Supplementary Materials

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Metropolis algorithm simulations with parallel annealing

Each nanomagnet is modelled as an infinitesimally thin compass needle with a uniform, linear magnetic moment density mL. The moment orients along the long axis of the island. The compass needles are exposed to a simulated external field in field cooled (FC) experiments, giving them an intrinsic energy of $\mathbf{B} \cdot \mathbf{m}$. The external field $\mathbf{B}$ is arbitrarily set in the x direction. Throughout the FC exeperiment, the field is reduced to zero from a maximum value of $B = 7.54 \times 10^{-6}$ T as the system evolves. In a parallel case, the systems are cooled without field entirely, referred to as the zero field cooled (ZFC) cases.

The interaction between compass needles are equivalent to two equal and opposite magnetic

charges with charges m/L placed at their ends [1, 2]. Their energy obeys Coulomb's law:

$$E_{ij} = \frac{\mu_0 m^2}{4\pi L^2}\Big[\frac{1}{|r_{ai} - r_{aj}|} - \frac{1}{|r_{ai} - r_{bj}|} - \frac{1}{|r_{bi} - r_{aj}|} + \frac{1}{|r_{bi} - r_{bj}|}\Big] \tag{1}$$

where $\mathbf{r}_{ai}$ and $\mathbf{r}_{bi}$ are the locations of the positive and negative magnetic charge on the ith nanomagnet, μ_0 is the magnetic permeability, $L = 400$ nm is the island length, and $m = MV$ is the magnetic moment of each nanomagnet with M being the saturation magnetization and V the nanomagnet volume. The magnetization was chosen to be $M = 362$ kA m^{-1} in agreement with previous studies on nanoislands fabricated in the same manner [2, 3]. Cayley trees of 5 layers (63 nanomagnets) and 6 layers (127 nanomagnets) were simulated over 3000 and 1000 replicas respectively. All dipolar interactions were accounted for.

We use the metropolis Monte Carlo method annealed with the parallel tempering method [4, 5] to bring the systems to equilibrium across a temperature range demonstrating their low energy behavior. First, 200 randomized spin configurations were decided at an evenly separated set of temperatures between 0 and 210K. Each configuration was then evolved with $5N$ potential spin updates. For each update this method chooses a random spin, calculates the energy difference $(\Delta E)_i$ in the system when that spin is flipped, and flips the spin with a probability of

$$P_{flip}(i) = \min\Big\{1, \exp\Big[-\frac{(\Delta E)_i}{k_B T}\Big]\Big\}, \tag{2}$$

where $k_B = 8.62 \times 10^{-5}$ eV K^{-1} is the Boltzmann constant and T is the temperature. The spin configurations that had been updated in parallel were then compared and potentially exchanged via the parallel tempering method. The probability of a pair of systems at adjacent temperatures, T_n and T_m, swapping is

$$P_{swap}(n, m) = \min\Big\{1, \exp\Big[-(\frac{1}{k_B T_m} - \frac{1}{k_B T_n})(E_n - E_m)\Big]\Big\}, \tag{3}$$

where E_n is the energy of configuration n. The Monte Carlo sweeps then begin again, repeating the process until 25N spin flips were attempted.

The average magnetic moment of the resulting spin configurations was then calculated for the FC and ZFC systems:

$$\bar{m} = \frac{1}{N} |\sum_i \mathbf{m}_i|. \tag{4}$$

These moments are then averaged over the system replicas and plotted as a function of temperature in Fig. S1. Error bars are defined by the standard deviation of the mean. The temperature at which these two curves diverge by more than this error is designated as T_c, the critical temperature at which glassy dynamics may emerge.

