Electronic Supplementary Material (ESI) for Lab on a Chip.

Supplementary Information

A. Derivation of Equation 1

To find a relation between the stratified flow properties (mainly the flow thickness and liquid flow rate), a hydrodynamic analysis is conducted. Naiver-Stokes and continuity equations are the main equations that are used in these cases. The centrifugal force ($F_\omega = \rho r\omega^2$) is considered as the exerted external force.We will neglect the Coriolis force as well as the minor forces exerted by the overlying air on the moving liquid.

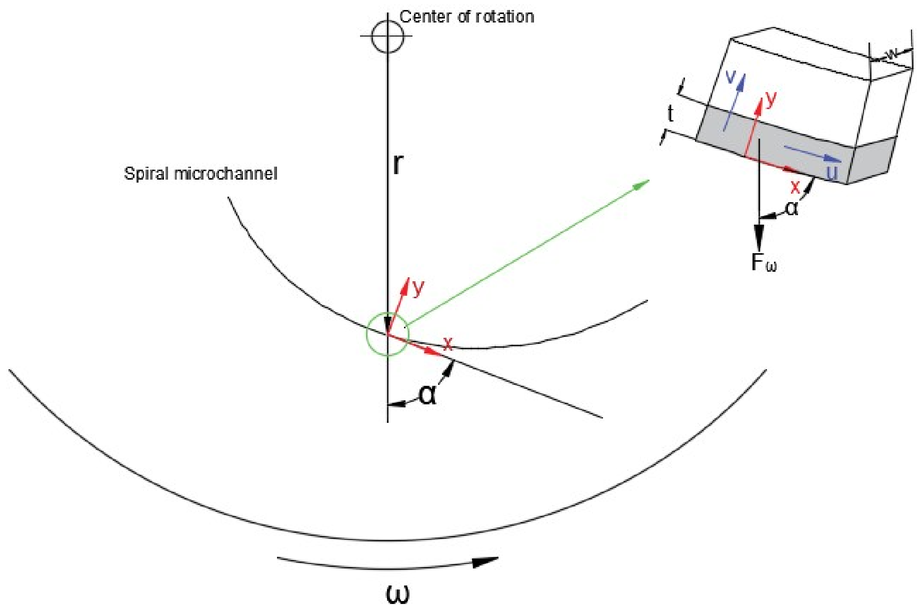

Figure1: A schematic view of the model for the hydrodynamic analysis of the stratified flow in spiral microchannel located on a rotating disc

For a Newtonian and incompressible liquid, the Naiver-Stokes equation in 2-dimensional format can be written as:

$$x - direction: \quad F_\omega \cos(\alpha) - \frac{\partial p}{\partial x} + \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} = \rho(\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y})$$

$$y - direction: \quad - F_\omega \sin(\alpha) - \frac{\partial p}{\partial y} + \frac{\partial \tau_{yy}}{\partial y} + \frac{\partial \tau_{xy}}{\partial x} = \rho(\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial x})$$

And the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

in a fully developed flow and steady state condition: : $\frac{\partial u}{\partial x} = \frac{\partial u}{\partial y} = 0$, $\frac{\partial u}{\partial t} = \frac{\partial v}{\partial t} = 0$

by neglecting the pressure and normal tension gradients : $\frac{\partial p}{\partial x} = \frac{\partial p}{\partial y} = 0$

The equations will be reduced to:

$$F_{\omega} \cos(\alpha) + \frac{\partial \tau_{yx}}{\partial y} = 0 \quad (1)$$

$$, \frac{\partial \tau_{yx}}{\partial x} = F_{\omega} \sin(\alpha)$$

Considering a correction factor for a stratified flow similar to the flow in open channels equation (1) will be converted to (White 2002):

$$\tau = F_{\omega} \cos(\alpha) \frac{b}{2t + w}(d - y)$$

For a Newtonian liquid: $\tau = \mu \frac{\partial u}{\partial y}$

By eliminating τ from these two equations:

$$\frac{\partial u}{\partial y} = \frac{b\rho F_{\omega} \cos(\alpha)}{(2t + w)\mu}(t - y)$$

