Supplementary Information

Rational Design of a High-Throughput Droplet Sorter

Simon S. Schütz, Thomas Beneyton, Jean-Christophe Baret and Tobias S. Schneider

ACCURACY OF THE APPROXIMATION FORMULA FOR SMALL RADII

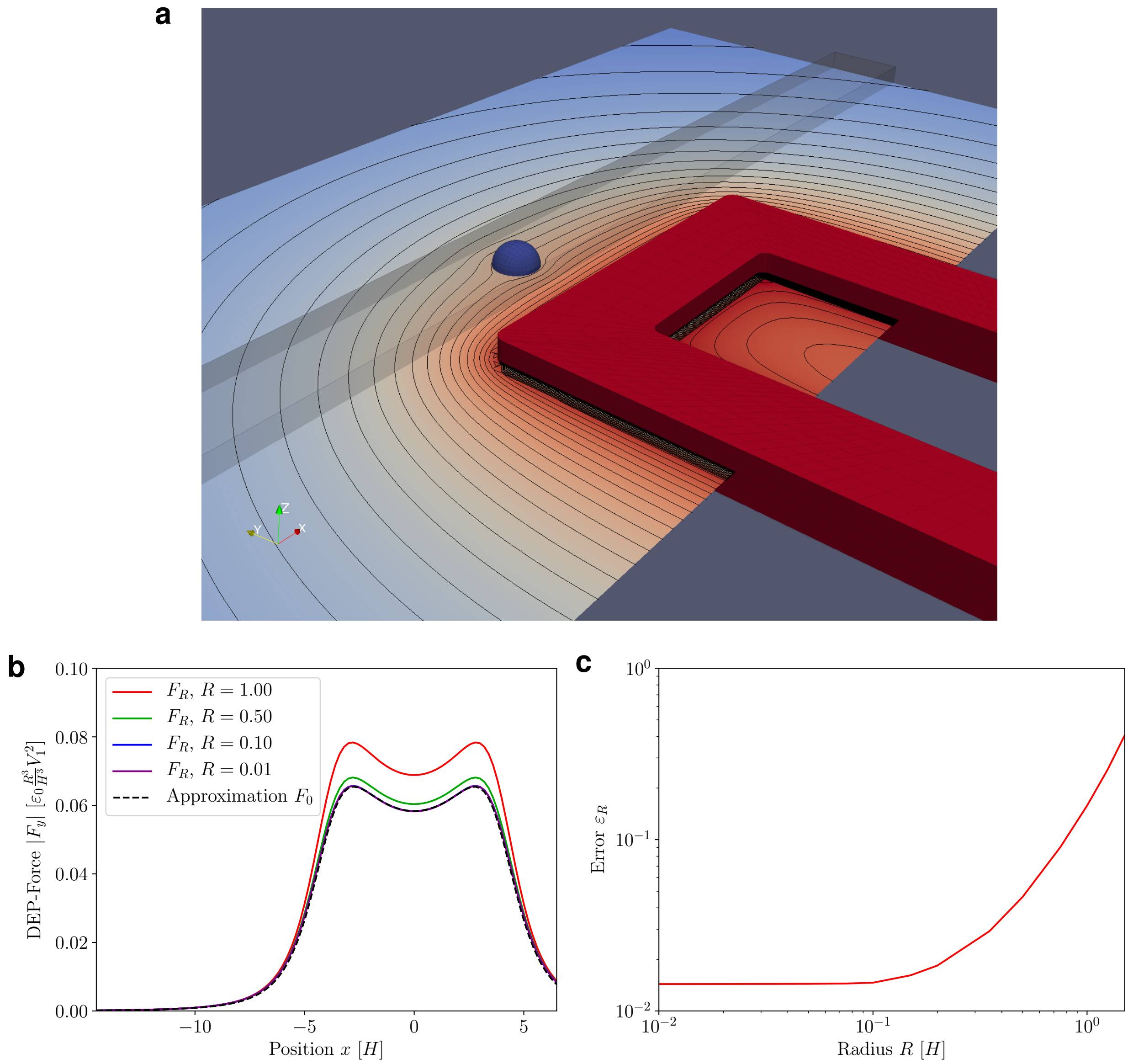
To estimate the deflection force $\vec{F}$ on the droplets, we use a relation by Pohl (equation (1) in the article), which is valid for spherical droplets whose radius R is small compared to the characteristic length scale of the electric field gradient. The relation is exact for electric fields with an azimutally symmetric potential $\varphi(r,\theta) := \varphi_0 + E_0 r P_1(\cos\theta) + E_0' r^2 P_2(\cos\theta)$, with r the radius and θ the polar angle from the droplet center, P_n the Legendre polynomials, and φ_0, E_0 and E_0' arbitrary constants. Since the actual droplet radius is similar to the size of the electrodes that create the field, Pohl's equation is only an approximation, and higher order terms affect the deflection force. Here, we investigate the accuracy of the relation for large droplets.