References

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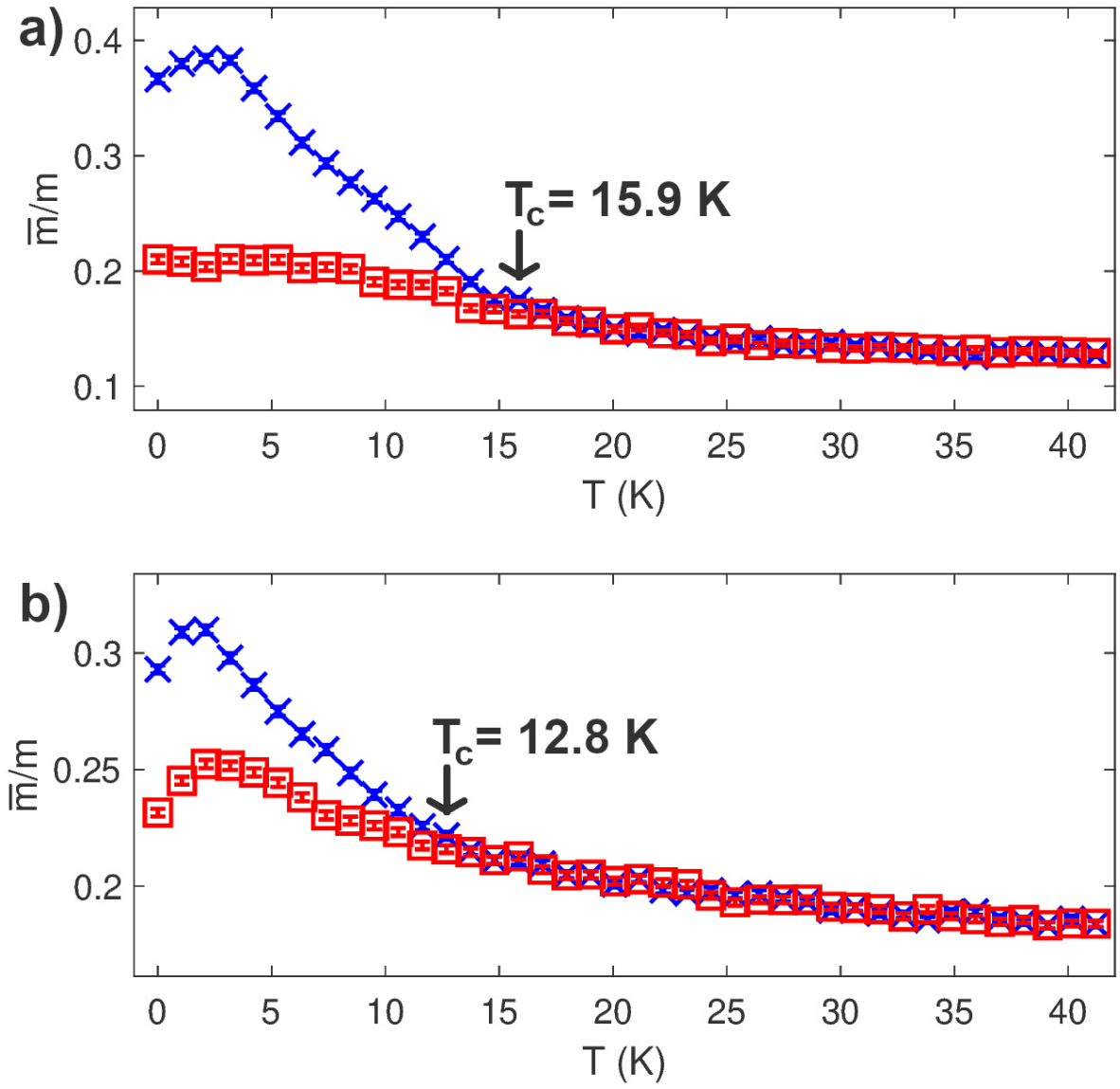

Figure S1. Mean magnetic moment of Cayley trees with a) $N = 127$ spins ($T_c = 15.9$ K) and b) $N = 63$ spins ($T_c = 12.8$ K). Blue x's represent FC systems while red squares represent ZFC systems. The y scale is set by the saturation magnetic moment of a single nano-magnet, m. Critical temperatures are determined by the points at which the ZFC and FC curves do no longer have overlapping error bars.