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The Nanoscopic Principles of Capacitive Ion Sensing Interfaces

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SI. 1. The meaning of electrochemical capacitance within Density Functional Theory

Density Functional Theory (DFT) is a quantum mechanical method that permits useful perceptions of chemical processes.[1, 2] In short, provided that a constant external potential is applied to a chemical system, DFT postulates that the total energy of the ground state of a chemical system is defined as a functional, $E[\rho]$, of the electron density, ρ. Note that a functional $F[]$ is a mathematical operator that runs over a function $f(a)$, such as $F[f(a)] = b$, where b is provided for a value of a in the function $f(a)$. Note also that because the number of electronic particles, N, in a chemical system is obtained if the electron density, $\rho(\vec{r})$, is known, thus in the DFT nomenclature, a chemical system is referred as an N-electron system which is a function of $\vec{r}$ such as $N = \int_{\mho} \rho(\vec{r}) d\vec{r}$, where $\mho$ in the integrand represents the spatial region where the N-electron system is stated. Therefore, if the electrons are considered non-interacting particles then the associated Hamiltonian is[1, 2]

$$E[\rho] = T_0[\rho] + \int_{\mho} \rho(\vec{r}) v(\vec{r}) d\vec{r} + \frac{1}{2} \iint_{\mho \mathrm{x} \mho} \frac{e^2}{4\pi\varepsilon} \frac{\rho(\vec{r})\rho(\vec{r}')}{|\vec{r}-\vec{r}'|} d\vec{r} d\vec{r}' + E_{xc}[\rho] + E_{nn}, \qquad \text{(SI. 1)}$$

where the dependency of $\rho(\vec{r})$ to the spatial coordinates is omitted (in some parts) for simplicity. $\varepsilon = \varepsilon_r \, \varepsilon_0$, where ε_0 is the vacuum permittivity constant and ε_r is the dielectric constant. T_0 is the kinetic energy and the second and third terms are the interaction energy of the electrons with an external potential $[v(\vec{r})]$ and the electron-electron Coulomb energy, respectively. E_{xc} is the exchange-correlation energy and E_{nn} is the inter-nuclear Coulomb interaction energy. In summary, because the energy is associated with the electron density, DFT provides an adequate description of a chemical system and additionally allows us to define associated changes in the system's capacitance (as will be shown below).

Suppose that we want to calculate the capacitance of a mesoscopic system that gets charged due to the insertion or removal of charge, that is, the total number of electrons in the system changes from N to N'. For instance, the capacitance of a mesoscopic chemical system can be calculated considering the constraint of a homogeneous potential variation in the entire volume of the system when the system is charged from N to $N' = N + \Delta N$ particles. Defining $\rho'(\vec{r})$, in Eqn. (SI. 1), as the electron density of the charged system, the equation for the effective Kohn-Sham potential is written as[3]

$$V'_{ef} = \frac{e^2}{4\pi\varepsilon} \sum_a \frac{Z_a}{|\vec{r} - \vec{R}_a|} + \int_\Omega \frac{e^2 \rho'(\vec{r}')}{4\pi\varepsilon |\vec{r} - \vec{r}'|} d\vec{r}' + \frac{\delta E_{xc}[\rho']}{\delta \rho'}, \qquad \text{(SI. 2)}$$

where $\vec{R}_a$ is the coordinate of the nuclei of the chemical system and Z_a is the atomic number associated with $\vec{R}_a$. Eqn. (SI. 2) allows us to directly associate energy changes with the capacitance of the system. Based on the presumptions that the potential varies homogeneously in the volume, then differences between effective potentials of the charged and the uncharged state densities can be written as[3]

$$V'_{ef} - V_{ef} = \frac{e^2}{4\pi\varepsilon_r\varepsilon_0} \left\{ \int_\Omega \frac{\rho'(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' - \int_\Omega \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' \right\} + \left\{ \frac{\delta E_{xc}[\rho']}{\delta \rho'} - \frac{\delta E_{xc}[\rho]}{\delta \rho} \right\}, \qquad \text{(SI. 3)}$$

and the difference is another constant that determines the energy of the system at a given state due to the changes from N to N'. The electrostatic potential of a given charge distribution $\rho(\vec{r})$ can be defined in classical mechanics (see any textbook on electromagnetism) as $V = \frac{1}{4\pi\varepsilon_r\varepsilon_0} \int_\Omega \frac{e\Delta\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' = q/C$, where q is the charge state whose changes are $q = (N' - N)e$ and C is the capacitance of the system. By analogy, we can observe that the term in Eqn. (SI. 3) can be rewritten as a function of the variation of the electron density as[3]

$$\frac{e^2}{4\pi\varepsilon_r\varepsilon_0} \int_\Omega \left\{ \frac{\rho'(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' - \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} d\vec{r}' \right\} = \frac{(N' - N)e^2}{C}, \qquad \text{(SI. 4)}$$

in which C is interpreted as the "classic" (or the geometric) capacitance and $q = (N' - N)e = \Delta N e$ correspond to a variation in the electron density such as $\mathcal{N} = \int_\mho \Delta\rho(\vec{r}) d\vec{r}$. This is the charge or electron density state varied during the application of an electrical external potential that can induce charge displacement in, for instance, a conductor where electrons are considered to behave as a "gas". This interpretation of C thus far ignores effects related to the occupation of the electronic structure (bands, HOMO, LUMO or accessible orbitals), where the contributions are significant for mesoscopic systems.[3] In additional to a "simple" electrostatic capacitance, then, we have a detectable additional term here that relates directly to the electron density within the probed mesoscopic element.[3] A more complete description of the system electrochemical capacitance is provided in Eqn. (SI. 5) below.

