Supplementary Information

for

Thermal Transport through Fishbone Silicon Nanoribbons: Unraveling the Role of Sharvin Resistance

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I. Correction factor calculated based on different approaches

We note that different approaches of defining the correction factor could be introduced while extracting the thermal conductivity of nanostructures with complex morphologies. Fig. S1 shows a comparison of the correction factor calculated based on equivalent volume, equivalent thermal resistance (the strategy adopted by Nomura et al.),^{1,2} and ANSYS simulation.



Figure S1. Comparison of the calculated correction factor based on equivalent volume, equivalent thermal resistance (the strategy adopted by Nomura et al.),^{1,2} and ANSYS simulation.

II. Effects of lateral size variation on phonon transport in nanoribbons with constant thickness

As we recently demonstrated, the measured thermal conductivity of Si nanoribbons shows a systematic increasing trend with ribbon width even when the limiting dimension (thickness, ~20 and 30 nm) is much smaller than the ribbon width and kept as a constant.³ This is later verified by Park et al. through varying the aspect ratio of the Si nanobeams with a constant thickness of ~78 nm, and their results show that the measured thermal conductivity (κ) of the nanobeam approaches the thin film value for an aspect ratio larger than 15.⁴ Therefore, even though phonon scattering at the top and bottom surfaces are highly diffusive, as the lateral dimension increases, phonons would still have a larger effective mean free path (mfp) owing to the boundary scattering from the two sides.

To provide a more quantitative picture of the effective phonon mfp enhancement as the lateral dimension expands, we calculate the reduction function F due to phonon-boundary scattering, which allows us to extract the nanoribbon phonon mfp as $\Lambda_{ribbon} = F \Lambda_{bulk}$. The Fuchs–Sondheimer reduction function⁵ developed by Chambers based on the kinetic theory is expressed as $F(w,t,\Lambda) = 1 - \sigma \langle w,t,\Lambda \rangle - \sigma \langle t,w,\Lambda \rangle$,⁶ where σ is given by

$$\sigma \langle w, t, \Lambda \rangle = \left(\frac{6}{4\pi \cdot wt}\right) \cdot \int_{0}^{w} dx \int_{0}^{t} dy \int_{-\arctan\left(\frac{y}{w-x}\right)}^{\arctan\left(\frac{t-y}{w-x}\right)} d\varphi \int_{0}^{\pi} \sin\theta \cos^{2}\theta \exp\left(\frac{x-w}{\Lambda \sin\theta \cos\varphi}\right) d\theta \, \text{.} \text{* MERGEFORMAT (1)}$$

Here w and t are the width and height of the cross-section, respectively; and Λ is the carrier mfp in bulk media, which is taken as 300 nm at room temperature for phonons in silicon.⁷ The integration is over all directions of azimuthal angle θ , radial angle φ , and from all locations in the cross-section. Note that in the derivation of the above equation, the specularity parameter at the free surface of the ribbon is set to 0, which dictates diffusive boundary scattering.



Figure S2. Calculated reduction function ratio of nanoribbons to thin films as a function of ribbon width.

We show the effect of phonon scattering at the two side surfaces using the reduction function ratio, i.e., F_{ribbon}/F_{film} , which is plotted in Fig. S2 as a function of ribbon width with a constant thickness of 34 nm. A continuous increasing trend could be observed as the ribbon width expands from 120 nm to 430 nm; and as a result, for fishbone ribbons with larger fin width, the effective phonon mfp is longer for phonons arriving at the constriction, which boosts the ballistic constriction resistance, i.e., Sharvin resistance, and leads to a lower thermal conductivity.

References:

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