We use a single electrode of length $L = 8H$ and a spherical droplet of radius R with a trajectory on the microchannel centerline. We calculate the electric field around the electrodes in the presence of the droplet (Supplementary Figure 1(a)), using a mesh of approximately 2,200 degrees of freedom. From the field at the droplet surface, we determine the cross-stream deflection force F_R along the droplet path, which we compare to the approximated force F_0 from equation (1). The approximation error is quantified by the integrated force difference along the droplet path,

$$\varepsilon_R := \frac{\langle |F_R - F_0| \rangle}{\langle |F_R| \rangle}, \tag{1}$$

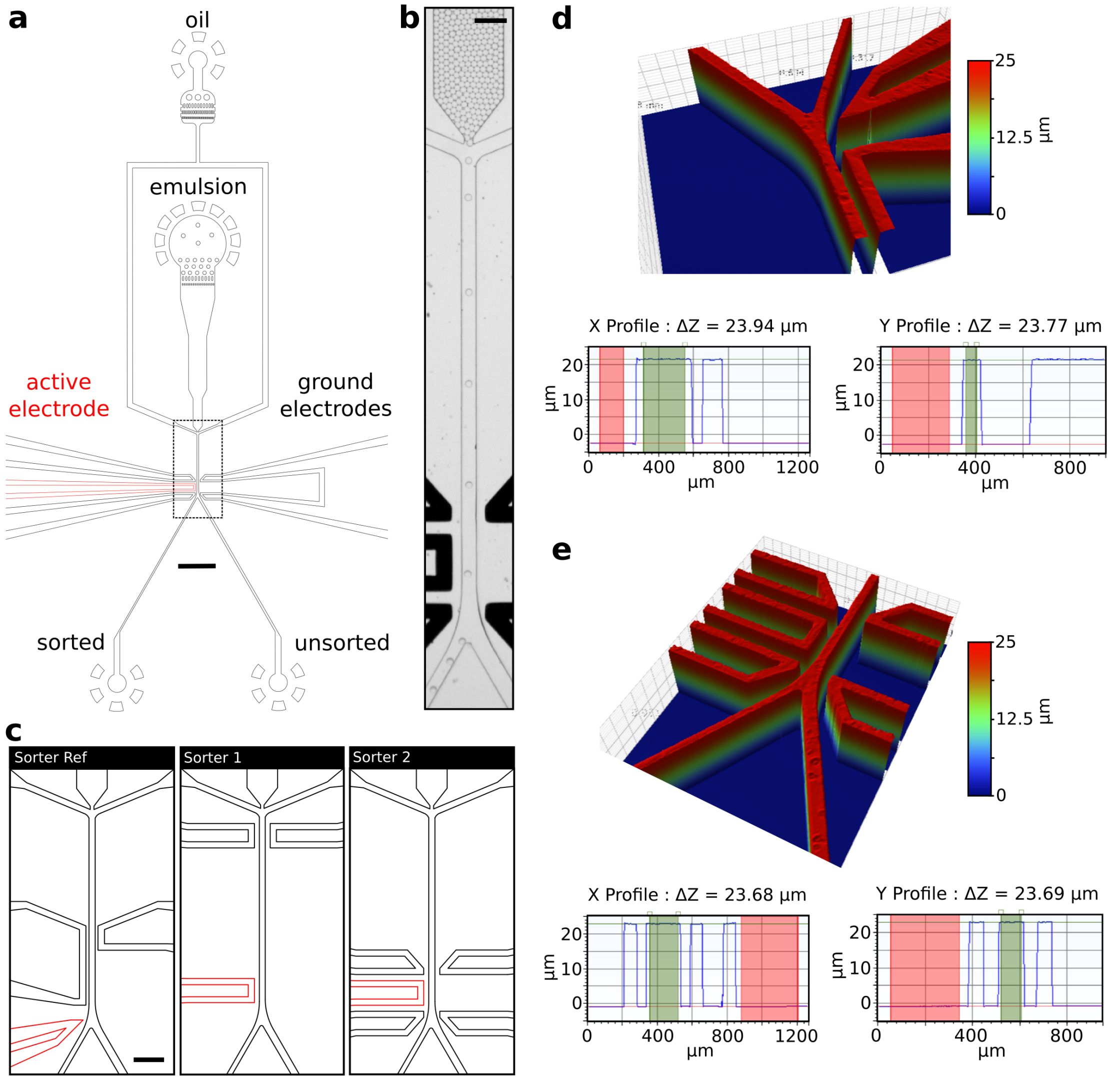
where $\langle ... \rangle$ denotes the spatial average along the droplet path.