Eqn. (SI. 5) is resolved by assuming that the variations in the electron density are gradual during charging (charge perturbation is reasonably small compared to the total electron density) and summarizes the associated variations of the energy by[3]

$$\Delta E = E[N'] - E[N] = \sum_{N+1}^{N'} \varepsilon_i + \frac{(N'-N)e^2}{2C} = (N'-N)\varepsilon_i + \frac{(N'-N)e^2}{2C}, \qquad \text{(SI. 5)}$$

where ε_i is the Kohn-Sham energy of the ith occupied state during charging of the system involving insertion of electron particles. Thus, it can be noted that $\sum_{N+1}^{N'} \varepsilon_i$ accounts for capacitive contributions coming from electronic states of the mesoscopic element (which are states cited in the mesoscopic volume, $\mho$) and that the second term accounts for the “classical” capacitive contribution. In terms of associated circuit elements, in the absence of Faradaic activity, the two terms of Eqn. (SI. 5) represent a classical electrostatic non-specific ionic capacitance C_{e_i} in series with a specific quantum (or chemical) capacitive contribution C_{q_i} that, in turn, largely accounts for the contribution of any change in the electronic structure. These two series non-Faradaic capacitive elements make up the electrochemical ionic capacitance C_{μ_i} such that $1/C_{\mu_i} = 1/C_{e_i} + 1/C_{q_i}$. This capacitance arises exclusively from the specific binding of the target analytes (i.e. ReO_4^-, SCN^- or I^-) to the MREI site. Together with the “baseline” capacitance C_t, C_{μ_i} then makes up the overall electrochemical capacitance $C_{\bar{\mu}}$ such that $C_{\bar{\mu}} = C_t + C_{\mu_i}$ (see Figure SI. 1). It is important to note that the C_{q_i} quantum term of C_{μ_i} is not to be confused with a quantum capacitance C_q that can be observed in mesoscopic interfaces in which Faradaic charging events are present (such as in redox-active SAMs in which we observe a redox capacitance). Furthermore, it should be noted that the C_{μ_i} response/element is an average of all MREI sites (as parallel $R_t C_{\mu_i}$ branches), whereby each MREI site can be thought of as a separate RC element.

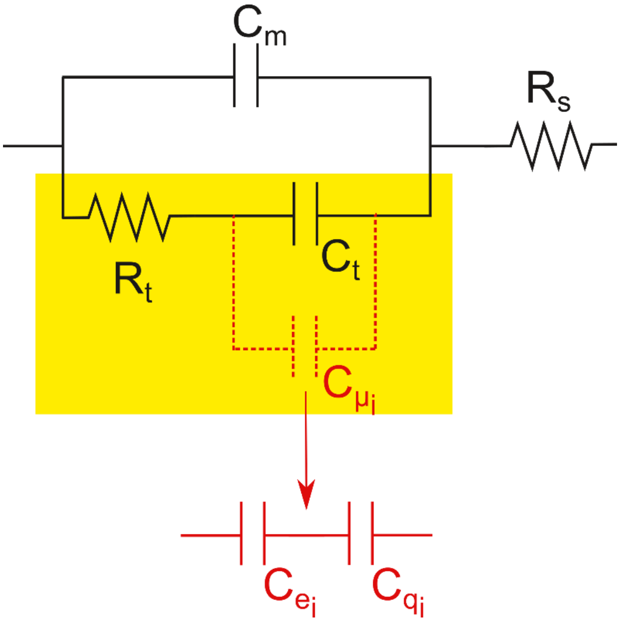

Figure SI. 1. Proposed equivalent circuit diagram that describes the mesoscopic ion-receptive interface. Of relevance to the discussion within this manuscript are only the elements that are contained within the yellow box, which make up the MREI sites. In the absence of any specific anion binding event the measured electrochemical capacitance is equivalent to $C_{\bar{\mu}} = C_t = C_0$ as described previously.[4] In the presence of specifically bound target anion an additional ionic capacitance C_{μ_i} is introduced (red elements; comprised of classical electrostatic and quantic contributions associated with the contribution of the electronic structure upon ion binding) such that the overall measured electrochemical capacitance is equivalent to $C_{\bar{\mu}} = C_t + C_{\mu_i}$.

It should be also noted that the presence of the external electrolyte environment influences all capacitive terms,[5, 6] acting as an external potential (the second term in Eqn. (SI. 1)). This contribution is constant under the experimental conditions used in this work and thus does not need to be further taken into account in the present study.

The energy variation associated with the electrochemical capacitance is $\Delta E = q^2/2C_{\overline{\mu}}$, which in the main text is associated with information which is stated in Eqn. (2) as the e^2/C_{μ_i} term. More discussion about $C_{\overline{\mu}}$ and its correspondence within DFT energy analysis can be obtained elsewhere.[3, 7, 8] In summary, $C_{\overline{\mu}}$, or more specifically, C_{μ_i} is a capacitance that can be resolved experimentally and provides information about the energy state as a function of the state of charge or its electron density upon ion binding.