In steady-state and with considering:

$$a = \frac{b\rho F_{\omega} \cos(\alpha)}{(2t + w)\mu}(t - y) \quad (2)$$

$$\rightarrow du = a(t - y)\, dy$$

("a" is independent of y)

$$\rightarrow u = -\frac{a}{2} y^2 + aty + c_1$$

Using no-slip boundary condition on the walls: $y = 0 \rightarrow u = 0 \rightarrow C_1 = 0$

$$\rightarrow \quad u = -\frac{a}{2}y^2 + aty$$

For calculation of the liquid flow rate (Q), the following equation can be used:

$$Q = \int_0^y u.dA = \int_0^y uwdy$$

$$\rightarrow Q = \int_0^t \left(-\frac{a}{2}y^2 + aty\right)w\,dy = \frac{a}{2}\left(-\frac{1}{3}t^3 + t^3\right) = \frac{a}{3}t^3$$

by replacing "a" from equation 2:

$$\rightarrow \frac{wt^4}{(2t+w)} = \frac{3\mu Q}{\cos(\alpha) F_\omega}$$

It should be noted that the angle α should be less than 90° to have a meaningful equation. In case of $\alpha = 90°$, the propulsion force is zero and the multiphase flow cannot be generated.

- White, Frank M. 2002. "Fluid Mechanics-5th." *McGraw-Hill,New York*.

B. Stratified to slug flow transition

Figure 2 shows the transition steps of a stratified flow (i) to slug flow (iii) through forming a liquid bridge (ii) in a 400 μm-wide micro chamber. The bubbles and liquid plugs are pushed into the microchannel by a sufficient centrifugal force.

Figure2. Detailed steps of the transition from stratified flow to slug flow, i) the thickness of the stratified flow is growing through accelerating the lab-disc ii) liquid thickness is reached to a critical value and the liquid climb to the upper wall of the microchannel due to dominance of the surface tension forces, iii) the air is trapped in the form of a bubble among the liquid plugs.

C) A multiplex design for the generation of slug flow or liquid plugs in radially-oriented spiral channels for the applications that require a large amount of the bubbles or liquid plugs.

The conventional disc real-estate allows for generating up to hundreds of thousands of bubbles and liquid plus, with a multiplex design of the spiral channels. Figure 5 shows an exemplary design for generating large number of plugs on a single disc.

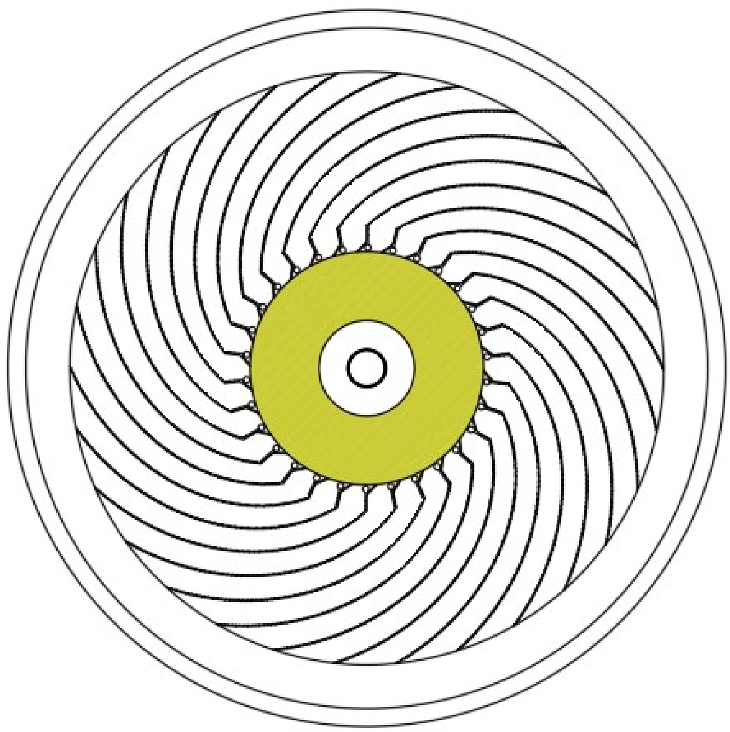

Figure 4: A multiplex design for the generation of slug flow or liquid plugs in radially-oriented spiral channels by implementing the presented technique.

