction of ice can then be calculated by normalizing  $\Delta \varepsilon_{ice}$  to its maximum value obtained at about 190 K. About 2 wt. % of the total cold-crystallized ice precipitated at 174 K,

which keeps almost unchanged initially but increases sharply just when increasing temperature above  $T^*$ . Within the next 5 K (heating rate: 0.7 K/min), non-isothermal cold-crystallization of water completes quickly.