SI. 2. Statistical mechanics and the ion occupancy of molecular films

Herein we examine the energy changes associated with specific anion recruitment from solution to a receptive surface from a statistical mechanical perspective and further we correlate the quantum (as stated above) and the statistical mechanics (as stated below) analysis. The receptive centres are contained in a molecular layer that intermediate electrons and ions between two reservoirs, that is the electrode and the electrolyte. We are assuming that the electron density of the receptors can be affected by both a mirror charge in the electrode and by the ionic occupancy, forming a mesoscopic element referred to by us in the main text as the mesoscopic receptor-electrode interfacial site (MREI). The recruitment of ions into MREI microstates is sensed by the energy state of the electrode that can be measured as a capacitive response. This occurs due to a change in the electron density of the MREI microstates which is detectable by a capacitive response through a local voltage decay within the MREI sites (see discussion below and in the main text).

Let us now consider a diffusive equilibrium situation. This equilibrium occurs between the concentration of anions, $[A]_{eq}$, in the solution phase and the average occupancy of MREI microstates, $\langle N_i \rangle$, constrained by the molecular coverage. Ionic binding events occur within a few nanometre thicknesses in the film such that generates a perturbation in the local electron density of the electrode. Hence it is assumed that $\langle N_i \rangle \rightleftharpoons [A]_{eq}$, where the general equilibrium is $N_a + [A]_{eq} \rightleftharpoons \langle N_i \rangle$; N_a is the total number of MREI microstate available for ions, which is assumed to be a constant, see Figure 1 of the main text. Note that N_a equates to the molecular coverage $\Gamma = N_a$, a value that can be controlled experimentally.

We aim to connect the occupancy of $\langle N_i \rangle$ sites with the ionic chemical capacitive (C_{μ_i}) response of the film, as experimentally obtained from impedance-derived capacitance spectroscopy at the static low frequency limit (for details see main text and below). At this low frequency the charges are relaxed and their state is independent of the rate of the reaction, i.e. the film is sampled at equilibrium. C_{μ_i} is a particular case of $C_{\overline{\mu}}$ as defined in in the main text. For the case of mesoscopic films, impedance-derived capacitance spectroscopy measurements do not show the presence of any mass diffusive process [see Figure SI. 6.1(a) below]. Therefore, the assumption concerning the equilibrium $\langle N_i \rangle \rightleftharpoons [A]_{eq}$ (at the low frequency limit) is plausible and self-consistent.

Now we invoke the Langmuir isotherm premises for the occupation of MREI microstates by ions. It will be considered that ions in MREI microstate are in equilibrium with ions in the solution phase at a fixed absolute temperature, T, and that the average volume of the molecular film, $\mho$, is kept constant during occupation, as is the total number of sites present. We additionally assume that no interaction exists between MREI microstates, meaning their occupation is energetically independent (but can follow different statistics, i.e. a Fermionic $\left[1 + e^{-\beta\Delta\mu_i}\right]^{-1}$ or Boltzmannian $e^{\beta\Delta\mu_i}$ depending on the degree of occupancy – see discussion in the main text and below). Each MREI microstate can be occupied solely by a single ion. The latter assumption allows us to state that the ion from solution that occupies MREI microstates has a favourable available energy ϵ such that the free-energy of the occupancy of a MREI microstate per a single ion can be defined as $E_i = -|\epsilon|$. Changes associated with the occupancy of E_i states can be detected by changes in the electron density of MREI microstates due to a variation of the free-energy associated with ions binding to E_i energy states.

According to the assumptions introduced in the previous paragraph, the occupation fraction of the molecular film is assumed to be $\Theta = \langle \mathcal{N}_i \rangle / N_a$, where N_a, is the total amount of available sites that equates to the molecular coverage, $\Gamma = N_a$. With these premises, now we can study the thermodynamics of the interface using statistical mechanics. Hence, note that the assumption of diffusive equilibrium requires that the chemical potential of ions (μ_s) in solution equates to the chemical potential (μ_i) of ions in the receptor centres so that $\mu_s = \mu_i$. Owing to this we can use the properties of the grand canonical ensemble, within statistical mechanics, and it can be shown that the ion occupancy follows

$$\langle \mathcal{N}_i \rangle = \left[\sum_{N=0}^{N_a} \sum_a N\, e^{-\beta\{[E_a[N] - \mu_i N]\}}\right] / \mathcal{Z}_{gr}, \tag{SI. 6}$$

where it can be noted that $\langle \mathcal{N}_i \rangle$ is limited to N_a and $\beta = 1/k_B T$, where k_B is the Boltzmann constant. E_a is the energy barrier for ions to bind to MREI sites. The denominator $\mathcal{Z}_{gr}$ is the *grand partition function* and is defined, for the current ion occupancy, as

$$\mathcal{Z}_{gr} = \sum_{N=0}^{N_a} \sum_s e^{-\beta\{[E_a[N] - \mu_i N]\}}, \tag{SI. 7}$$