D. Implanting slug flow for the mixing the miscible liquids

The slug flow is regularly utilized for the mixing of the miscible liquids based on the formation of a secondary flow inside the liquid plugs. We designed a model comprising two inlet chambers connected by same-sized microchannels (hydraulic diameter=200 µm) to the radial channel. For evaluating the mixing capability of the presented model, 100µl of two differently colored dyed-water samples were introduced into the inlet chambers and the disc was rotated at 700 rpm for the generation of the slug flow. Through the radial channel (hydraulic diameter=1mm), the solution of the liquids was introduced into the spiral channel (hydraulic diameter=600µm) in the form of a slug flow. After 30s, we stopped the disc with a deceleration rate of 1000 rpm/s for analysis of the color changing of the mixture. Figure3a shows the colors of the liquids before and after the mixing captured by a benchtop scanner. It should be noted that the inlet chambers were refilled by the initially colored liquids for a better comparison. Based on the color changes, the liquids were mixed based on the circulation of the slug flow. Figure 3-b shows the conditions of the solution inside a plug in different times. The internal circulations inside the liquid plug are obviously observed in these figures. The induced convection mixing leads to mixing of the liquids.

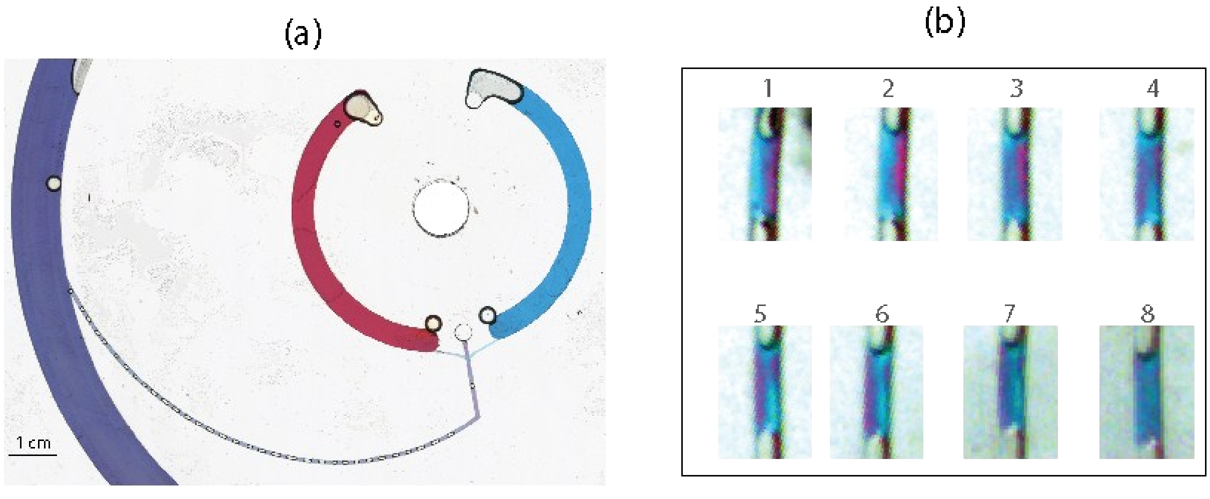

Figure 3: Utilizing the centrifugal induced slug flow for mixing 2 miscible liquids: a) the model design comprising 2 inlet chambers connected to the inlet channel. The colors of the inlet liquids and the mixture is shown after stopping the disc. b) The recirculation motion inside a liquid plug during the rotation of the disc (the channel width= 600µm).