We find that the approximation accurately describes the force for small R, but underestimates the force for larger R, as shown in Supplementary Figure 1(b,c). The approximation error ε_R is on the order of few percent and increases for $R > 0.1H$. At $R = 0.5H$, when the droplet diameter equals the channel height, the relative error is $\varepsilon_R = 4.6\%$: For typical droplet sizes and channel layouts, the approximation conservatively underestimates the sorting efficiency, with a relative error on the order of 5%.



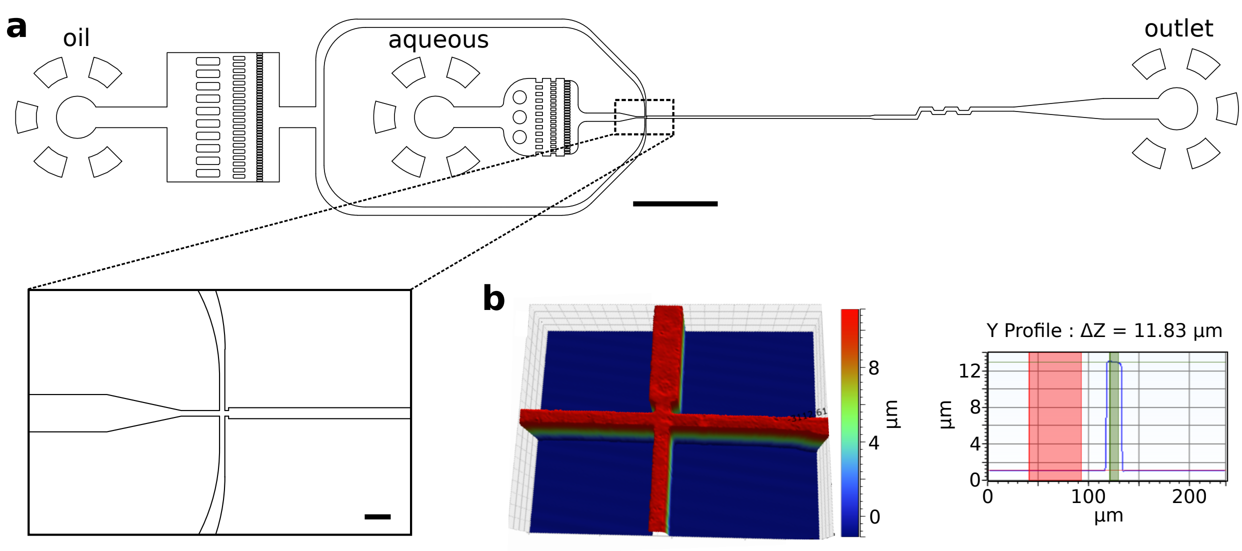
Supplementary Fig. 1: Deflection force on a droplet with finite radius. (a) Droplet ($R = 0.5H$) in a microchannel next to an active electrode. Equipotential contour lines around the sorting electrode show the distortion of the field due to the finite-size droplet. (b) Deflection force of the approximation formula, and the full 3D simulation, for different droplet radii R. (c) Relative error of the approximation formula against droplet radius R. The error ε_R for $R \to 0$ does not converge to zero due to the limited numerical precision of the simulation.

DESIGN OF THE SORTING DEVICES



Supplementary Fig. 2: Sorting devices. (a) General schema of the sorting devices. 8 pL w/o droplets are reinjected and spaced by two streams of fluorinated oil before entering the sorting area. Droplets are sorted by applying AC field (20kHz; 0 to 1000 V_{pp}). Scale bar is 1 mm. (b) Bright-field image of the device under operation. Scale bar is 100 μm. (c) Schema of the sorting area for each tested sorter. Active electrodes are in red. Scale bar is 250 μm. (d-e) 3D-profile of the sorting area of Sorter Ref (d) or Sorter 2 (e) obtained with a 3D optical microscope (Contour GT-K, Bruker).