which is obtained by considering that

$$\mathcal{Z}_{gr} = \sum_{N=0}^{N_a} e^{-\beta N \mu_i}\, \mathcal{Z}(N), \tag{S. 8}$$

where $\mathcal{Z}(N)$ is the *canonical partition function* for the occupancy of N MREI microstates in the film, which leads to

$$\mathcal{Z}(N) = g(N, N_a) e^{N\beta|\epsilon|}. \tag{SI. 9}$$

Note that Eqn. (SI. 9) considers that the ingress of ions into MREI microstates can possess some configurational degeneracy. Thus, the definition of $g(N, N_a)$ is important. We assume a configurational degeneracy (which satisfies the statistics of occupation of the sites) such that

$$g(N, N_a) = \frac{N_a!}{(N_a - N)!N!}. \tag{SI. 10}$$

Thus, the *grand partition function* can now be written as

$$\mathcal{Z}_{gr} = \sum_{N=0}^{N_a} \frac{N_a!}{(N_a - N)!N!} e^{-N\beta\{\mu_i - |\epsilon|\}} = \left[1 + e^{\beta\{\mu_i - |\epsilon|\}}\right]^{N_a}, \tag{SI. 11}$$

from which the fraction of occupancy of MREI sites in the molecular film can be determined as

$$\Theta = \frac{\langle \mathcal{N}_i \rangle}{N_a} = \frac{1}{N_a} \times \frac{1}{\beta} \frac{\delta}{\delta \mu_i} \ln \mathcal{Z}_{gr} = \frac{e^{\beta \mu_i} e^{\beta |\epsilon|}}{1 + e^{\beta \mu_i} e^{\beta |\epsilon|}} = \frac{1}{1 + e^{-\beta(\mu_i - E_i)}}. \tag{SI. 12}$$

Note that $e^{\beta|\epsilon|}$, in Eqn. (SI. 12), is the *canonical partition function* for one single ion binding to a MREI microstate.

Eqn. (SI. 12) directly corresponds to the mainstream Langmuir-adsorption isotherm as used previously[9]

$$\Theta = \frac{\langle \mathcal{N}_i \rangle}{N_a} = \frac{K[A]_{eq}}{1 + K[A]_{eq}}, \tag{SI. 13}$$

where K (in M^{-1}) is the binding (association) constant (related to the $N_a + [A]_{eq} \rightleftharpoons \langle N_i \rangle$ reaction) and $[A]_{eq}$ is the equilibrium concentration of ions in the electrolyte. As discussed in the main text, the investigated ions are the anions perrhenate, iodide and thiocyanate, following previous work.[9]

Comparing Eqn. (SI. 12) and (SI. 13), it can be demonstrated that $\mu_i - E_i = -k_B T \ln[K[A]_{eq}]$, where $\Delta\mu_i = \mu_i - E_i$ is associated with the free-energy required for ions to ingress into MREI microstates as a function of variations in $[A]_{eq}$, as experimentally controlled. In summary

$$\Delta\mu_i = -k_B T \ln\left[K[A]_{eq}\right], \tag{SI. 14}$$

which allows us to directly compare the microscopic statistical model to the macroscopic variable ($[A]_{eq}$). Note also that $K[A]_{eq} = \Theta/(1 - \Theta)$ which allows us to compare the microscopic statistical model with the results of the previous analysis[9] where Eqn. (SI. 13) was applied. The potential difference (see Figure 1) within MREI microstates, V_G, is related to a free-energy, $-\Delta\bar{\mu}/e$, per unit of electron charge of the electrode compensating the ionic ingress. This charge distribution is required to compensate the occupancy of a single MREI microstate, such that $\mathcal{N} V_G = -\Delta\bar{\mu}/e$, where $\mathcal{N} = \int_{\mho} \Delta\rho(\vec{r}) d\vec{r}$ is the variation in the local electron density of an occupied MREI microstate (see more discuss below and in the main text).

In summary, Eqn. (SI. 14) demonstrates that the energy associated with the ingress of ions into MREI microstates is a function of $[A]_{eq}$. Note that because E_i is a constant, $\Delta\mu_i$ only depends on the ion occupancy and on local potential and electron charge distribution per MREI microstate. Both $\Delta\bar{\mu}$ and $\Delta\mu_i$ are dependent on the chemical characteristics of MREI microstates. In the limit of low dilution, that is when $[A]_{eq}$ is tending to null, changes in $\Delta\bar{\mu}$ or $\Delta\mu_i$ are predicted to be independent of the chemical properties of the film (and will only be impacted by the baseline capacitance C_t – see also additional discussion provided in SI. 6 below).