E. The viability test for the cultured Bacterial cells

To determine the viability of bacterial cells, the sample was collected from the microfluidic system after 12 hours and was cultured in Eosin methylene blue agar (EMB) medium and then kept at a temperature of 37 °C for 24 hours. Escherichia coli growth colonies exhibiting a greenish metallic sheen by reflected light (fig5 a). Also, we used the LIVE/DEAD BacLight kit to viability assay. In this regard, 100 µL of bacteria solution was mixed with dye (1µL). For 15 min, the mixture was kept in darkness at room temperature to be stained. The stained bacteria were observed with fluorescence microscopy to determine the viability. In this study, the viable and non-viable bacteria in the sample are shown by fluorescent green and red color. The images are shown significant viability of bacteria in the sample (Fig. 5b)

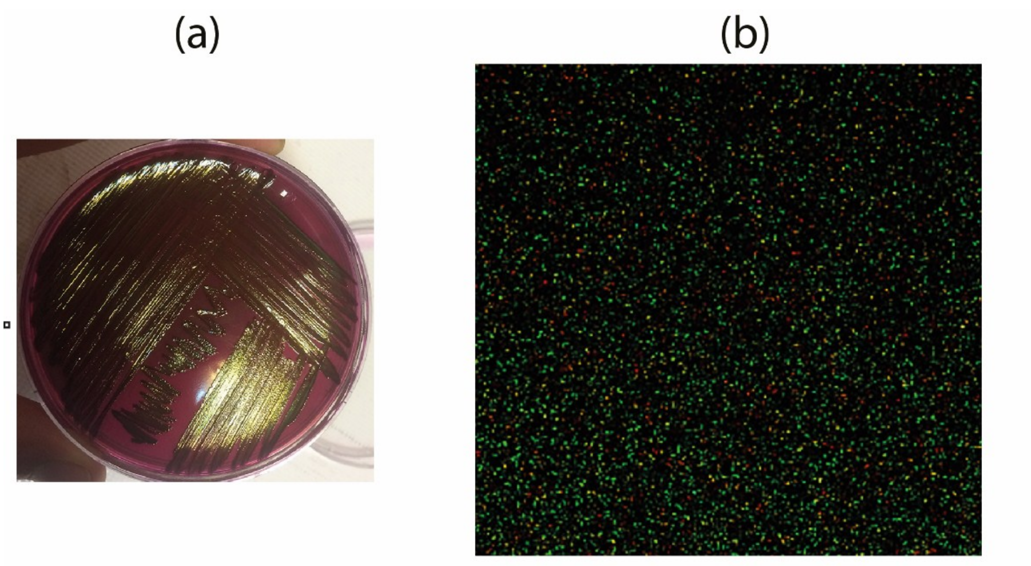

Figure 6: Images of the viability test of the extracted E.coli bacteria on: a) EMB medium and b) fluorescence microscopy

F. Measuring the stratified thickness by image processing

The thickness of the stratified flows formed during the experiments is measured by analysis of the captured images by the high-speed camera platform. Figure 6 shows a post-processed image in ImageJ software.

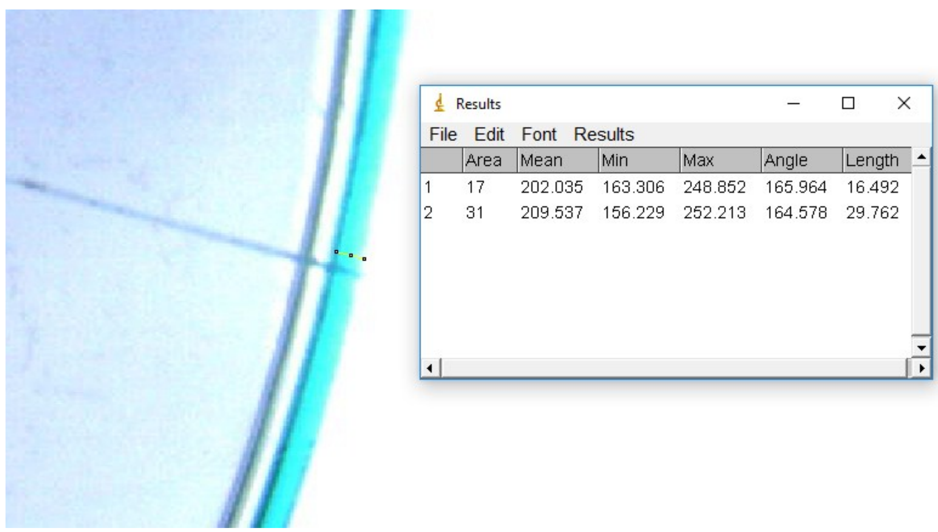

Figure 6: Measuring the stratified flow thickness for the experiments by ImageJ software.