DROPLET GENERATION

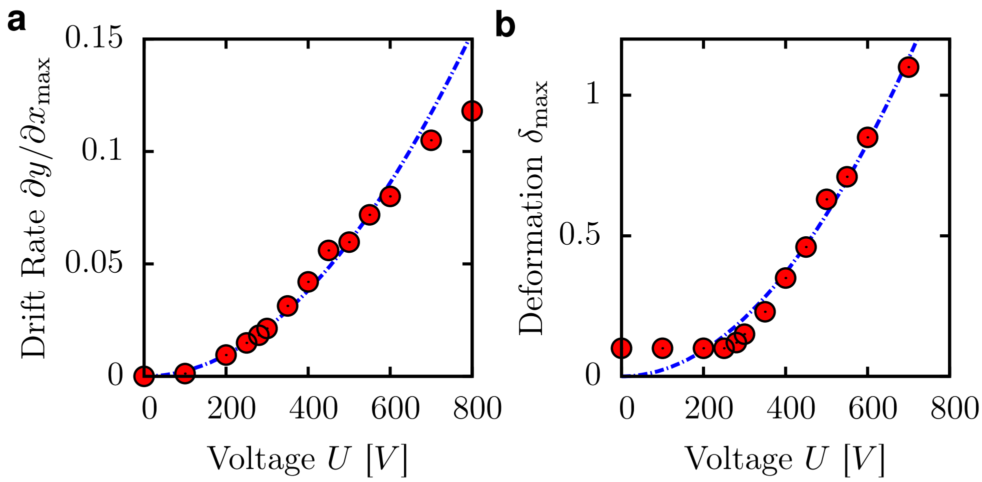


Supplementary Fig. 3: Dropmaker device. (a) Schema of the dropmaker device. The design was taken from Pekin et al., Lab Chip **11** (2011), pp. 2156–2166. 8 pL w/o droplets are generated by the flow-focusing of an aqueous stream with two streams of fluorinated oil containing surfactant. Droplets are collected off-chip from the outlet into a glass vial. The nozzle dimensions are 15x15x12 μm. Scale bar is 1 mm (60 μm for the zoom-in). (b) 3D-profile of the nozzle area obtained with a 3D optical microscope (Contour GT-K, Bruker).

IMAGE PROCESSING AND DATA ANALYSIS

Droplet trajectories and droplet deformations are obtained from the recorded high-speed movies using an home-made matlab routine. In brief, a series of 500 successive frames is averaged to determined the background intensity. Then each frame is processed by first subtracting the calculated background to enhance the contrast of the droplet. Distance calibration is performed using the channel width (W): $y = 0$ corresponds to the center of the channel; $x = 0$ corresponds to the position of the tip of the sorting wall. The image is then filtered using a median filter and the contrast is adjusted. A thresholding is performed using a parameter tuned to optimize the number of droplets detected on a single binarized frame. Each filled structured in the frame is detected and for those having a size sufficiently large (i.e. larger than 50 pixels) the mean position of the structure is determined. The droplet deformation is then computed by first calculating R^2 the square of the maximum distance between any point of the structure and the center of the structure. The final deformation is normalized by dividing the R^2 using the radius r of a disk having an area equal to the total number of pixels of the structure (its area). In order to obtain a deformation parameter linear in dimension, the final deformation is computed as $\delta = (R^2/r^2 - 1)$. A value of $\delta = 0$ corresponds to a disk while any value larger than one corresponds to an elongated structure.

DROPLET DRIFT RATE AND DEFORMATION IN THE EXPERIMENT



Supplementary Fig. 4: Maximum drift rate and deformation of droplets in the optimized design of Sorter 2. (a) Maximum drift rate (cross-stream displacement per downstream displacement) of droplets at different electrode voltages U. At constant downstream advection velocity, the cross-stream drift velocity U_{drift} is proportional to the drift rate ($U_{\mathrm{drift}} \propto \partial y/\partial x$). With the electric field proportional to the electrode voltage ($E \propto U$), we find a scaling $U_{\mathrm{drift}} \propto E^2$ (blue fit curve) that matches the theoretical predicition (eq. (1) in the main article). (b) Maximum deformation of droplets at different electrode voltages U. The deformation δ_{max} grows with the squared strength of the electric field (blue fit curve), matching both our numerical results and previous analytical predictions by Sherwood, J. Fluid Mech. **188**, 133 (1988). The small but finite deformation at very low voltages is due to hydrodynamic droplet deformation at the channel junction.