SI. 3. The ionic chemical capacitance and occupation fraction

The ionic capacitance is calculated as $C_{\mu_i} = e^2(d\langle\mathcal{N}_i\rangle/d\mu_i)_{T,\mho}$ and can be determined for any ion receptive SAM providing that the derivative $(d\langle\mathcal{N}_i\rangle/d\mu_i)_{T,\mho}$ is obtainable. In other words, the MREI microstates available for ions in the molecular film are governed by the derivative of the fraction of occupied sites, $\langle\mathcal{N}_i\rangle$, with respect to the chemical potential of the ions that ingress in the film, μ_i. The derivative $(d\langle\mathcal{N}_i\rangle/d\mu_i)_{T,\mho} = N_a(d\Theta/d\mu_i)_{T,\mho}$ is obtained by invoking the result attained in Eqn. (SI. 12), which leads to[3, 7]

$$\left(\frac{d\langle\mathcal{N}_i\rangle}{d\mu_i}\right)_{T,\mho} = \beta N_a\Theta(1-\Theta). \qquad \text{(SI. 15)}$$

Assuming the Langmuir process is obeying the conditions of Henry's law for diluted concentrations (equivalent to $K[A]_{eq} \ll 1$ and where the occupation of the states is exclusively proportional to $[A]_{eq}$), for sake of simplicity, we can consider the following approximation $\Theta(1-\Theta) \simeq \Theta$ in Eqn. (SI. 15). Therefore, C_{μ_i} can be written as

$$C_{\mu_i} = e^2\left(\frac{d\langle\mathcal{N}_i\rangle}{d\mu_i}\right)_{T,\mho} = \beta N_a e^2\Theta, \qquad \text{(SI. 16)}$$

where it can be noted that C_{μ_i} is directly associated to Θ, as observed experimentally. The term $e^2\Theta$ in Eqn. (SI. 16) can be interpreted as the variation in the local electron density (as will be further discussed below). In other words, $e^2\Theta$ corresponds to the redistribution of the electron density accompanying the ion ingress into MREI microstates, i.e. it is the variation of the electron density that accommodates the free-energy associated with the spontaneous ionic ingress of anions according to changes in $[A]_{eq}$. Eqn. (SI. 15), which corresponds to any situation beyond the low occupancy conditions, can be rewritten as

$$C_{\mu_i} = e^2\left(\frac{\langle\mathcal{N}_i\rangle}{d\mu_i}\right)_{T,\mho} = \beta N_a e^2\Theta(1-\Theta) = \beta N_a(e\Theta)^2 e^{\beta\Delta\mu_i}, \qquad \text{(SI. 17)}$$

noting that $(1-\Theta) = \Theta e^{\beta\Delta\mu_i}$.

In terms of the energy of individual MREI microstates, Eqn. (SI. 16), considering low occupation fraction for the sake of simplicity, can be rewritten as $\beta N_a\left(e^2/C_{\mu_i}\right)$ and thus

$$\beta N_a\left(\frac{e^2}{C_{\mu_i}}\right) = e^{\beta\Delta\mu_i}, \qquad \text{(SI. 18)}$$

where e^2/C_{μ_i} can be identified as an individual energy occupancy of a MREI microstate (more details below) and the total available energy occupancy $N_a(e^2/C_{\mu_i})$ decreases as Θ increases.

From Eqn. (SI. 16) the actual occupancy fraction of the MREI sites can be calculated, at low occupancy (a valid assumption, as can be seen below), from the known constants $\beta N_a e^2$ via $\Theta =$

$C_{\mu_i}/(\beta N_a e^2)$. With the help of N_a, obtained from thiol stripping as shown previously[9], as well as C_{μ_i} at $[A]_{eq}$ = 50 mM, Θ can be determined as 0.53‰, 0.26‰ and 0.36‰ for ReO_4^-, SCN^- and I^-, respectively. This corresponds to a receptor-to-ion ratio of 1910, 3880 and 2820, respectively, indicating that the vast majority of MREI sites remain unoccupied, even at high $[A]_{eq}$. This confirms that the bound anions do not interact with each other and each anion binding event is independent. Furthermore, this indicates that a significantly higher response could be expected if more MREI sites could be occupied. This could be achieved, for example, by raising the potential of the ionic (electrolyte) reservoir or increasing the binding energy E_i (i.e. designing more effective receptors).

SI. 4. Electric field-effects and the ion sensing mechanism

In this section we intend to demonstrate that the potential decay, V_G, in MREI microstates is related to a mesoscopic field-effect associated with the sensitivity of C_{μ_i} to the ions. This is also related to the variation in the electron density associated with $\mathcal{N} = \int_{\mho} \Delta\rho(\vec{r})d\vec{r}$ within MREI microstates. This electron density variation $\mathcal{N} = \int_{\mho} \Delta\rho(\vec{r})d\vec{r}$ is the electric charge compensation (opposed to the ion charge) for the ingress of an ion charge which is in turn associated to the variation of $\Delta\bar{\mu}$. The changes in $\Delta\bar{\mu}$ correlate to $[A]_{eq}$ as stated in Eqn. (SI. 18).

Owing to the fact that the macroscopic potential of the electrode, V, is kept constant (as controlled experimentally), a distribution of charge is required to attain the changes in the electrochemical potential of the electrons in the electrode such as $\Delta\bar{\mu} = -V\Delta q$, where it can be identified that $\Delta q = e\mathcal{N}$. It implies that the local charge distribution in the MREI microstates is directly associated with the grand canonical equipartition function as stated previously. The mesoscopic situation allows "free" electronic charge to follow the probabilistically distribution associated with the equipartition function. To detail this process, we will denote an occupied individual MREI microstates with $\mathcal{N}e$ charge that can be identified as an electrochemical non-Faradaic "dipole" in which the local potential difference is $\Delta V_{ei} = V_G$, where V_G states for a local gate voltage. Now observing that $\Delta\bar{\mu} = -\mathcal{N}eV_G$, we can demonstrate that MREI microstates corresponds to the thermodynamic microstates that correlate with quantum mechanical states through $\mathcal{N} = \int_{\mho} \Delta\rho(\vec{r})d\vec{r}$.

Let us now prove that V_G is a voltage associated with individual mesoscopic capacitors within MREI microstates. For this, let us guess a fictitious distance between the charge density $e\mathcal{N}$ and the ion charge. This fictitious situation resembles the length of a bridge or of a channel of a field-effect transistor (see Figure 1d of the main text). Accordingly, we can name the voltage of the channel V_c, which is dependent on the electron density in the channel, $\mathcal{N}_c$. Note that this is a mesoscopic effect associated with how MREI microstates are electronically coupled to electrode states in terms of their common and shared electron densities (in a non-Faradaic situation). We can thus relate V_c to V_G via

$$V_c = V_G - e\mathcal{N}_c/C_{e_i}, \qquad \text{(SI. 19)}$$

where $q_c = e\mathcal{N}_c$ is the charge in the MREI "channel" which is hypothetically representing a kind of non-Faradaic bridge between the most probable location of $e\mathcal{N}$ distribution in the electrode side of the MREI microstate and the location of its opposite charge distributed within an ionically occupied MREI site. Hence, $e\mathcal{N}_c/C_{e_i}$ is the potential of electrons in this fictitious bridge. Accordingly, the capacitance in the gate can be written as

$$\frac{dq_c}{dV_G} = \frac{dq_c}{dV_c}\frac{dV_c}{dV_G}. \tag{SI. 20}$$

Now by applying the derivative of Eqn. (SI. 19) with respect to V_G we see that $dV_c/dV_G = 1 - [(1/C_{e_i})(dq_c/dV_G)]$ and Eqn. (SI. 19) can be rewritten as

$$\frac{dq_c}{dV_G} = \frac{dq_c}{dV_c}\left(1 - \frac{1}{C_{e_i}}\frac{dq_c}{dV_G}\right). \tag{SI. 21}$$

By assuming that dq_c/dV_G is a total equivalent capacitance (of the two series capacitances C_{e_i} and C_{q_i}) of MREI site, it can be noted that dq_c/dV_G is truly what was defined as electrochemical ionic capacitance, C_{μ_i}. Therefore, by observing that dq_c/dV_c (associated with MREI site) is the quantum capacitance, C_q, we obtain

$$\frac{1}{C_{\mu_i}} = \frac{dV_G}{dq_c} = \left(\frac{C_{q_i}+C_{e_i}}{C_{q_i}C_{e_i}}\right) = \frac{1}{C_{e_i}} + \frac{1}{C_{q_i}}. \tag{SI. 22}$$

and thus, we demonstrate that V_G is directly associated with the electrochemical capacitance of the interface. C_{μ_i} is thus a particular type of non-Faradaic electrochemical capacitance that serve as a mesoscopic capacitive gate for ions possessing an associated V_G and a distribution of charge such as $e\int_{\mho} \Delta\rho(\vec{r})d\vec{r} = e\mathcal{N}$.

In summary, individual C_{μ_i} elements associated with single MREI sites are capacitive transducer element for ion sensing which uses MREI microstate elements in the sensing interface connecting electronic (as detector) and ionic (as receptor) states. Occupied MREI microstates generate a separation of charge which was herein referred to as MREI electrochemical "dipole". To keep the MREI state charged a free-energy associated with the electron density distribution is required. This free-energy can be interpreted as a variation in the electrochemical potential per electron as $-\Delta\bar{\mu}/e = \mathcal{N}V_G$, where it can be noted that $\mathcal{N}V_G$ is the distributed potential localized in an MREI site. Alternatively, due to the inherently quantum nature of $\mathcal{N}$ and uncertainties associated with the $\int_{\mho} \Delta\rho(\vec{r})d\vec{r}$ character of the charge density, the occupation of a MREI site can be equivalently assumed as $V_G = -\Delta\bar{\mu}/e\mathcal{N}$, where now $\Delta\bar{\mu}$ is the chemical potential per amount of $e\mathcal{N}$ distributed charge in the MREI site. This process generates a detectable capacitive change – associated with a field-effect and due to its inherently meso and nanoscopic characteristics it is potentially more sensitive, selective and tuneable at a molecular scale.

The impact of an anion binding to the receptive film could potentially be screened by cation compensation from solution. However, we have previously shown that the concentration of counter cations has little impact on the measured capacitance of these films.[9] Furthermore, it has been shown that, in solution-phase, there are no counterion effects on binding for these receptors.[10] These observations suggest that there is no significant ion pairing between the bound anion and counterions from solution. The proposed model of charge screening through a mirror charge is thus reasonable.

SI. 5. Energy of the microstates in terms of electrochemical events

In the preceding sections we noted that $\Delta\bar{\mu} = -e\mathcal{N}V_G$ is associated with the free-energy per MREI site. Also note that owing to the fact that R_t is constant, thus $i_{nf} = V_G/R_t$ is the mesoscopic non-Faradaic current needed to compensate the charging of the MREI sites. $i_{nf}V_G$ is the energy per unit of time that the electrode spends to maintain the macroscopic V (OCP) constant for each ionic occupancy event, as required for equilibrium achievement, according to changes in $[A]_{eq}$.

Now, let us demonstrate that this free-energy is connected to C_{μ_i}. For this, note that variations in the electrochemical potential of the MREI gates can be stated as $\Delta\bar{\mu} = \bar{\mu}(N + \mathcal{N}) - \bar{\mu}(\mathcal{N}) = qV_G$, where the initial chemical state is $\bar{\mu}(N)$ and it is varied by changes in the charge density, $\mathcal{N}$. Note that $q = e\mathcal{N}$ and that $1/C_{\mu_i} = V_G/q$, and if we assume $\mathcal{N} = 1$ for the single compensation of charge in a single MREI site and additionally consider the negative elementary charge we have $-eV_G = \Delta\bar{\mu}$ which, when combined with $1/C_{\mu_i} = V_G/q$ provides $e^2/C_{\mu_i} = \Delta\bar{\mu}$, demonstrating that this is energy to compensate the charge of a MREI site associated to the ion binding reaction. This energy is part of Eqn. (2) of the main text.

In summary, MREI microstates as detailed herein, in terms of a comprehensive mesoscopic description, is what is referred by electrochemists as a non-faradaic charging process. In terms of energy, the charging of a MREI microstates element in the interface is $-\Delta\bar{\mu} = e^2/C_{\mu_i} = eV_G$, which can be alternatively written in terms of free-energy per electron, which give us $-\Delta\bar{\mu}/e = V_G = e/C_{\mu_i}$. The analysis made herein is self-consistent. The resulting statistics are equivalent by using quantum mechanics or thermodynamics.

SI. 6. Impedance-derived capacitance spectroscopy and circuit parameters

The experimental data provided herein was obtained utilizing a halogen-bonding foldamer self-assembled monolayer. The surface coverage and thickness of this film are strongly indicative of densely packed films in which the individual receptors adopt an upright conformation as schematically depicted in Figure 1*a*. The anion binding site is constituted of four convergent iodo-triazole moieties which interact with the anion via halogen-bonding interactions. Importantly, this non-covalent interaction is fully reversible enabling sensor reuse after a simple washing step. The anion binding site is furthermore preorganized by triethyleneglycol-amide appendages which restrict the rotational freedom of the foldamer core; the conformation of the receptor (and the film) is thus expected to be unaffected by anion binding. For further information the interested reader is referred to previous works in which the chemical aspects of this and related systems including detailed anion binding studies[9-11] are discussed in more detail. Importantly, the developed models are expected to be translatable to any ion-receptive interface sampled in a non-Faradaic format. The selectivity and binding strength (and thus response) is, according to supramolecular principles (e.g. complementarity), chemically tuneable through the design and synthesis of appropriate ion receptors.

Electrochemical impedance-derived capacitive analyses were performed utilizing an Autolab potentiostat (Metrohm) equipped with a frequency response analysis (FRA32) module using a three-electrode system: a gold disk working electrode, a platinum wire counter electrode and a Ag|AgCl reference electrode. Impedance-derived capacitive data were acquired at the open-circuit potential (ca. 0 V versus Ag|AgCl), at frequencies ranging from 1 kHz to 1 Hz, with a sinusoidal perturbation of 10 mV (peak-to-peak). It should be noted that the OCP does not change significantly upon anion binding (typically <30 mV). Derived spectra were also checked to satisfy the Kramers-Kronig relationship of time-invariance. Capacitive Nyquist diagrams were obtained by converting the complex impedance (Z^*) to complex capacitance (C^*), using the relationship $C^*(\omega) = 1/j\omega Z^*(\omega)$, where ω is the angular frequency and j is $\sqrt{-1}$. C_{μ_i} can be obtained by fitting of the data to an electric equivalent circuit or graphically as the diameter of the semicircular region in the capacitive Nyquist plot [Figure 2(b)] or at the low frequencies of capacitive Bode plots as shown in Figure 3(b). All experiments were carried out in pure aqueous solution containing 100 mM electrolyte (100 mM NaCl, titrated with 100 mM sodium salts; constant ionic strength. The preparation and analysis of the receptive films has recently been reported.[9]

Figure SI. 2(a) illustrates the traditional impedance response of foldamer monolayers for two different situations, i.e. the response of a halogen-bonding foldamer film obtained for the zero concentration and for 50 mM of perrhenate ion (ReO_4^-). As expected, no changes in the impedance response is detectable for variations in the concentration of the ion, which is in agreement with the assumption that there is no ion diffusion limiting process – which can be noted by the fact that no Warburg diffusive process is observable in the impedance spectra. Additionally, the impedance does not change because R_t is also constant. The meaning of this is that the dominant changes are in the energy state associated only with the capacitive elements of the interface – there is no ionic transport at the lower frequency, only a single occupancy of the capacitive states. Therefore, an impedance-derived capacitive spectroscopic method [Figure SI. 2(b)] is more appropriate than the mainstream impedance analysis [Figure SI. 2(a)]. Undeniably, Figure SI. 2(b) reveals that there are significant variations occurring in the interface as a function of ion concentration. The response that resembles a semi-circle in the diagram of Figure 2(b) is associated with R_t in series with the C_t that is the capacitance present without the existence of a specifically binding ion in solution. Note in Figure SI. 3 that R_t is invariant with the concentration of ions.

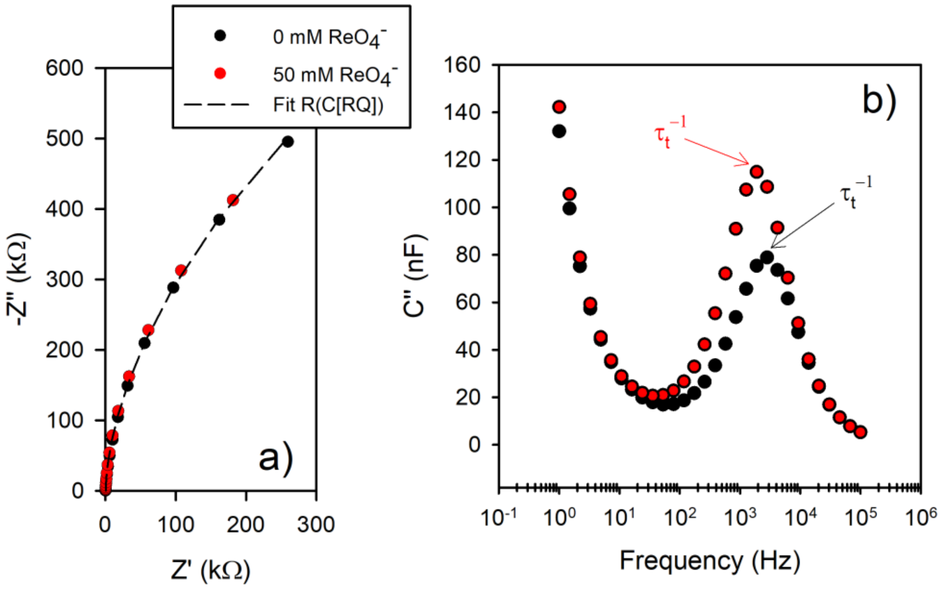

Figure SI. 2. (a) Impedance response of a halogen foldamer film at the zero- and 50-mM concentration of perrhenate ion. (b) Plot of the capacitive response of the same film. This figure (b) shows that significative differences can be observed whereas the impedance representation of the data suggests that there are no changes upon anion binding.

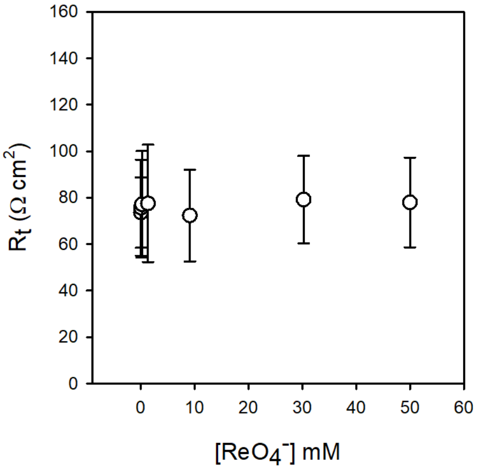

Figure SI. 3. Plot of R_t as a function of the concentration of ReO$_4^-$. An equivalent behavior is observed for the other ions (not shown). The invariance of R_t demonstrates that only C_{μ_i} varies during the reaction of the ions with the foldamer film.

Since $C_m \ll C_{\overline{\mu}}$, the R_t was calculated from the raw data via $\tau_t = RC_{\overline{\mu}}$, whereby τ_t is the relaxation time constant that considers all the series resistive element. Therefore, observe that $R = R_s + R_t$, where R_t can be obtained via $R_t = \tau_t / C_{\overline{\mu}} - R_s$. τ_t was obtained as the inverse of the frequency corresponding to the of the peak in the Bode plot of the imaginary capacitance C'', as shown in Figure SI. 2(b). Figure SI. 4 shows a simulated and experimental data using this model.

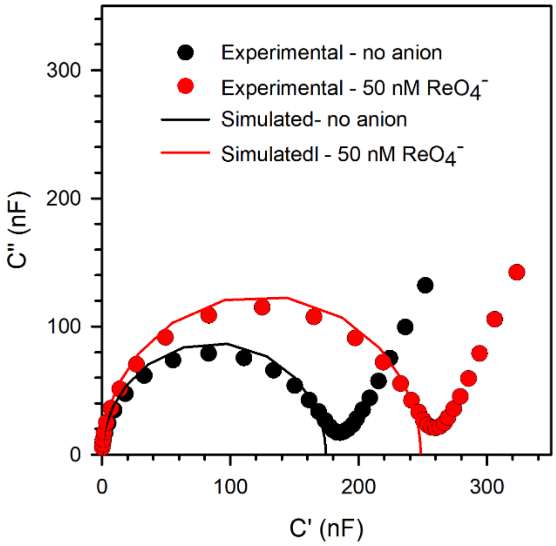

Figure SI. 4. Experimental and simulated data for ReO_4^- binding at 50 nM. As can be seen, the simulated data (proposed circuit) shows good agreement with experimental results.

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